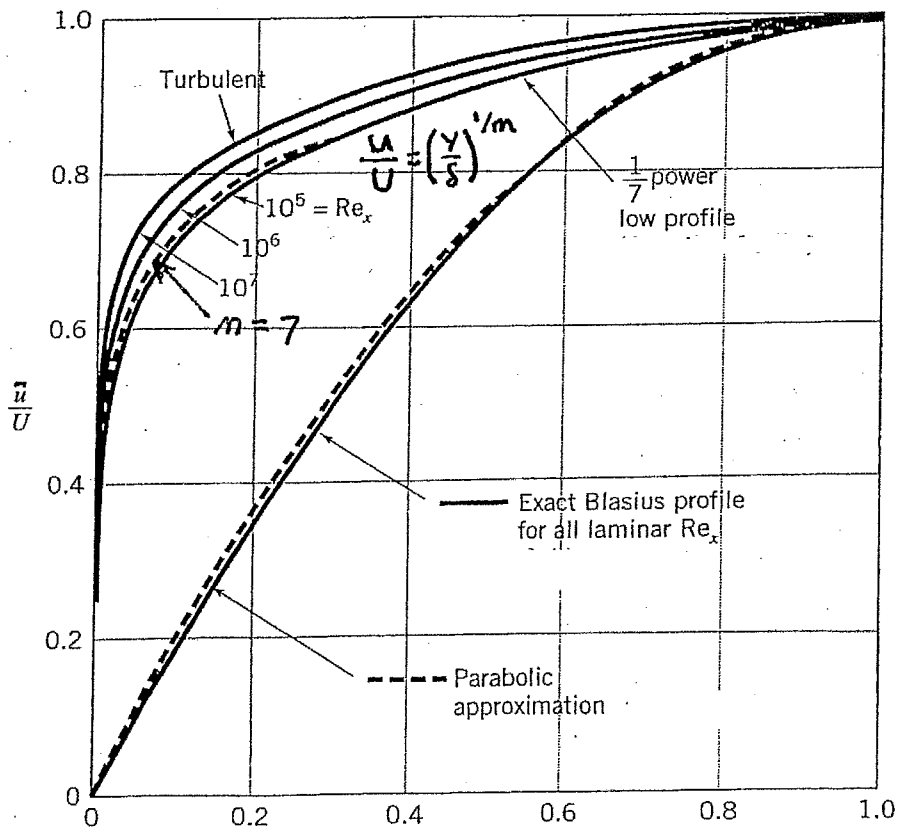
# Turbulent boundary layer




$$\frac{\bar{u}}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

for $\mathrm{Re}_x \approx 10^5$

$$\tau_w = \frac{d}{dx}\int_0^\delta \rho(U\bar{u} - \bar{u}^2)dy = \rho U^2 \frac{d}{dx}\int_0^\delta \left[\left(\frac{y}{\delta}\right)^{\frac{1}{7}} - \left(\frac{y}{\delta}\right)^{\frac{2}{7}}\right]dy = \frac{7}{72}\rho U^2 \frac{d\delta}{dx}$$

$$\tau_w = 0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} \quad \text{(empirical)} \qquad \frac{d\delta}{dx} = 0.231\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}$$

$$\delta = 0.37x/\mathrm{Re}_x^{1/5} \qquad c_f = 0.0576/\mathrm{Re}_x^{1/5} \qquad C_F = 0.074/\mathrm{Re}_L^{1/5}$$

