

Pipe flow analysis

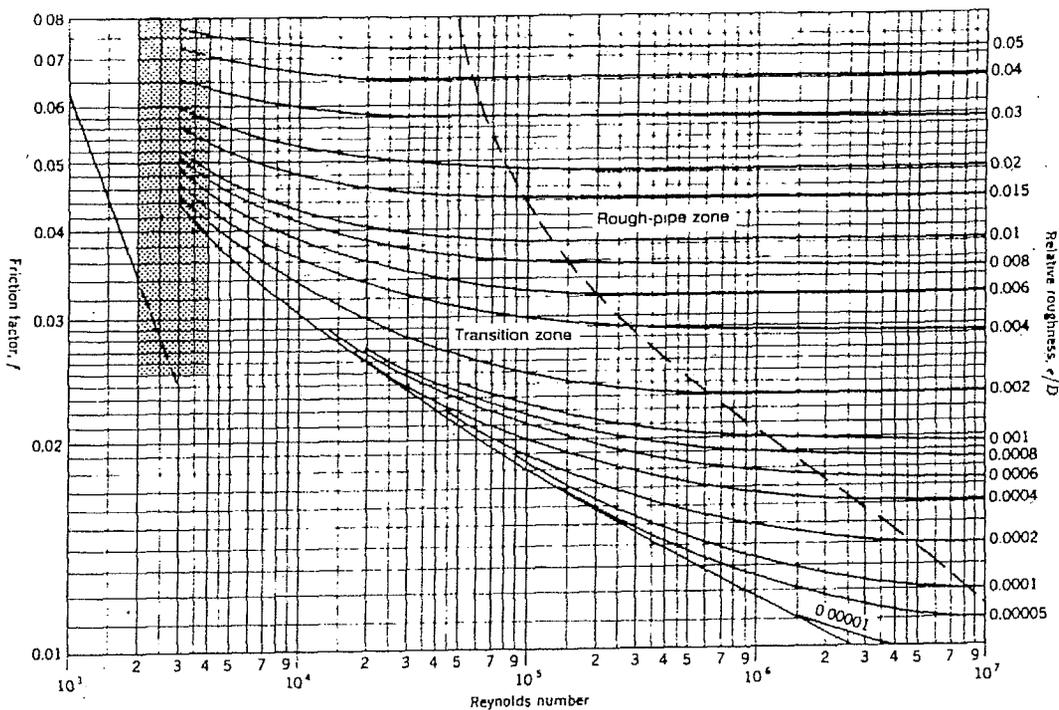
Internal
Flow 3

Energy equation:

$$\frac{\dot{W}_s}{\dot{m}} + \left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) = h_f$$

Friction loss: $h_f = f \frac{L V^2}{D 2}$ $f = f(\text{Re}, \frac{e}{D})$

Minor losses: $h_f = K \frac{V^2}{2}$ valves, fittings, bends, etc.



Problems:

Given: \dot{q}, D

Find: Diving force (p_1, h, \dot{W}_s)

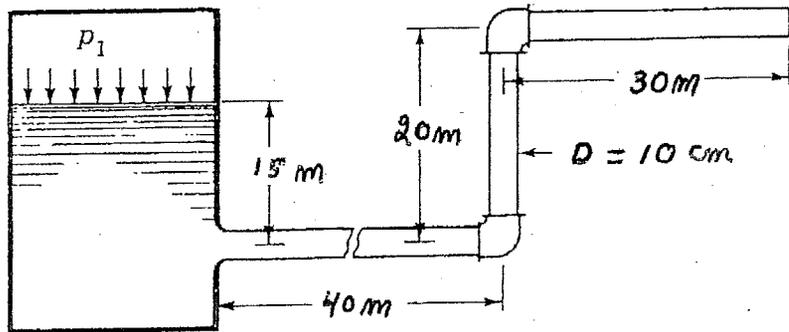
Given: p_1 or h or \dot{W}_s, D

Find: \dot{q} (iterative solution)

Given: p_1 or h or \dot{W}_s, \dot{q}

Find: D (iterative solution)

Example: Water ($\nu = 1.14 \times 10^{-6} \text{ m}^2 / \text{s}$) flows from the tank and through the pipe shown. The pipe is made of new commercial steel whose roughness is $e = 0.046 \text{ mm}$.



