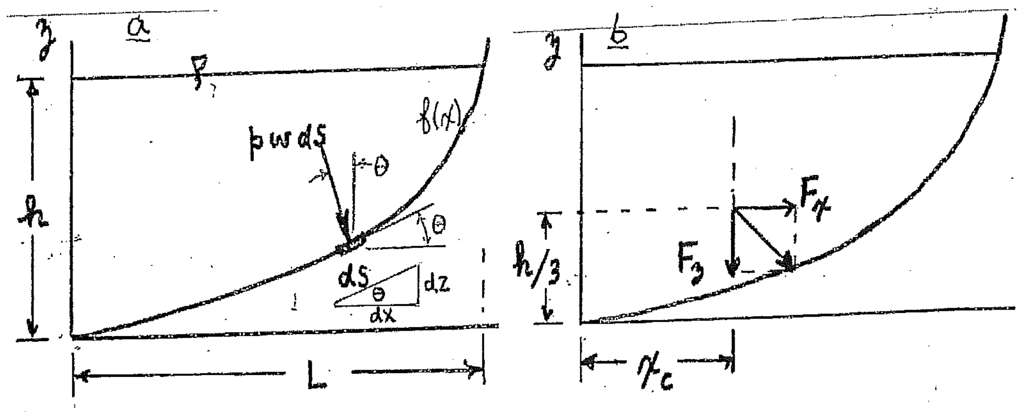
Forces on Curved Surfaces



Horizontal Force F_x

$$dA = wds \qquad dF = pwds$$

$$dF_x = pwds\sin\theta = pwdz \qquad dz = ds\sin\theta$$

$$p = \gamma(h-z)$$

$$F_x = \int_0^h pwdz = \int_0^h \gamma(h-z)wdz = \gamma\left(hz - \frac{z^2}{2}\right)_0^h w = \gamma\frac{h^2}{2}w = \gamma\frac{h}{2}(hw)$$

$$p_c = \gamma h/2 \qquad A_x = hw$$

$F_x = p_c A_x$ Acts at $z_c = h/3$

A_x = hw = vertically projected surface

F_x = force on vertically projected surface

Vertical force F_z

$$dF_z = -pwds\cos\theta = -pwdx \qquad dx = ds\cos\theta$$

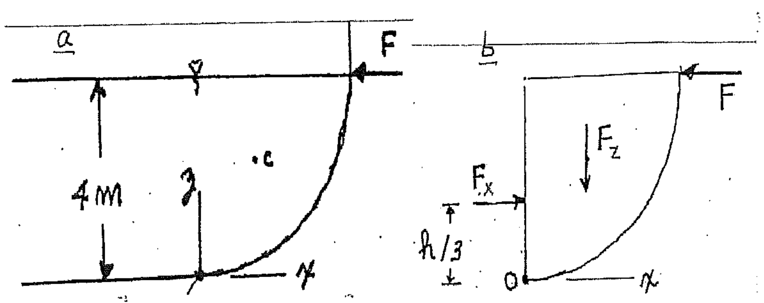
$$p = \gamma(h - z)$$

$$F_z = -\int_0^L pwdx = -\int_0^L \gamma(h-z)wdx = -\int_0^L \gamma d\forall = -\gamma\forall$$
$$= \text{weight of the liquid above the surface}$$

$$(M_y)_x = -\int_0^L xpwdx = -\int_0^L x\gamma(h-z)dx = -\int_0^L x\gamma d\forall = -x_c\gamma\forall$$