# FLAT PLATE DRAG

$$C_D = C_F = \frac{F_N}{\frac{1}{2}\rho U_e^2 w L}$$

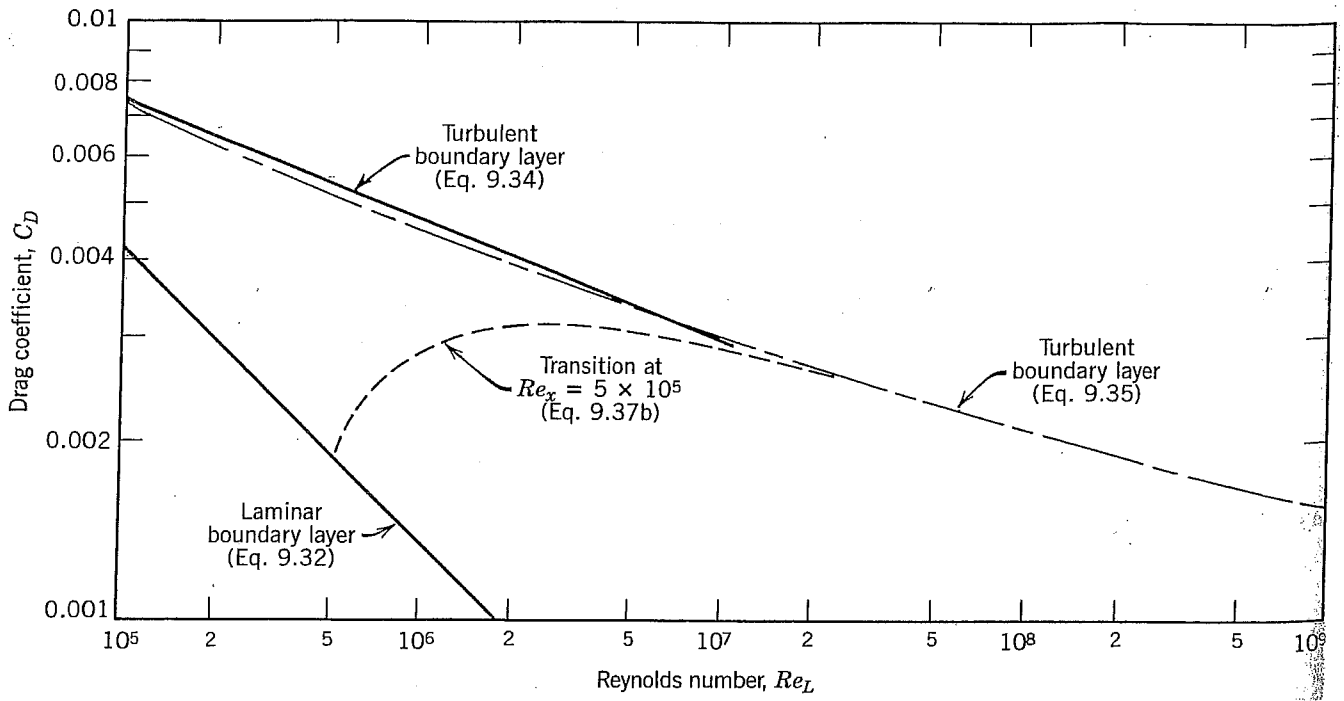
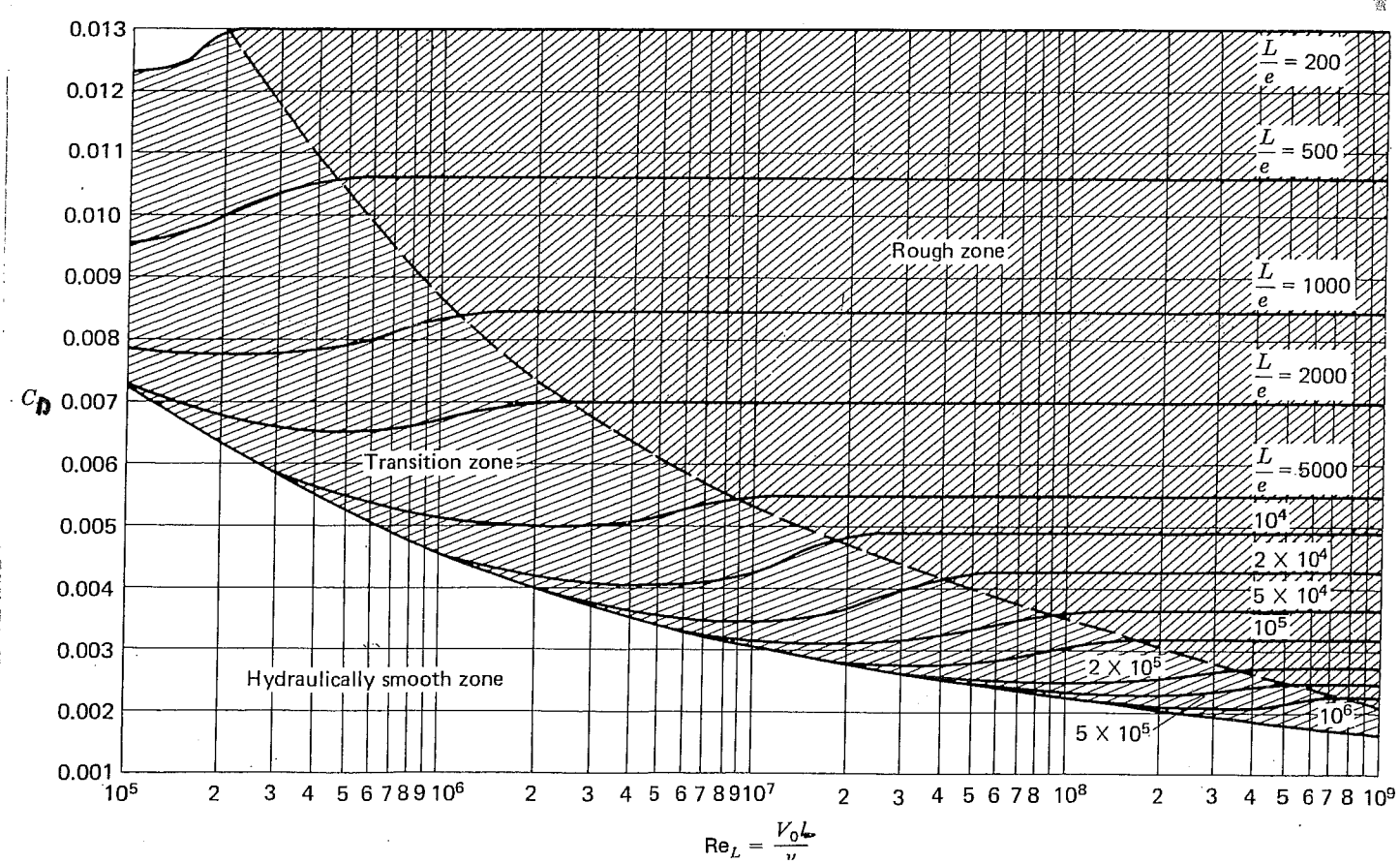