Solve the following problems:

a) If the pipe diameter is 10 cm, what is the pressure p_1 to cause a flow rate of $\dot{q} = 0.05 \text{ m}^3 / \text{s}$ through the pipe.

Modified Bernoulli equation between the surface 1 and the pipe exit 2.

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) = h_f$$

$$V_1^2 \ll V_2^2 \quad \text{and} \quad p_2 = p_a$$

$$p_1 - p_a = \rho \left[\frac{V^2}{2} + g(z_2 - z_1) + h_f \right]$$

$$A = \pi (0.1)^2 / 4 = 7.85 \times 10^{-3} \text{ m}^2$$

$$V = \dot{q} / A = 0.05 / 7.85 \times 10^{-3} = 6.37 \text{ m/s}$$

$$\text{Re} = VD / \nu = 6.37(0.1) / 1.14 \times 10^{-6} = 5.59 \times 10^5$$

$$e/D = 0.0046/10 = 0.00046 \quad \text{From diagram } f = 0.0175$$

$$L/D = (40 + 20 + 30) / 0.1 = 900$$

$$h_f = f(L/D)V^2 / 2 = 0.0175(900)6.37^2 / 2 = 320 \text{ m}^2 / \text{s}^2$$

$$p_1 - p_a = 998[6.37^2 / 2 + 9.8(5) + 320] = 389,000 \text{ N/m}^2$$

b) What is the flow rate \dot{q} in a 10cm diameter pipe if the pressure $p_1 - p_a = 200kPa$?

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) = h_l$$

$$V_1^2 \ll V_2^2 \quad \text{and} \quad p_2 = p_a$$

$$\frac{p_1 - p_a}{\rho} + g(z_1 - z_2) = \frac{V_2^2}{2} + f \frac{L}{D} \frac{V_2^2}{2} = \left(1 + f \frac{L}{D}\right) \frac{V_2^2}{2}$$

$$L/D = (40 + 20 + 30)/0.1 = 900$$

$$(1 + 900f) \frac{V^2}{2} = \frac{200000}{998} + 9.8(-5) = 151.4$$

$$V = \sqrt{\frac{302.8}{1 + 900f}} \quad \text{Re} = \frac{VD}{\nu} \quad f = f(\text{Re}, e/D)$$

Iterate:

$$\text{Re} = 100,000 \rightarrow f = 0.0203 \rightarrow V = 3.96 \rightarrow \text{Re} = 348,000$$

$$\text{Re} = 348,000 \rightarrow f = 0.0182 \rightarrow V = 4.17 \rightarrow \text{Re} = 366,000$$

$$\text{Re} = 366,000 \rightarrow f = 0.0181 \rightarrow V = 4.18 \rightarrow \text{Re} = 367,000$$

$$\dot{q} = VA = 4.18 \frac{\pi}{4} (0.1)^2 = 0.0328 m^3 / s$$

c) If the pressure $p_1 - p_a = 200kPa$, what is the diameter of the pipe to carry a flow rate of $\dot{q} = 0.05m^3s$.

$$\frac{p_1 - p_a}{\rho} + g(z_1 - z_2) = \frac{V^2}{2} + f \frac{L}{D} \frac{V^2}{2} = (1 + f \frac{L}{D}) \frac{V^2}{2}$$

$$(1 + 900f) \frac{V^2}{2} = \frac{200000}{998} + 9.8(-5) = 151.4$$

$$V = \sqrt{\frac{302.8}{1 + \frac{90}{D}f}} \quad \text{Re} = \frac{VD}{\nu} \quad f = f(\text{Re}, e/D)$$

$$\dot{q} = V \frac{\pi}{4} D^2 \quad D = \sqrt{\frac{4\dot{q}}{\pi V}}$$

Iterate:

Assume $V=4.0$ $D = \sqrt{\frac{4\dot{q}}{\pi V}} = \sqrt{\frac{4(0.05)}{\pi(4)}} = 0.126$

$$\text{Re} = \frac{VD}{\nu} = \frac{4(0.126)}{1.14 \times 10^{-6}} = 442,000$$

$$f = f(\text{Re}, e/D) = 0.0179$$

$$V = \sqrt{\frac{302.8}{1 + \frac{90}{D}f}} = \sqrt{\frac{302.8}{1 + \frac{90}{0.126}(0.0179)}} = 4.69$$

$$V = 4.69 \rightarrow D = 0.117 \rightarrow \text{Re} = 481,000 \rightarrow f = 0.0177 \rightarrow V = 4.55$$

$$V = 4.55 \rightarrow D = 0.118 \rightarrow \text{Re} = 471,000 \rightarrow f = 0.0178 \rightarrow V = 4.56$$

$$D = 0.118m$$

MINOR LOSSES

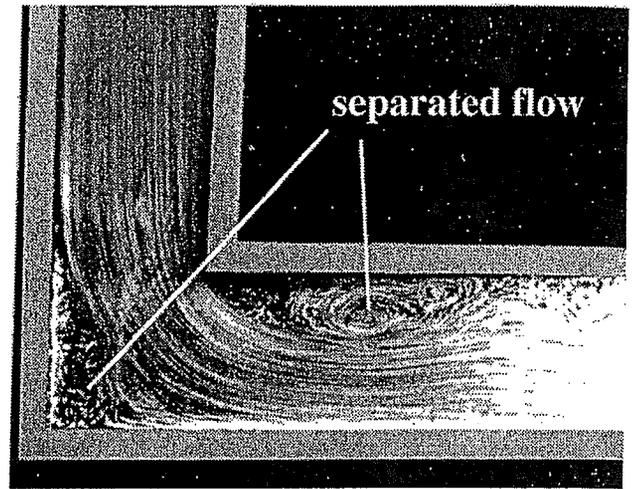
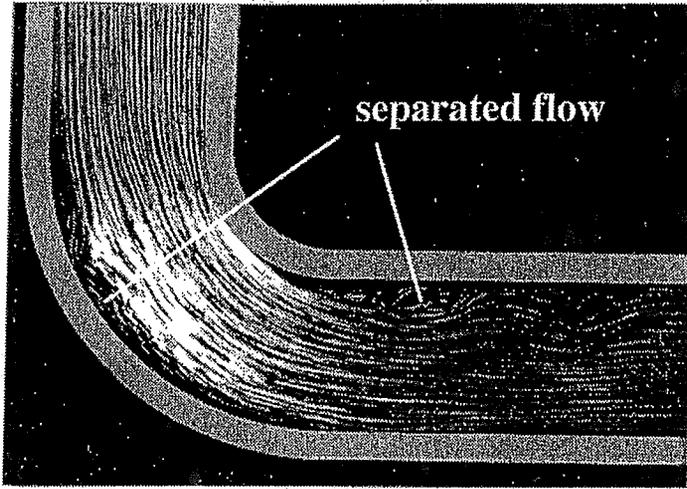


FIGURE 9.11 Flow through round and square bends showing regions of separation. From *Visualized Flow*, Japan Soc. of Mech. Engineers, Pergamon Press, 1988

$$(\text{minor losses}) = K \frac{\bar{V}^2}{2g}$$

TABLE 9.2 Typical Loss Coefficients for Pipe Entrances and Exits (D is the diameter of the pipe, and r is the radius of the rounded entry)