$$x_{cp} = (M_y)_x / F_z = -x_c\gamma\forall/(-\gamma\forall) = x_c$$

Example: Find the force F to hold the quarter-circular door



$$x_c = 0.424R$$
$$w = 3m$$

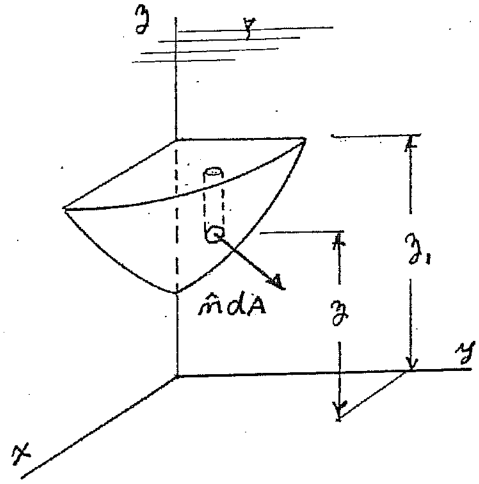
$$\sum M_0 = hF - \frac{h}{3}F_x - x_c|F_z| \qquad F = F_x/3 + x_c|F_z|/h$$

$$F_x = p_c A = \gamma\frac{h}{2}A = 9800\frac{4}{2}(4*3) = 235{,}000N$$

$$F_z = \gamma\forall = \gamma\frac{\pi R^2}{4}w = 9800\frac{\pi 4^2}{4}3 = 369{,}000N$$

$$F = F_x/3 + x_c|F_z|/h$$
=235000/3+0.424(369000)/4=117,000N

Force on downward facing surface



The level z_1 is at the top of the surface for which the force is to be found.

$$d\vec{F} = -pdA\hat{n}$$
$$dF_x = d\vec{F}\cdot\hat{i} = -p(\hat{n}dA)\cdot\hat{i} = -pdA_x = -pdydz$$
$$dF_y = d\vec{F}\cdot\hat{j} = -p(\hat{n}dA)\cdot\hat{j} = -pdA_y = -pdxdz$$
$$dF_z = d\vec{F}\cdot\hat{k} = -p(\hat{n}dA)\cdot\hat{k} = -pdA_z = pdxdy$$
$$p = p_1 + \gamma(z_1 - z)$$

$$F_x = -\int pdA_x = -\int[p_1 + \gamma(z_1 - z)]dA_x = -p_1A_x - \gamma(z_1 - z_{cx})A_x = -p_{cx}A_x$$

$$F_y = -\int pdA_y = -\int[p_1 + \gamma(z_1 - z)]dA_y = -p_1A_y - \gamma(z_1 - z_{cy})A_y = -p_{cy}A_y$$

$$F_z = -\int pdA_z = \int[p_1 + \gamma(z_1 - z)]dA_z = p_1A_z + \gamma\forall'$$

A_x, A_y, A_z = projected areas on y,z; x,z and x,z planes

Centroidal pressures

$$p_{cx} = p_1 + \gamma(z_1 - z_{cx})$$
$$p_{cy} = p_1 + \gamma(z_1 - z_{cy})$$

Displaced volume $\int(z_1 - z)dA_z = \forall'$

Example: A 1m diameter sphere is submerged with its center at d=10m depth. Determine the force on the top, bottom and side hemispheres.

Force on the top hemisphere = weight of the water above the sphere
Volume =cylinder minus a hemispherical volume

$$F_{top} = -\gamma \forall = -\gamma [\pi R^2 d - \frac{1}{2}\frac{4}{3}\pi R^3]$$

$$= -9800[\pi (0.5)^2 (10) - \frac{1}{2}\frac{4}{3}\pi (0.5)^3] = -74{,}400N$$

Force on the bottom hemisphere

$$F_{bottom} = \gamma d A_z + \gamma \forall' = \gamma \left[d\pi R^2 + \frac{1}{2}\frac{4}{3}\pi R^3 \right]$$

$$= 9800\left[10\pi (0.5)^2 + \frac{1}{2}\frac{4}{3}\pi (0.5)^3 \right] = 79{,}530N$$

Net upward force

$$F_{bottom} + F_{top} = 79530 - 74400 = 5130N$$

Force on the left hemisphere

$$F_x = p_c A_x = \gamma d \pi R^2 = 9800(10)\pi (0.5)^2 = 76{,}970N$$

Force on the right hemisphere equal, but opposite in direction, to that on the right.