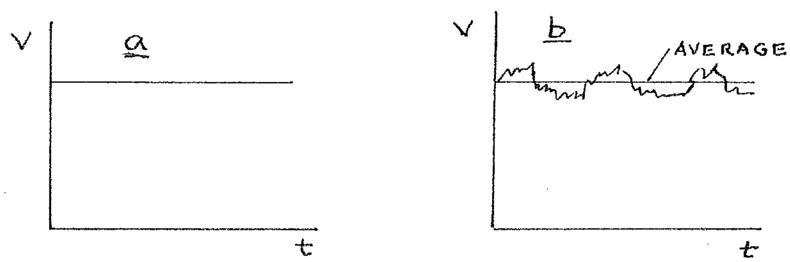
# Turbulent Flows

Laminar flows - particle paths are smooth curves
Laminar implies that the flow moves in layers or laminae which slide smoothly over one another with no erratic motions.

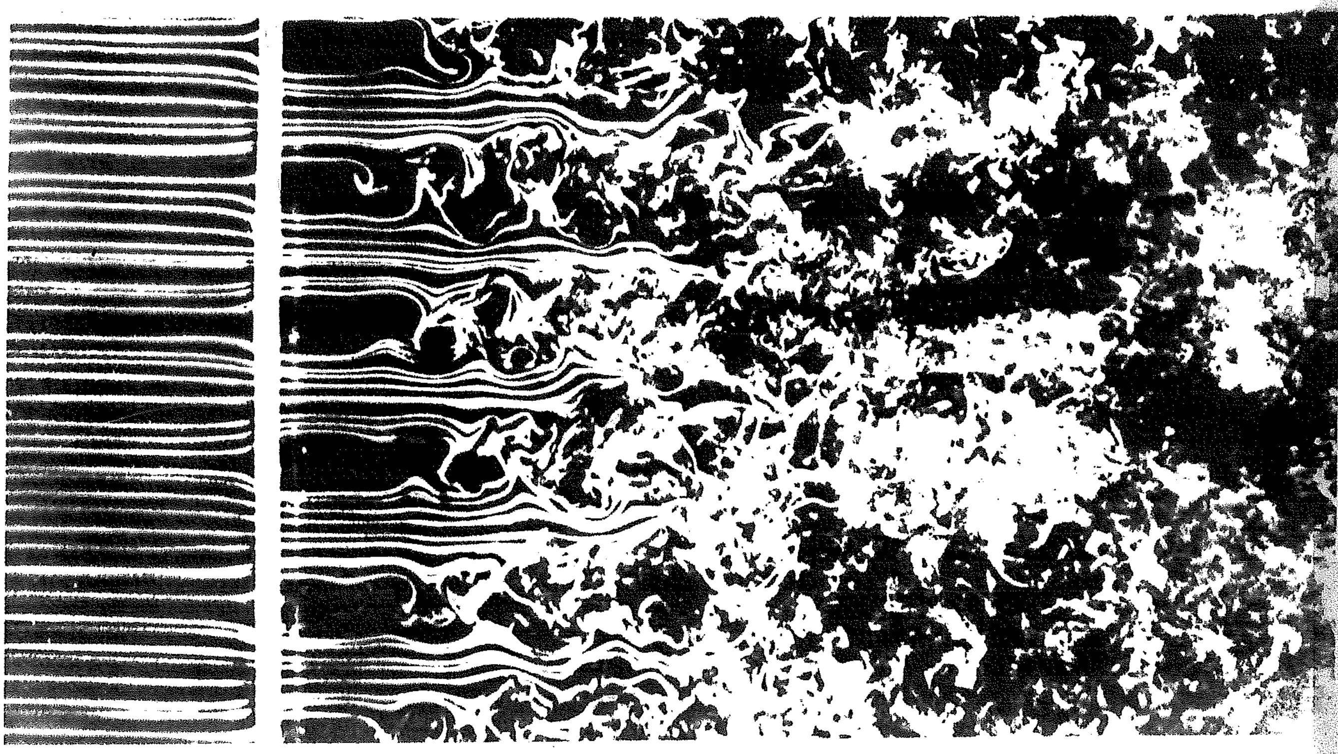
Turbulent flows have unsteady three-dimensional finite motions superimposed on the overall flow. The flow variations are erratic and non-repetitive.

A flow can become turbulent, either by natural transition or by forced transition where disturbances are introduced into the flow by a geometric device such as surface roughness, vibrations, or noise.

