First Law of Thermodyamics

$$dE = dQ + dW$$

For a moving system: $\dfrac{DE}{Dt} = \dot{Q} + \dot{W}$

$\dot{Q} =$ heat transfer rate to the system
$\dot{W} =$ rate of work done on the system

$\varepsilon = e + V^2/2 + gz =$ energy per unit mass

$E = \int \varepsilon dm = \int \varepsilon \rho d\forall =$ energy in system

$B = \int \beta d\forall$ where $\beta =$ property per unit volume

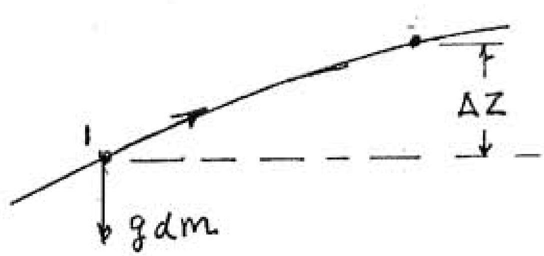
Then $B = E$ and $\beta = \rho\varepsilon = \rho(e + V^2/2 + gz)$.

Work: $dW = F_s ds$ for force F_s in the direction ds

$$dW = F_s ds = (F\cos\theta)ds = \vec{F}\cdot d\vec{s}$$

$$dW/dt = \dot{W} = \vec{F}\cdot d\vec{s}/dt = \vec{F}\cdot\vec{V}$$

Gravity: $W_{1-2} = -gdm(z_2 - z_1) = (potential)_1 - (potential)_2$



Potential energy $= gzdm$ and $gz =$ potential per unit mass

Work:

Pressure: $d\vec{F}_p = -pdA\hat{n}$

$$d\dot{W} = d\vec{F}_p \cdot \vec{V} = -p\vec{V} \cdot \hat{n}dA$$

For the whole system surface $\dot{W}_p = -\int p\vec{V} \cdot \hat{n}dA$

Shear stress: $d\vec{F}_f = \vec{\tau}dA$ tangent to the surface element dA

$$d\dot{W}_\tau = d\vec{F}_\tau \cdot \vec{V} = (\vec{\tau}dA) \cdot \vec{V} \quad \text{and} \quad \dot{W}_\tau = \int \vec{\tau} \cdot \vec{V}dA$$

$\dot{W}_\tau = 0$: $\vec{V} = 0$ for a solid surface

$\vec{\tau} \cdot \vec{V} = 0$ for $\vec{\tau} \perp \vec{V}$ or $\vec{V} \perp CS$

Neglect friction work. There can be friction work in the flow, but not on the CS.

Shaft Work: $\dot{W}_s$ = work done by a shaft or arm sticking through the control surface and delivering work to the CV.

Reynolds transport theorem: $\dfrac{DB}{Dt} = \dfrac{\partial}{\partial t} \int_{CV} \beta d\forall + \int_{CS} \beta\vec{V} \cdot \hat{n}dA$

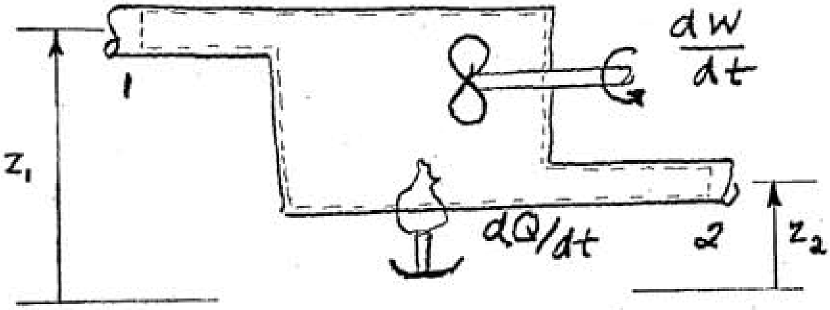
$$\dot{Q} + \dot{W} = \frac{DE}{Dt} = \frac{\partial}{\partial t} \int_{CV} \varepsilon\rho d\forall + \int_{CS} \varepsilon\rho\vec{V} \cdot \hat{n}dA$$

$$\dot{W} = \dot{W}_s - \int p\vec{V} \cdot \hat{n}dA = \dot{W}_s - \int (p/\rho)\rho\vec{V} \cdot \hat{n}dA$$

$$\dot{Q} + \dot{W}_s = \frac{\partial}{\partial t} \int_{CV} \varepsilon\rho d\forall + \int_{CS} (\varepsilon + \frac{p}{\rho})\rho\vec{V} \cdot \hat{n}dA$$

where $\varepsilon = e + V^2/2 + gz$

Steady flow through a devise with one entrance and on exit



$$\dot{Q}+\dot{W}_s = \int_{A_2} (e+\frac{V^2}{2}+gz+\frac{p}{\rho})\rho u dA - \int_{A_1} (e+\frac{V^2}{2}+gz+\frac{p}{\rho})\rho u dA$$

