Internal Viscous Flows

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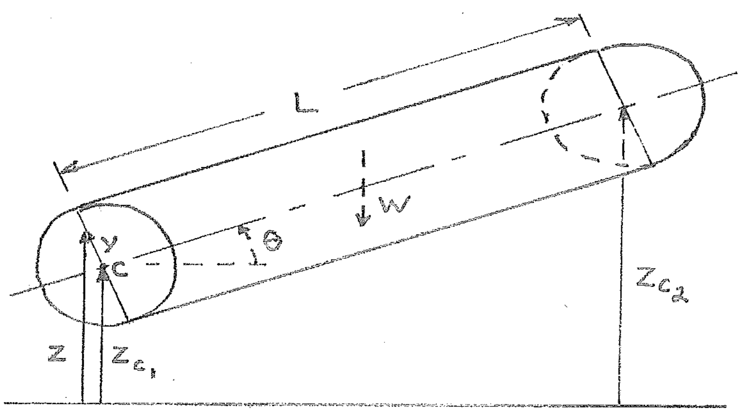
A fluid is forced through the duct by a pressure difference created by a pump, blower, pressurized a tank, or from height of liquid in a tank. Frictional forces, originating from the no-slip condition at the duct wall, resists the flow.

Friction forces dissipate some of the mechanical energy content of the fluid flow. Characterize these losses and incorporate the formulation into the energy equation which can be applied to problems involving flow through pipes.

First Law of Thermodynamics for a control volume

$$\dot{Q} + \dot{W}_s = \int\left(\frac{p}{\rho} + \frac{V^2}{2} + gz + e\right)\rho\vec{V}\cdot d\vec{A} \qquad \text{for steady flow}$$

Shear force work $\vec{\tau}_s \cdot \vec{V}dA$ does not appear in the equation provided $\vec{V}$ is perpendicular to the control surface. Effects of the friction appear as changes in the internal energy which could result in a heat transfer.



$$\dot{Q} = \left(\int\left(\frac{p}{\rho} + \frac{V^2}{2} + gz + e\right)\rho\vec{V}\cdot d\vec{A}\right)_2$$

$$-\left(\int\left(\frac{p}{\rho} + \frac{V^2}{2} + gz + e\right)\rho\vec{V}\cdot d\vec{A}\right)_1 \qquad \text{for no shaft work}$$

Incompressible flows = liquids or gases at low speeds
Pressure/Gravity term:
Pressure is hydrostatic across a parallel flow

$$p = p_c + \rho g(z_c - z) \qquad p_c = \text{pressure at the centroid}$$

$$\int(\frac{p}{\rho} + gz)\rho\vec{V}\cdot d\vec{A} = \int(\frac{p_c}{\rho} + gz_c)\rho u dA = (\frac{p_c}{\rho} + gz_c)\dot{m}$$

Drop the subscript – understand p and z are at the centroid
For flow of a gas the gravity term is negligible.

Kinetic energy term:

$$\int\frac{V^2}{2}\rho\vec{V}\cdot d\vec{A} = \int\frac{V^2}{2}\rho u dA \cong \frac{1}{2}\rho\int u^3 dA = \frac{1}{2}\lambda\rho\bar{u}^3 A = \lambda\frac{\bar{u}^2}{2}\dot{m}$$

where $\lambda = (\int u^3 dA)/\bar{u}^3$ and $\dot{m} = \rho A\bar{u}$.

Energy equation becomes:

$$(\frac{p_1}{\rho} + \lambda_1\frac{\bar{V}_1^2}{2} + gz_1) - (\frac{p_2}{\rho} + \lambda_2\frac{\bar{V}_2^2}{2} + gz_2) = (e_2 - e_1) - dQ/dm$$

where e is taken constant across the flow

-States that the difference or loss in mechanical energies (pressure potential, kinetic energies and gravitational potential) equal to a rise in internal thermal energy and a heat transfer from the control volume.
-Friction in the flow tends to warm the fluid which can also result in a heat transfer from the fluid.
-$\dot{Q}$ can also account for heat transfer due to direct heating or cooling the flow, not be considered here.