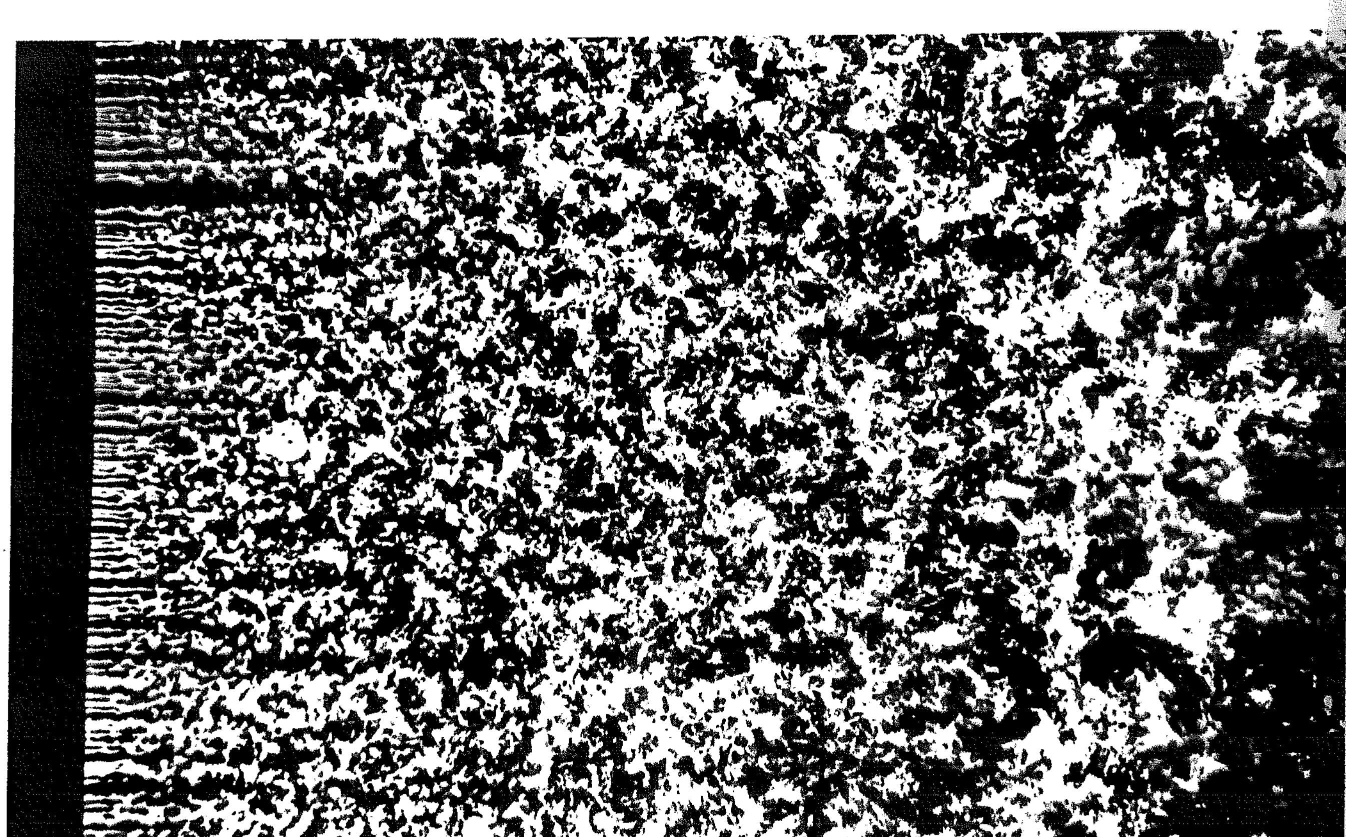



Velocity Variations: a) Laminar, b) Turbulent

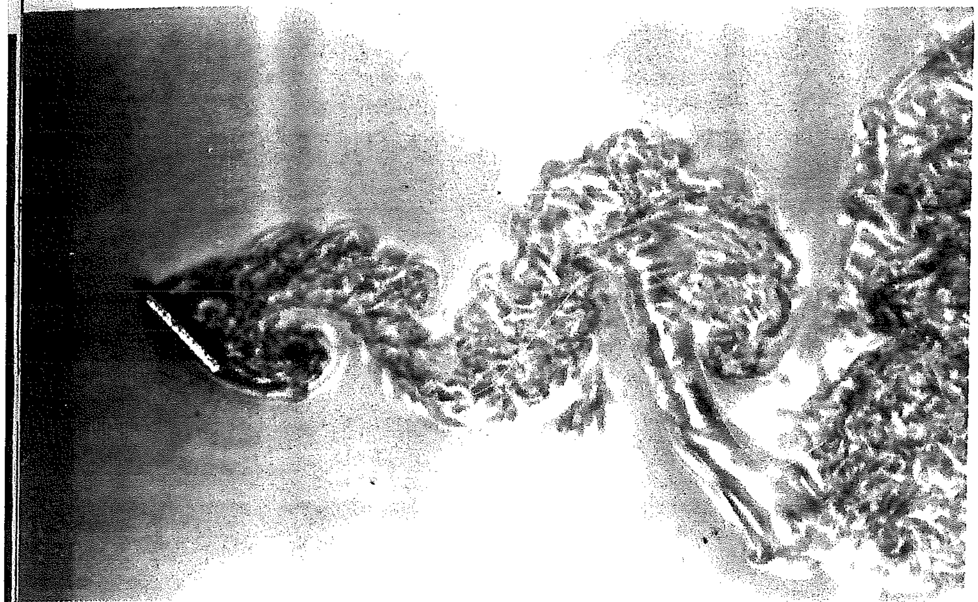
Turbulent flow structures (eddies) can be relatively large, comparable to the size of the problem, down to relatively small. For example, turbulent flow in a 10cm pipe can have flow structures from 0.1mm to 3cm. There is a spectrum of lengths and amplitudes in the velocity variations.



