Shock wave

A shock wave (stationary or moving) is a wave in supersonic flow where flow properties change very rapidly. Inside the wave viscous and heat transfer effects are important. However, a shock wave is very thin and, for practical purposes, can simply be treated as a discontinuity.

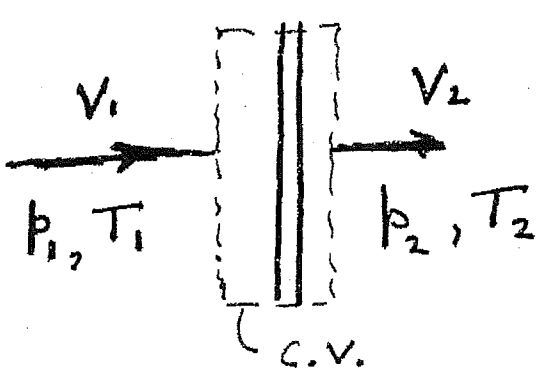
Shock waves can be normal, perpendicular to the flow direction, or oblique, inclined to the flow direction.

Shock waves are compressive – density, pressure and temperature increase across the shock.

Prandtl-Meyer waves are expansive, occur over longer distances and are isentropic.

Shock waves occur because the flow is supersonic and disturbances cannot be communicated upstream. A supersonic flow moves along without knowing that it has to adjust to an object in the flow, an increasing pressure field, or a wall turning into the flow. Finally adjustments takes place to the conditions and the changes occur very rapidly via a shock.

Normal shock



Continuity:

$$\rho_1 V_1 A = \rho_2 V_2 A$$

Momentum:

$$p_1 A - p_2 A = \rho_2 V_2^2 A - \rho_1 V_1^2 A$$

Energy:

$$h_1 + V_1^2/2 = h_2 + V_2^2/2$$

Shock wave equations

Energy: $h_1 + V_1^2/2 = h_2 + V_2^2/2$ $\qquad h_{0_1} = h_{0_2}$

$$c_p T_1 + V_1^2/2 = c_2 T_2 + V_2^2/2 \qquad c_p = \gamma R/(\gamma - 1)$$

$$T_1\left(1 + \frac{\gamma - 1}{2}\frac{V_1^2}{\gamma R T_1}\right) = T_2\left(1 + \frac{\gamma - 1}{2}\frac{V_2^2}{\gamma R T_2}\right) \qquad a = \sqrt{\gamma R T}$$

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2}M_1^2}{1 + \frac{\gamma - 1}{2}M_2^2} \qquad T_{0_1} = T_{0_2}$$

Momentum: $p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$

$$p_1\left(1 + \frac{\gamma V_1^2}{\gamma p_1/\rho_1}\right) = p_2\left(1 + \frac{\gamma V_2^2}{\gamma p_2/\rho_2}\right) \qquad a = \sqrt{\gamma p/\rho}$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

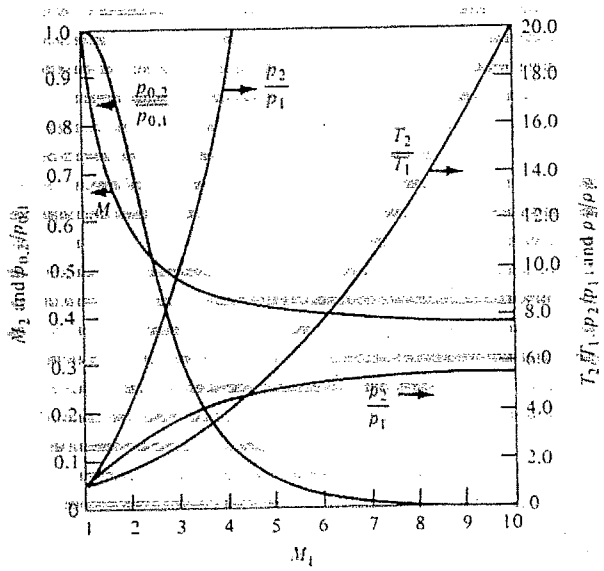
Continuity: $\rho_1 V_1 = \rho_2 V_2 \quad \rightarrow \frac{p_1}{RT_1}V_1 = \frac{p_2}{RT_2}V_2 \qquad p = \rho R T$