Loss in mechanical energies

$$(\frac{p_1}{\rho} + \lambda_1\frac{\bar{V}_1^2}{2} + gz_1) - (\frac{p_2}{\rho} + \lambda_2\frac{\bar{V}_2^2}{2} + gz_2) = h_l$$

where h_l = loss of mechanical energies per unit mass

Momentum equation can give information on the loss of mechanical energies

$$\sum F_x = \int_{CS} V_x \rho \vec{V} \cdot d\vec{A}$$

For the control volume containing the fluid in the pipe

$$\int_1 p dA - \int_2 p dA - \int \tau_w \pi D dx - \gamma \forall \sin\theta = -\int_1 \rho u^2 dA + \int_2 \rho u^2 dA$$

Wall shear stress τ_w only depends on x

For parallel or near-parallel flow pressure variations are only due to hydrostatic effects.

$$p = p_c - \gamma(z - z_c) = p_c - \gamma y \cos\theta$$
$$\int p dA = p_c A - \gamma \cos\theta \int y dA = p_c A$$

where $\int y dA = 0$ since y is measured from the centroid of the cross-section.

The volume $\forall$ of the fluid in the control volume is $\forall = AL$ and $L \sin\theta = z_{c2} - z_{c1}$.

The momentum equation can then be written

$$p_{c1} A - p_{c2} A - \rho g A(z_{c2} - z_{c1}) - \int \tau_w \pi D dx = -\int_1 \rho u^2 dA + \int_2 \rho u^2 dA$$

Dividing this equation by ρA and adding and subtracting terms for $\lambda \overline{V}^2/2$ and putting $\int u^2 dA = \kappa \overline{V}^2 A$ we obtain

$$\left(\frac{p_c}{\rho} + \lambda \frac{\overline{V}^2}{2} + g z_c\right)_1 - \left(\frac{p_c}{\rho} + \lambda \frac{\overline{V}^2}{2} + g z_c\right)_2 =$$

$$\frac{\pi D}{\rho A} \int_0^L \tau_w dx + \left(\lambda \frac{\overline{V}^2}{2} - \kappa \overline{V}^2\right)_1 - \left(\lambda \frac{\overline{V}^2}{2} - \kappa \overline{V}^2\right)_2$$

The energy loss h_l is identified by comparing the result for the momentum equation with the energy equation

$$h_l = \frac{\pi D}{\rho A}\int_0^L \tau_w dx + (\lambda\frac{\overline{V}^2}{2} - \kappa\overline{V}^2)_1 - (\lambda\frac{\overline{V}^2}{2} - \kappa\overline{V}^2)_2$$

Only consider fully developed flow formulation:
-Velocity profile does not changed with distance and the velocity terms cancel.
-Wall shear τ_w is constant on the inside pipe surface.

For fully developed flow

$$h_l = \frac{\pi D}{\rho A}\tau_w L = 4\frac{L}{D}\frac{\tau_w}{\rho} = 4c_f\frac{L}{D}\frac{\overline{V}^2}{2} = f\frac{L}{D}\frac{\overline{V}^2}{2}$$

where the friction factor $f = 4c_f$ and $c_f = \tau_w/(\rho\overline{V}^2/2)$ is the wall shear stress coefficient

Energy loss:

-is in terms of the kinetic energy $\overline{V}^2/2$
-proportional to the length of pipe L
-Inversely proportional to the diameter D

