

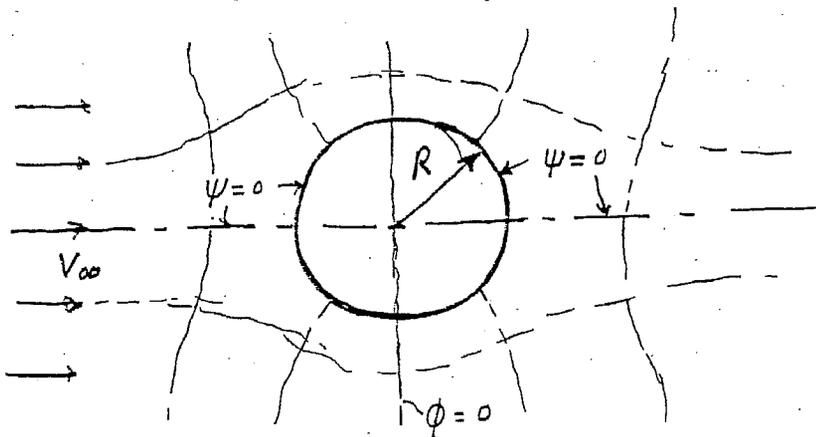
Flow over a circular cylinder

Uniform flow over a doublet:

$$\psi = V_{\infty}y - \kappa \frac{y}{r^2} = V_{\infty}y \left(1 - \frac{\kappa/V_{\infty}}{r^2}\right) = V_{\infty}y \left(1 - \frac{R^2}{r^2}\right)$$

$\psi = 0$ for $y = 0$ and $r = \sqrt{\kappa/V_{\infty}} = R = \text{radius}$

$$\phi = V_{\infty}x + \kappa \frac{x}{r^2} = V_{\infty}x \left(1 + \frac{R^2}{r^2}\right)$$

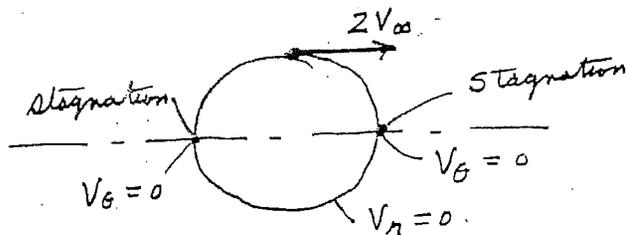


$$\phi = V_{\infty} \left(r + \frac{R^2}{r}\right) \cos \theta$$

$$v_r = \frac{\partial \phi}{\partial r} = V_{\infty} \left(1 - \frac{R^2}{r^2}\right) \quad v_{\theta} = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -V_{\infty} \left(1 + \frac{R^2}{r^2}\right) \sin \theta$$

On the cylinder: $r = R$, $v_r = 0$, $v_{\theta} = -2V_{\infty} \sin \theta$

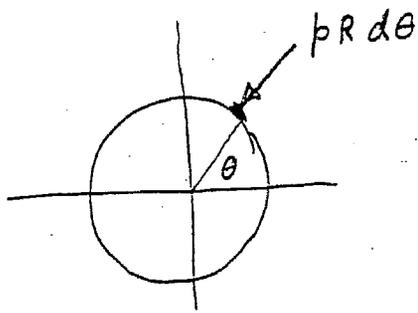
$$|v_{\theta}|_{\max} = 2V_{\infty} \text{ at } \theta = 90^{\circ}$$



$$p = p_0 - \frac{1}{2} \rho V^2$$

$$p_s = p_0 - 2\rho V_{\infty}^2 \sin^2 \theta$$

Drag force



$$\begin{aligned}
 F_x &= - \int_0^{2\pi} (pRd\theta) \cos\theta \\
 &= - \int_0^{2\pi} (p_0 - 2\rho V_\infty^2 \sin^2\theta) \cos\theta R d\theta \\
 &= -R(p_0 \sin\theta - \frac{2}{3}\rho V_\infty^2 \sin^3\theta) \Big|_0^{2\pi}
 \end{aligned}$$