$$\frac{\gamma p_1}{\sqrt{\gamma R T_1}}\frac{V_1}{a_1} = \frac{\gamma p_2}{\sqrt{\gamma R T_2}}\frac{V_2}{a_2} \qquad a = \sqrt{\gamma R T}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{T_2}{T_1}}\frac{M_1}{M_2}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{T_2}{T_1}}\frac{M_1}{M_2}$$

$$\frac{1+\gamma M_1^2}{1+\gamma M_2^2} = \sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}\frac{M_1}{M_2} \qquad M_2^2 = \frac{2+(\gamma-1)M_1^2}{2\gamma M_1^2-(\gamma-1)}$$



$$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma-1}(M_1^2-1)$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

$$\frac{s_2-s_1}{R} = \ln\frac{(T_2/T_1)^{\gamma/(\gamma-1)}}{p_2/p_1} = \ln\frac{(T_{02}/T_{01})^{\gamma/(\gamma-1)}}{p_{02}/p_{01}} = -\ln\frac{p_{02}}{p_{01}}$$

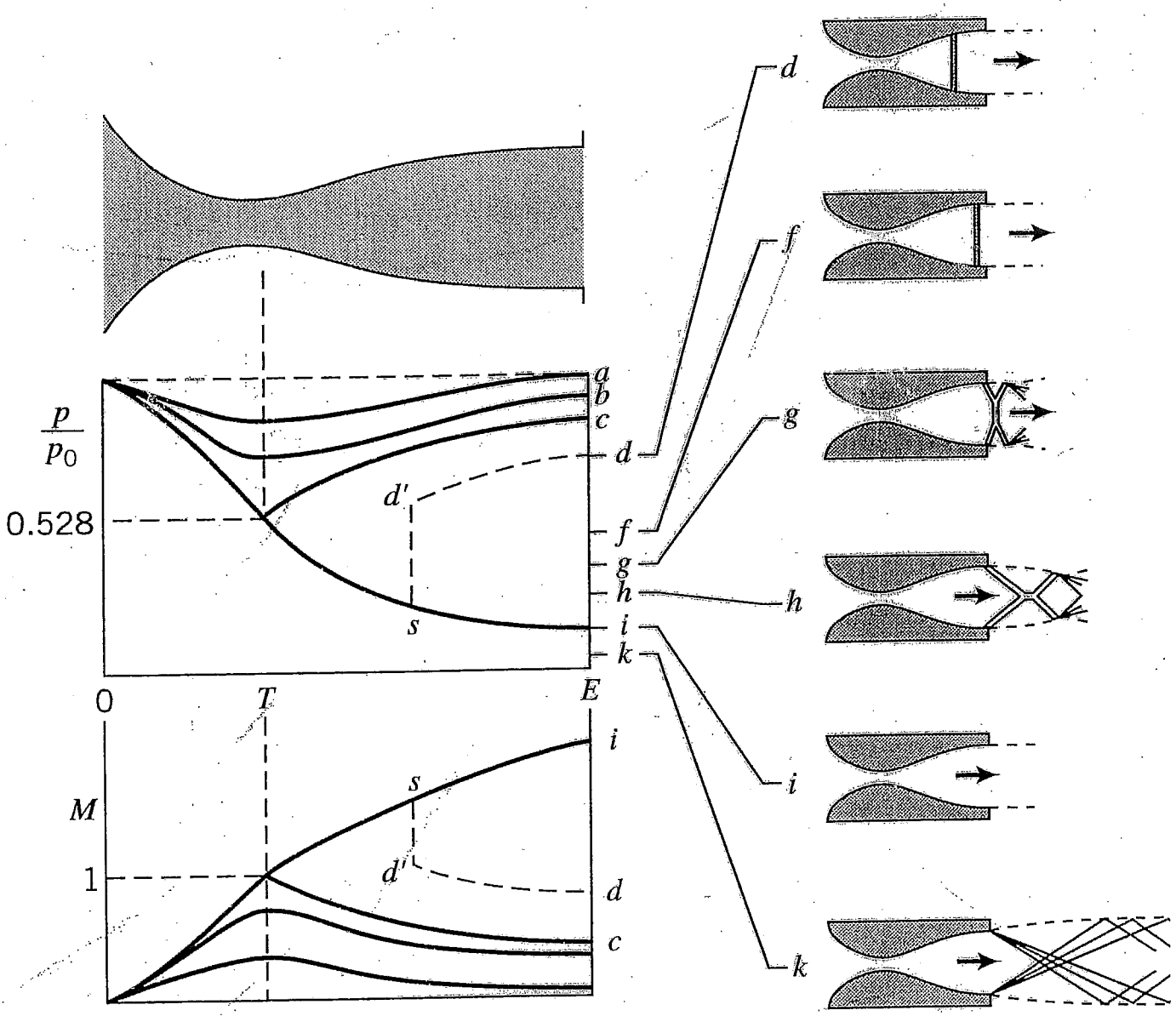
p_{02}/p_{01} is a measure of the loss through a shock