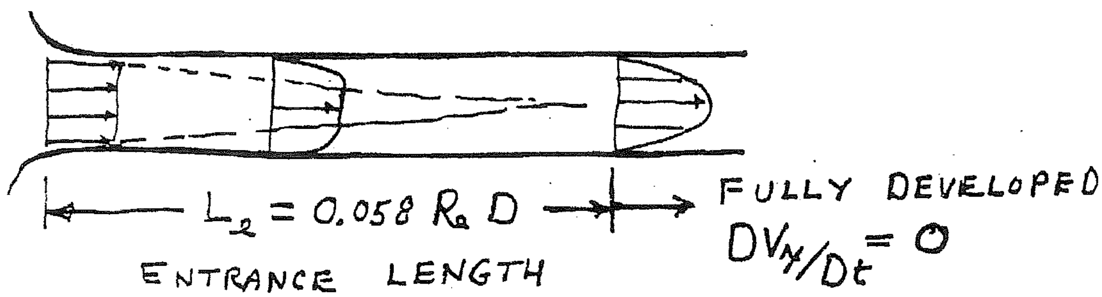
Energy equation for pipe flows

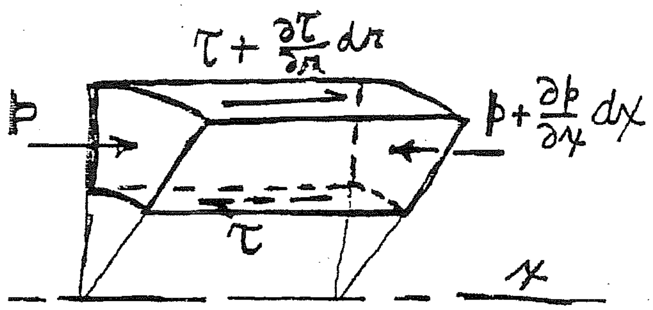
$$\left(\frac{p}{\rho} + \frac{V^2}{2} + gz\right)_1 - \left(\frac{p}{\rho} + \frac{V^2}{2} + gz\right)_2 = h_l = f\frac{L}{D}\frac{V^2}{2}$$

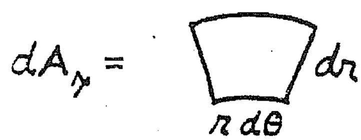
Profile factor, $\lambda = (\int u^3 dA)/\overline{u}^3$, neglected and $V = \overline{V}$
Modified Bernoulli Equation=Bernoulli Equation with loss

Laminar flow

Solve for the velocity profile to determine τ_w and f:
- circular pipe with constant cross-section
- steady incompressible flow
- fully developed, velocity profile constant with distance
- balance between pressure and friction forces







$\sum F_x = 0$

$$prd\theta dr - (p + \frac{\partial p}{\partial x}dx)rd\theta dr - \tau rd\theta dr + (\tau + \frac{\partial \tau}{\partial r}dr)rd\theta dr = 0$$

$$r\frac{\partial \tau}{\partial r} + \tau = r\frac{\partial p}{\partial x} \quad \text{or} \quad \frac{\partial r\tau}{\partial r} = r\frac{\partial p}{\partial x}$$

$\partial p / \partial x$ independent of r and dp/dx = constant
τ independent of x

$$\frac{dr\tau}{dr} = r\frac{dp}{dx}$$

Solution to $\dfrac{dr\tau}{dr} = r\dfrac{dp}{dx}$

$$r\tau = \frac{dp}{dx}\frac{r^2}{2} + C_1 \qquad C_1 = 0 \text{ for } \tau_{r=0} = \text{finite}$$

$$\tau = \frac{1}{2}\frac{dp}{dx}r$$

$$\tau = \mu\frac{du}{dr} \quad \text{and} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{dp}{dx}r$$

$$u = \frac{1}{4\mu}\frac{dp}{dx}r^2 + C_2 \qquad u(R) = 0 \text{ gives } \quad C_2 = -\frac{R^2}{4\mu}\frac{dp}{dx}$$

$$u = -\frac{1}{4\mu}\frac{dp}{dx}(R^2 - r^2) \quad \text{parabolic profile}$$

$$u = u_c(1 - r^2/R^2) \quad \text{where } u_c = u(0) = -\frac{R^2}{4\mu}\frac{dp}{dx}$$

Flow Rate: $\dot{q} = \int u dA = \int_0^R \left[-\dfrac{1}{4\mu}\dfrac{dp}{dx}(R^2 - r^2) \right] 2\pi r dr$

$$\dot{q} = \left(-\frac{\pi}{2\mu}\frac{dp}{dx}\right)\left(R^2\frac{r^2}{2} - \frac{r^4}{4}\right)_0^R = -\frac{\pi R^4}{8\mu}\frac{dp}{dx} = -\frac{\pi D^4}{128\mu}\frac{dp}{dx}$$

Relation between pressure difference and flow rate

$$\dot{q} = \frac{\pi D^4}{128\mu}\frac{p_1 - p_2}{L}$$

Average velocity = $\bar{u} = \dfrac{\dot{q}}{\pi R^2} = -\dfrac{R^2}{8\mu}\dfrac{dp}{dx} = \dfrac{1}{2}u_c$