**Fig. 9.8** Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.



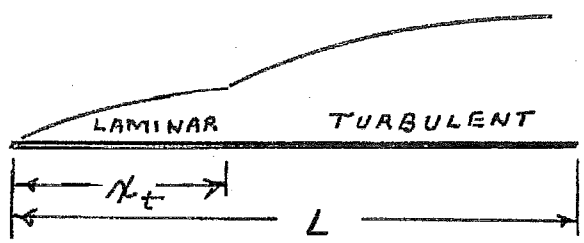
**Transition**

$$\mathrm{Re}_t \approx 5x10^5 \quad \rightarrow \quad x_t = 5x10^5 \frac{\nu}{U}$$

For U=10m/s

| | | |
|---|---|---|
| Water: | $\nu = 1.0x10^{-6}$ | $x_t = 0.05m$ |
| Air: | $\nu = 1.5x10^{-5}$ | $x_t = 0.75m$ |
| Oil: | $\nu = 2.0x10^{-4}$ | $x_t = 2.00m$ |

Drag with transition




Laminar: $c_f = 0.664/\mathrm{Re}_x^{1/2}$

Turbulent: $c_f = 0.0576/\mathrm{Re}_x^{1/5}$

Drag on one side $F_D = \int_0^L \tau_w w dx = \int_0^{x_t} \tau_{w_l} w dx = \int_{x_t}^L \tau_{w_t} w dx$

$$F_D = \frac{1}{2}\rho U^2 w \left[ .664 \int_0^{x_t} \left(\frac{\nu}{Ux}\right)^{\frac{1}{2}} dx + .0576 \int_{x_t}^{L} \left(\frac{\nu}{Ux}\right)^{\frac{1}{5}} dx \right]$$

$$F_D = \frac{1}{2}\rho U^2 w \left[ .664 \left(\frac{\nu}{U}\right)^{\frac{1}{2}} 2x_t^{1/2} + .0576 \left(\frac{\nu}{U}\right)^{\frac{1}{5}} \frac{5}{4}(L^{4/5} - x_t^{4/5}) \right]$$

$$C_F = \frac{F_D}{0.5\rho U^2 wL} = \frac{1.328}{\mathrm{Re}_L^{1/2}}\left(\frac{x_t}{L}\right)^{1/2} + \frac{.072}{\mathrm{Re}_L^{1/5}} - \frac{.072}{\mathrm{Re}_L^{1/5}}\left(\frac{x_t}{L}\right)^{1/5}$$

$$C_F = \frac{.072}{\mathrm{Re}_L^{1/5}} + \frac{1.328\mathrm{Re}_t^{1/2} - .072\mathrm{Re}_t^{1/5}}{\mathrm{Re}_L} = \frac{.072}{\mathrm{Re}_L^{1/5}} - \frac{938}{\mathrm{Re}_L}$$