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2}\frac{p_2}{p_1}\frac{p_1}{p_{01}} = f(M_1)$$

Pitot tube pressure p_{02} in a flow with $M_1 = 2$ and $p_1 = 20kPa$.
From isentropic table $p_1/p_{01} = 0.128$ for $M_1 = 2$.
From the shock table $M_2 = 0.577$ *and* $p_{02}/p_{01} = 0.721$ for M_1

$$p_{01} = p_1/.128 = 20/.128 = 156kPa$$
$$p_{02} = .721p_{01} = 112kPa$$

Operation of a converging-diverging nozzle



a,b) p_b slightly lower than p_0 and $p_e = p_b$. Subsonic flow through the nozzle.

c) p_b to just cause sonic flow at the throat, $p_e = p_b$.

d) p_b to cause normal shock in the divergent section, $p_e = p_b$.

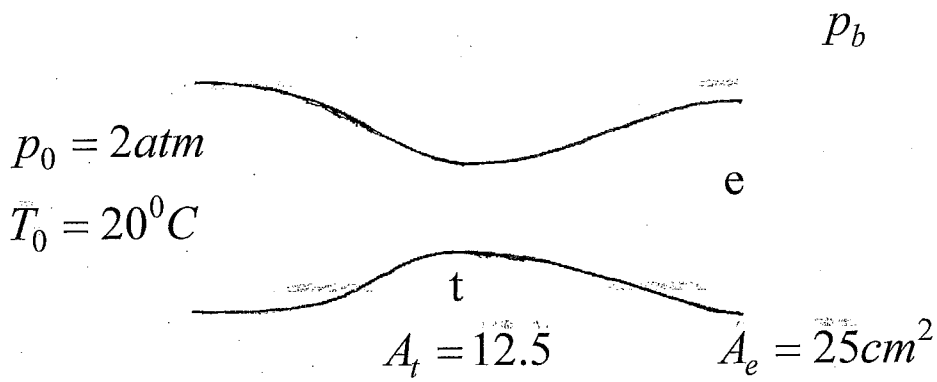
f) p_b to cause normal shock at exit plane, $p_{e2} = p_b$.

g,h) $p_e < p_b$, shocks occur in the jet

i) p_b to cause smooth supersonic flow in nozzle and jet

k) $p_e > p_b$, expansion waves occur in the jet

Nozzle with shocks



a) Find the back pressure p_b to cause a normal shock at the exit plane. The flow before the exit plane would be isentropic and supersonic.