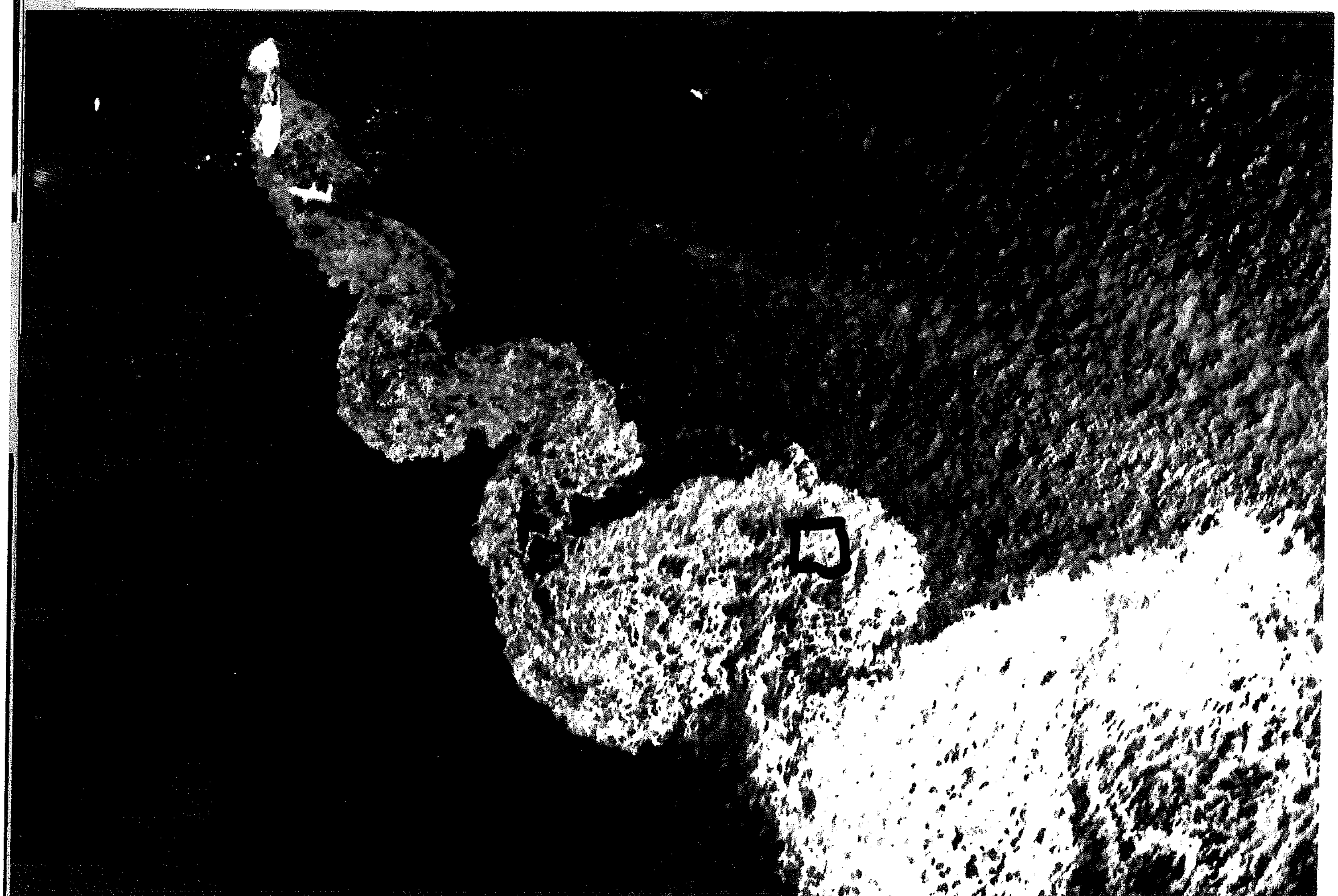
**152. Generation of turbulence by a grid.** Smoke wires show a uniform laminar stream passing through a $^1\!/_{16}$-inch plate with ¾-inch square perforations. The Reynolds number is 1500 based on the 1-inch mesh size. Instability of the shear layers leads to turbulent flow downstream. *Photograph by Thomas Corke and Hassan Nagib*