Profile factor: $\lambda = (\int u^3 dA)/\bar{u}^3 A$ where $\bar{u} = (\int u dA)/A$

Gravity term: $\int zudA = z_c \bar{u} A$ where z_c distance to centroid

$$\dot{Q}+\dot{W}_s = (e_2+\lambda_2\frac{\bar{u}_2^2}{2}+gz_{c2}+\frac{p_2}{\rho_2})\rho_2\bar{u}_2 A_2$$
$$-(e_1+\lambda_1\frac{\bar{u}_1^2}{2}+gz_{c1}+\frac{p_1}{\rho_1})\rho_1\bar{u}_1 A_1$$

For *gases* neglect the gravity term
Uniform properties and velocity over the cross section

$$\dot{Q}+\dot{W}_s = \dot{m}[(h_2+V_2^2/2)-(h_1+V_1^2/2)]$$

$h = e + p/\rho$ = enthalpy, and $\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$

Incompressible Flow

$\dot{Q} = 0$: No forced heat transfer
Little frictional effects
Internal energy e = constant

One dimensional flow at entrance and exit

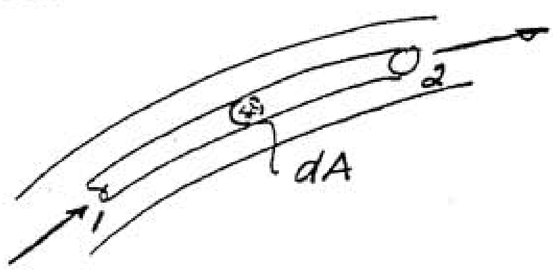
$$\dot{W}_s = \dot{m}[(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2) - (\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1)]$$

Dimensions: $\dot{W}_s \sim N \cdot m/s \sim watt$

$$\frac{dW_s}{dm} = (\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2) - (\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1)$$

where $dW_s / dm \sim (N \cdot m/s)/(kg/s) \sim N \cdot m/kg$

Flow in a Stream Tube

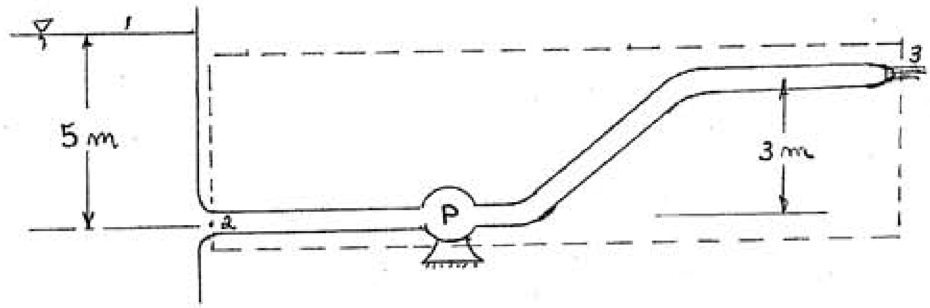


With no shaft work we obtain Bernoulli's equation

$$(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2) = (\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1)$$

The sum of the pressure potential, kinetic energy and gravitational potential is constant along a flow stream for frictionless flow.

Example: Determine the Power from the Pump



Flow rate of water $\dot{q} = 0.095 m^3 / s$.
Diameter of pipe = $d_2 = 10cm$
Diameter of nozzle exit = $d_3 = 6cm$ and $p_3 = p_a$

$$\dot{W}_s = \dot{m}[(\frac{p_3}{\rho} + \frac{V_3^2}{2} + gz_3) - (\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2)]$$

$$= \dot{m}[-\frac{p_2 - p_a}{\rho} + \frac{V_3^2 - V_2^2}{2} + g(z_3 - z_2)]$$

$$V_2 = \dot{q} / A_2 = 12.1 m/s \quad \text{and} \quad V_3 = \dot{q} / A_3 = 33.6 m/s$$

Pressure p_2 determined from Bernoulli's equation

$$(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2) = (\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1)$$

p_1 is atmospheric and $V_1^2 << V_2^2$

$$p_2 - p_a = \rho[-V_2^2 / 2 + g(z_1 - z_2)] = 998[-12.1^2 / 2 + 9.8(5)] = -24{,}060 Pa$$

$$\dot{W}_s = (998)(0.095)[-\frac{-24060}{998} + \frac{(33.6)^2 - (12.1)^2}{2} + (9.82)(3)] = 105.2 kw$$

Solution option: Use Bernoulli equation to eliminate $p_2 - p_a$

$$\dot{W}_s = \dot{m}[(\frac{p_3}{\rho} + \frac{V_3^2}{2} + gz_3) - (\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1)]$$