Isentropic table:

$$\frac{A_{e1}}{A^*} = \frac{25}{12.5} = 2 \quad \rightarrow M_{e1} = 2.2, \frac{p_{e1}}{p_{01}} = 0.0935$$

$$p_{e1} = .0935 p_{01} = .0935(2) = 0.187atm$$

Shock table:

$$M_{e1} = 2.2 \quad \rightarrow M_{e2} = 0.547, \frac{p_{e2}}{p_{e1}} = 5.48$$

$$p_b = p_{e2} = 5.48 p_{e1} = 5.48(.187) = 1.025atm$$

b) Find the back pressure to cause a shock at $A' = 18.75cm^2$.

Isentropic table: $\frac{A_1^{'}}{A_1^*} = \frac{A_1^{'}}{A_t} = \frac{18.75}{12.5} = 1.5 \quad \rightarrow M_1^{'} = 1.86$

Shock table: $M_1^{'} = 1.86 \quad \rightarrow M_2^{'} = .604, p_{02} / p_{01} = .786$

Isentropic table: $M_2^{'} = .604 \quad \rightarrow A_2^{'} / A_2^* = 1.19$

$$A_2^* = A_2^1 / 1.19 = 18.75 / 1.19 = 15.8$$

Isentropic table: $A_e / A_2^* = \frac{25}{15.8} = 1.58 \rightarrow M_e = .40, \frac{p_e}{p_{02}} = .896$

$$p_{02} = .786 p_{01} = 1.57 \qquad p_b = p_e = .896 p_{o2} = 1.41atm$$

Table B.6 One-dimensional normal-shock relations†

M_1	M_2	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{(p_0)_2}{(p_0)_1}$	$\frac{(A^*)_2}{(A^*)_1}$	M_1	M_2	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{(p_0)_2}{(p_0)_1}$	$\frac{(A^*)_2}{(A^*)_1}$
1.00	1.000	1.000	1.000	1.000	1.000	1.80	0.617	3.613	1.532	0.813	1.228
1.02	0.980	1.047	1.013	1.000	1.000	1.82	0.612	3.698	1.547	0.804	1.239
1.04	0.962	1.095	1.026	1.000	1.000	1.84	0.608	3.783	1.562	0.795	1.252
1.06	0.944	1.144	1.039	1.000	1.000	1.86	0.604	3.869	1.577	0.786	1.273
1.08	0.928	1.194	1.052	0.999	1.000	1.88	0.600	3.957	1.592	0.777	1.286
1.10	0.912	1.245	1.065	0.999	1.000	1.90	0.596	4.045	1.608	0.767	1.307
1.12	0.896	1.297	1.078	0.998	1.000	1.92	0.592	4.134	1.624	0.758	1.319
1.14	0.882	1.350	1.090	0.997	1.010	1.94	0.588	4.224	1.639	0.749	1.339
1.16	0.868	1.403	1.103	0.996	1.000	1.96	0.584	4.315	1.655	0.740	1.352
1.18	0.855	1.458	1.115	0.995	1.000	1.98	0.581	4.407	1.671	0.730	1.373
1.20	0.842	1.513	1.128	0.993	1.010	2.00	0.577	4.500	1.688	0.721	1.391
1.22	0.830	1.570	1.140	0.991	1.015	2.02	0.574	4.594	1.704	0.711	1.411
1.24	0.818	1.627	1.153	0.988	1.010	2.04	0.571	4.689	1.720	0.702	1.430
1.26	0.807	1.686	1.166	0.986	1.013	2.06	0.567	4.784	1.737	0.693	1.447
1.28	0.796	1.745	1.178	0.983	1.017	2.08	0.564	4.881	1.754	0.683	1.467
1.30	0.786	1.805	1.191	0.979	1.022	2.10	0.561	4.978	1.770	0.674	1.485
1.32	0.776	1.866	1.204	0.976	1.027	2.12	0.558	5.077	1.787	0.665	1.504
1.34	0.766	1.928	1.216	0.972	1.022	2.14	0.555	5.176	1.805	0.656	1.522
1.36	0.757	1.991	1.229	0.968	1.026	2.16	0.553	5.277	1.822	0.646	1.551
1.38	0.748	2.055	1.242	0.963	1.032	2.18	0.550	5.378	1.839	0.637	1.570
1.40	0.740	2.120	1.255	0.958	1.037	2.20	0.547	5.480	1.857	0.628	1.667
1.42	0.731	2.186	1.268	0.953	1.051	2.22	0.544	5.583	1.875	0.619	1.614
1.44	0.723	2.253	1.281	0.948	1.057	2.24	0.542	5.687	1.892	0.610	1.642
1.46	0.716	2.320	1.294	0.942	1.063	2.26	0.539	5.792	1.910	0.601	1.667
1.48	0.708	2.389	1.307	0.936	1.068	2.28	0.537	5.898	1.929	0.592	1.686
1.50	0.701	2.458	1.320	0.930	1.083	2.30	0.534	6.005	1.947	0.583	1.712
1.52	0.694	2.529	1.334	0.923	1.077	2.32	0.532	6.113	1.965	0.575	1.739
1.54	0.687	2.600	1.347	0.917	1.081	2.34	0.530	6.222	1.984	0.566	1.767
1.56	0.681	2.673	1.361	0.910	1.090	2.36	0.527	6.331	2.003	0.557	1.798
1.58	0.675	2.746	1.374	0.903	1.095	2.38	0.525	6.442	2.021	0.549	1.825
1.60	0.668	2.820	1.388	0.895	1.110	2.40	0.523	6.553	2.040	0.540	1.852
1.62	0.663	2.895	1.402	0.888	1.125	2.42	0.521	6.666	2.060	0.532	1.886
1.64	0.657	2.971	1.416	0.880	1.128	2.44	0.519	6.779	2.079	0.523	1.912
1.66	0.651	3.048	1.430	0.872	1.136	2.46	0.517	6.894	2.098	0.515	1.945
1.68	0.646	3.126	1.444	0.864	1.147	2.48	0.515	7.009	2.118	0.507	1.977
1.70	0.641	3.205	1.458	0.856	1.156	2.50	0.513	7.125	2.138	0.499	2.009
1.72	0.635	3.285	1.473	0.847	1.169	2.52	0.511	7.242	2.157	0.491	2.041
1.74	0.631	3.366	1.487	0.839	1.185	2.54	0.509	7.360	2.177	0.483	2.073
1.76	0.626	3.447	1.502	0.830	1.20	2.56	0.507	7.479	2.198	0.475	2.104
1.78	0.621	3.530	1.517	0.821	1.214	2.58	0.506	7.599	2.218	0.468	2.139