**153. Homogeneous turbulence behind a grid.** Behind a finer grid than above, the merging unstable wakes quickly form a homogeneous field. As it decays downstream, it provides a useful approximation to the idealization of isotropic turbulence. *Photograph by Thomas Corke and Hassan Nagib*



**172. Wake of an inclined flat plate.** The wake behind a plate at 45° angle of attack is turbulent at a Reynolds number of 4300. Aluminum flakes suspended in water show its characteristic sinuous form. *Cantwell 1981. Reproduced, with permission, from the* Annual Review of Fluid Mechanics, *Volume 13. © 1981 by Annual Reviews Inc.*



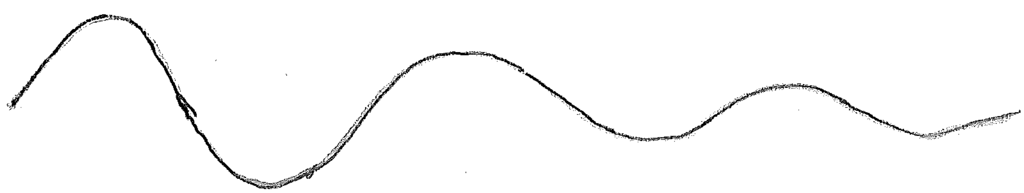
**173. Wake of a grounded tankship.** The tanker *Argo Merchant* went aground on the Nantucket shoals in 1976. Leaking crude oil shows that she happened to be inclined at about 45° to the current. Although the Reynolds number is approximately $10^7$, the wake pattern is remarkably similar to that in the photograph at the top of the page. *NASA photograph, courtesy of O. M. Griffin, Naval Research Laboratory.*

**Transition**

All flows have small disturbances from sound and vibrations which occur as oscillations in the pressure and inertia forces.