	K
Entrance type	
Re-entrant	0.78
Square-edged	0.5
Rounded ($r/D = 0.02$)	0.28
Rounded ($r/D = 0.06$)	0.15
Rounded ($r/D \geq 0.15$)	0.04
Exit type	
Abrupt	1.0

TABLE 9.3 Typical Loss Coefficients for Pipe Fittings

Valve or fitting	K
Gate valve (open)	0.20
Globe valve (open)	6.4
Angle valve (open)	—
Ball valve (open)	—
Standard 45° elbow	0.35
Standard 90° elbow	0.75
Long-radius 90° elbow	0.45
Standard tee (flow through run)	0.4
Standard tee (branch flow)	1.5

MINOR LOSSES

LOSSES IN VALVES, FITTINGS, BENDS, EXPANSIONS, CONTRACTIONS, ETC.

THESE DEVICES CAUSE DISTURBANCES AND TURBULENCE WHICH RESULTS IN LOSSES OF MECHANICAL ENERGIES OF THE MEAN FLOW.

$$h_l = K \frac{V^2}{2}$$

K = LOSS COEFFICIENT

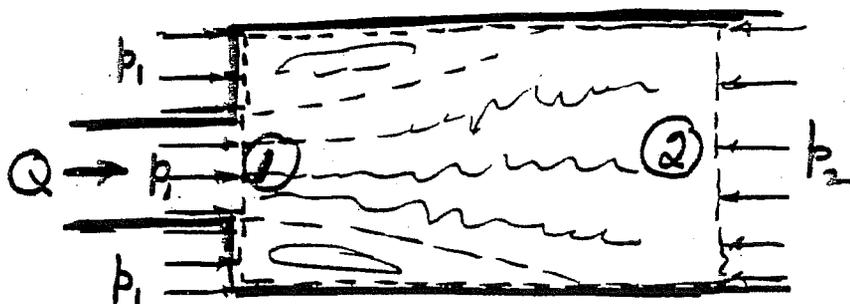
FREE JET



$$h_l = V_1^2/2 \Rightarrow K=1$$

$$\begin{aligned} & \cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + \cancel{gz_1} \\ &= \cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2} + \cancel{gz_2} + h_l \end{aligned}$$

SUDDEN EXPANSION



$$Q = V_1 A_1 = V_2 A_2$$

$$\sum F_x = \iint V_x \rho \vec{v} \cdot d\vec{A}$$

$$p_1 A_2 - p_2 A_2 = \rho V_2^2 A_2 - \rho V_1^2 A_1$$

$$h_l = (V_1^2/2 + p_1/\rho) - (V_2^2/2 + p_2/\rho)$$

$$h_l = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2} \quad \therefore K = \left(1 - \frac{A_1}{A_2}\right)^2$$

d) If the pipe diameter is 10cm, what is the pressure p_1 to cause a flow rate of $\dot{q} = 0.05m^3/s$ through the pipe. Include minor losses: sharp edged entrance $K = 0.5$, 90 deg elbow $K = 0.75$.

Modified Bernoulli equation between the surface 1 and the pipe exit 2.

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) = h_l$$

$$V_1^2 \ll V_2^2 \quad \text{and} \quad p_2 = p_a$$

$$p_1 - p_a = \rho \left[\frac{V^2}{2} + g(z_2 - z_1) + h_l \right]$$

$$V = \dot{q} / A = 6.37m/s$$

$$Re = VD/\nu = 559,000$$

$$e/D = 0.00046 \quad \text{From diagram } f = 0.0175$$

$$L/D = 900$$

$$h_l = f \frac{L}{D} \frac{V^2}{2} + \sum K \frac{V^2}{2} = 320 + (0.5 + 0.75 + 0.75) \frac{6.37^2}{2} = 361m^2/s^2$$

$$p_1 - p_a = 998 \left[\frac{6.37^2}{2} + 9.8(5) + 361 \right] = 430,000N/m^2$$

compared to $p_1 - p_a = 389,000N/m^2$ without minor losses