Table B.6 *Continued*

M_1	M_2	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{(p_0)_2}{(p_0)_1}$	$\frac{(A^*)_2}{(A^*)_1}$	M_1	M_2	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{(p_0)_2}{(p_0)_1}$	$\frac{(A^*)_2}{(A^*)_1}$
2.60	0.504	7.720	2.238	0.460	2.177	2.84	0.485	9.243	2.496	0.376	2.657
2.62	0.502	7.842	2.260	0.453	2.208	2.86	0.484	9.376	2.518	0.370	2.704
2.64	0.500	7.965	2.280	0.445	2.246	2.88	0.483	9.510	2.541	0.364	2.751
2.66	0.499	8.088	2.301	0.438	2.280	2.90	0.481	9.645	2.563	0.358	2.794
2.68	0.497	8.213	2.322	0.431	2.318	2.92	0.480	9.781	2.586	0.352	2.841
2.70	0.496	8.338	2.343	0.424	2.359	2.94	0.479	9.918	2.609	0.346	2.894
2.72	0.494	8.465	2.364	0.417	2.396	2.96	0.478	10.055	2.632	0.340	2.948
2.74	0.493	9.592	2.396	0.410	2.445	2.98	0.476	10.194	2.656	0.334	2.990
2.76	0.491	8.721	2.407	0.403	2.482	3.00	0.475	10.333	2.679	0.328	3.043
2.78	0.490	8.850	2.429	0.396	2.522						
2.80	0.488	8.980	2.451	0.389	2.566						
2.82	0.487	9.111	2.473	0.383	2.613						

†For a perfect gas with $k = 1.4$.