$F_x = 0$ no drag because no friction

Pressure coefficient:

$$p_s = p_0 - \frac{1}{2}\rho V^2 = p_0 - 2\rho V_\infty^2 \sin^2\theta$$

$$p_0 = p_\infty + \frac{1}{2}\rho V_\infty^2$$

$$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - 4\sin^2\theta$$

$$p_{\min} = p_\infty - \frac{3}{2}\rho V_\infty^2$$

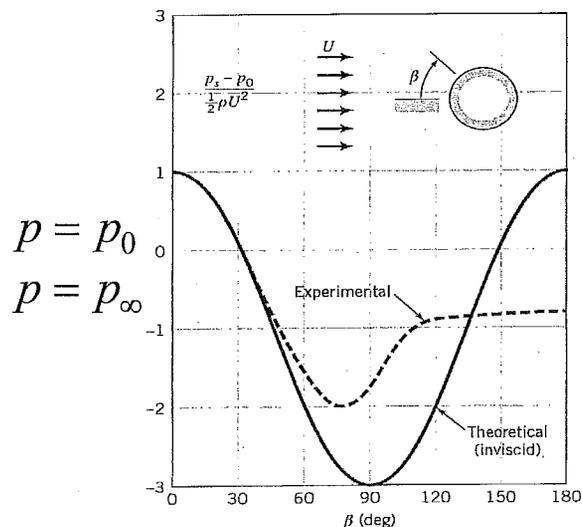


FIGURE 7.11 A comparison of the inviscid pre

Viscous effects significantly modifies the flow field for blunt objects.

Rotating cylinder → Cylinder with clockwise vortex

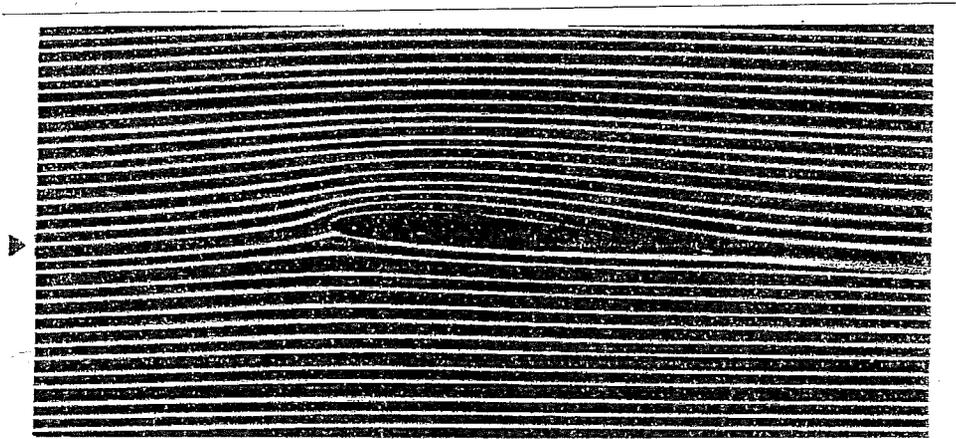
$$\phi = V_{\infty} \left(r + \frac{R^2}{r} \right) \cos \theta - \frac{\Gamma \theta}{2\pi}$$

$$v_r = \frac{\partial \phi}{\partial r} = V_{\infty} \left(1 - \frac{R^2}{r^2} \right) = 0 \text{ for } r = R$$

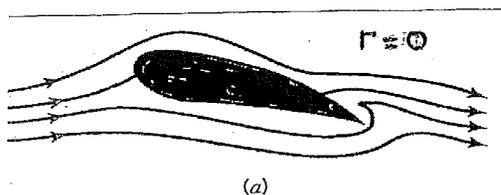
$$v_{\theta} = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -V_{\infty} \left(1 + \frac{R^2}{r^2} \right) \sin \theta - \frac{\Gamma}{2\pi} = -2V_{\infty} \sin \theta - \frac{\Gamma}{2\pi} \text{ for } r = R$$

$$p_s = p_0 - \frac{1}{2} \rho \left(-2V_{\infty} \sin \theta - \frac{\Gamma}{2\pi} \right)^2$$

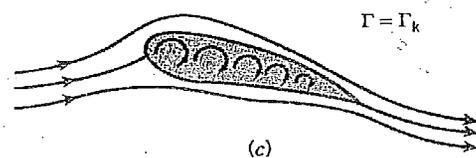
$$F_y = - \int_0^{2\pi} (pR d\theta) \sin \theta = \rho V_{\infty} \Gamma$$



Note trailing edge flow – leaves at trailing edge angle



Ideal flow – no circulation



Circulation for trailing edge condition. $Lift = \rho V_{\infty} \Gamma$