**Example** A sail boat has a flat plate keel which is 1.5m deep and 1.0m long. Calculate the drag on the keel for the boat speed of a) 7.25m/s and b) 1.75m/s. The transition Reynolds number is $\mathrm{Re} = 5x10^5$.

a) $x_t = \frac{\nu}{U}\mathrm{Re}_t = \frac{10^{-6}}{7.25}5x10^5 = 0.067m$

$x_t << L \quad \rightarrow \quad$ neglect laminar region

$$\mathrm{Re}_L = \frac{7.25(1)}{10^{-6}} = 7.25x10^6$$

$$C_F = \frac{.072}{\mathrm{Re}^{1/5}} = 0.00306$$

$$F_D = 2C_F\frac{1}{2}\rho U^2 wL = 2(.00306)(.5)(998)(7.25^2)(1.5)(1)$$

$$F_D = 241N$$

b) $x_t = \frac{\nu}{U}\mathrm{Re}_t = \frac{10^{-6}}{1.75}5x10^5 = 0.286m$

$$\mathrm{Re}_L = \frac{1.75(1)}{10^{-6}} = 1.75x10^6$$

$$C_F = \frac{.072}{\mathrm{Re}_L^{1/5}} - \frac{938}{\mathrm{Re}_L} = .00522 - .00188 = 0.00334$$

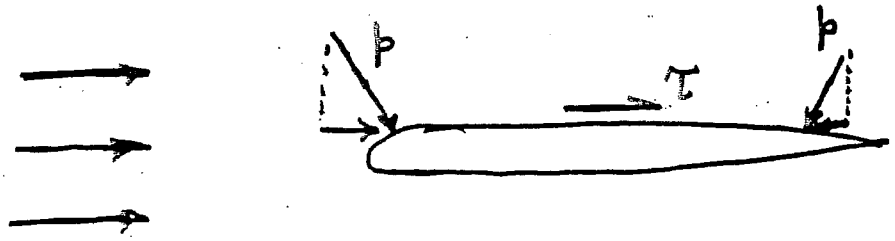
$$F_D = 2C_F\frac{1}{2}\rho U^2 wL = 2(.00334)(.5)(998)(1.75^2)(1.5)(1)$$

$$F_D = 15.3N$$

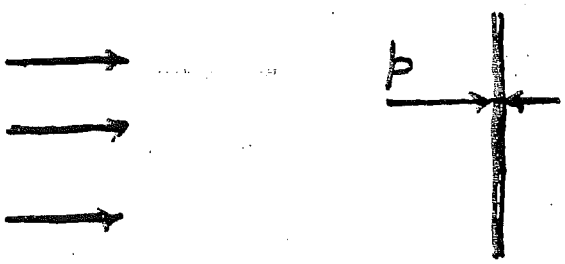
# DRAG

ALL FRICTION DRAG (Viscous Drag)

FRICTION + PRESSURE DRAG

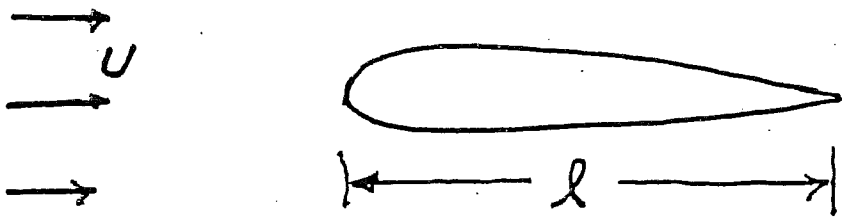



ALL PRESSURE DRAG



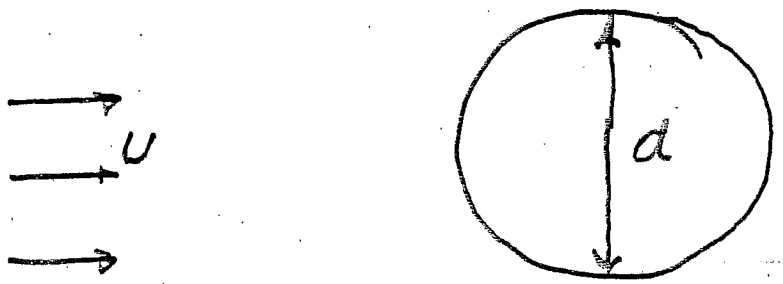
# DRAG

## STREAMLINED OBJECTS




$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \underbrace{bl}_{\text{PLAN AREA}}}$$

## BLUNT OBJECTS




$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \underbrace{bd}_{\text{CROSS-SECTIONAL AREA } \perp \text{ TO FLOW}}}$$