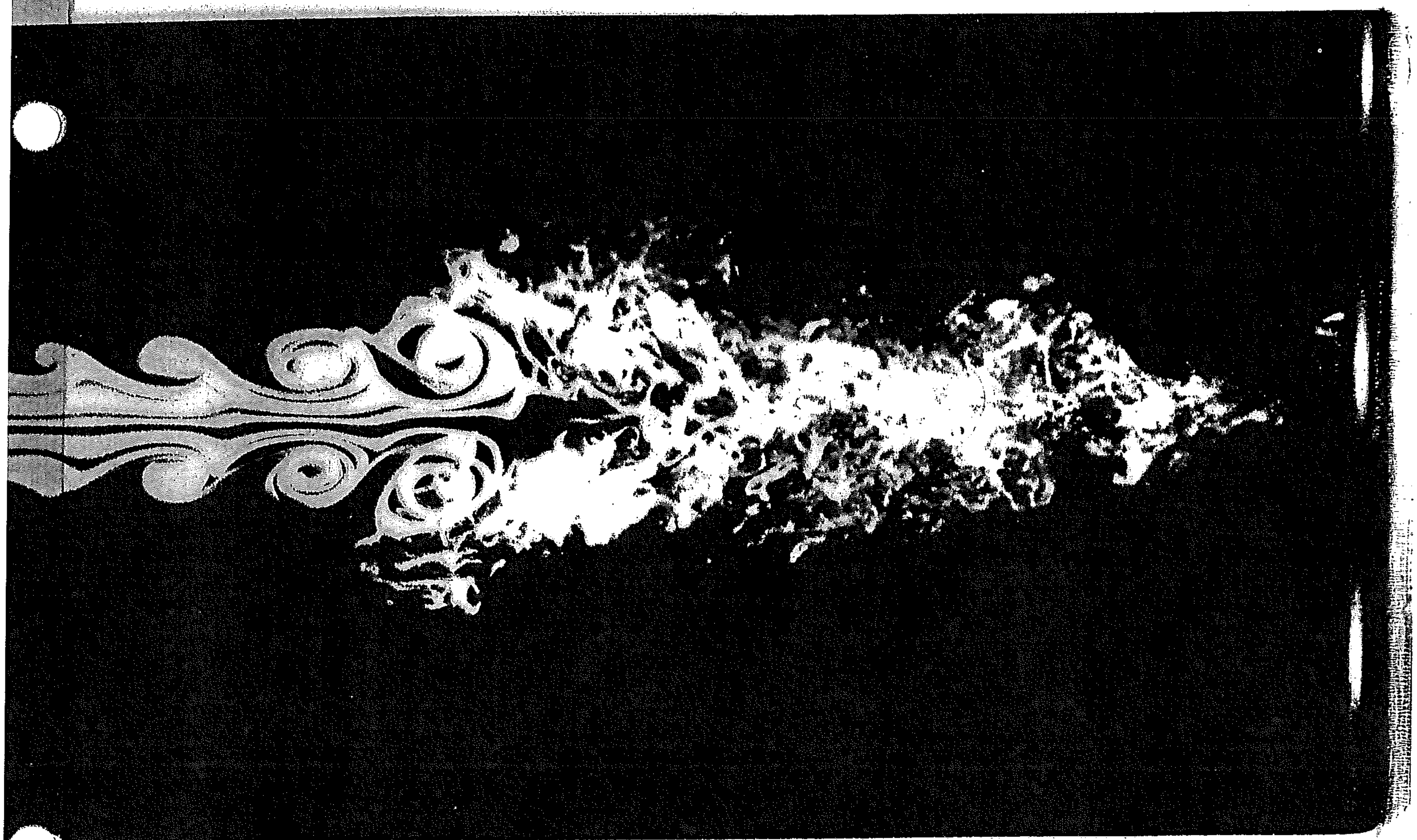
Transition is initiated when disturbances in the flow began to grow. These disturbances can gain energy from the overall mean flow to feed their growth.

Viscosity in the fluid tends to attenuate the disturbances and keep the flow stable. With relatively large viscous forces disturbances are attenuated and flows tend to be laminar.



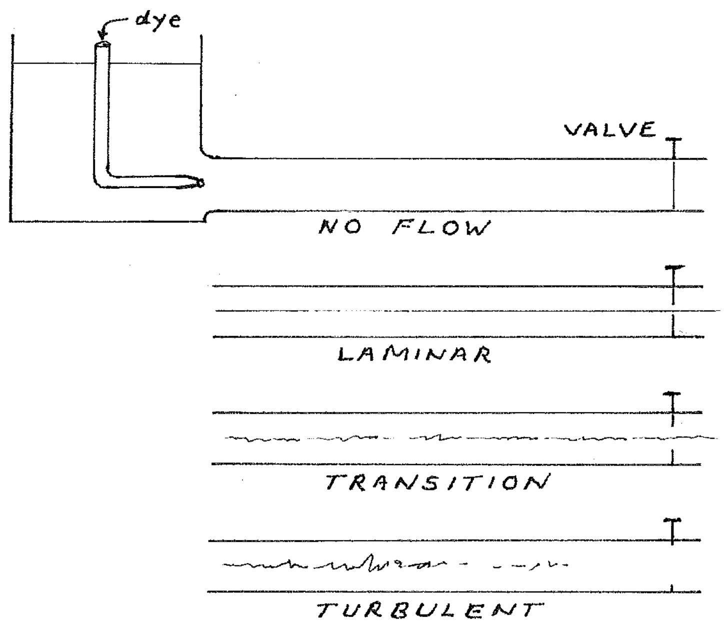
Instability in the flow occurs when viscous flows cannot keep oscillations in the inertia and pressure forces under control. With relatively small viscous forces disturbances grow and random events occur which leads to transition

# 5. Instability



**102. Instability of an axisymmetric jet.** A laminar stream of air flows from a circular tube at Reynolds number 10,000 and is made visible by a smoke wire. The edge of the jet develops axisymmetric oscillations, rolls up into vortex rings, and then abruptly becomes turbulent. *Photograph by Robert Drubka and Hassan Nagib*

**Transition experiment** by Sir Osborn Reynolds. The flow velocity from the tank is controlled by the valve near the end of the pipe.




Transition process depends on the relative magnitudes of the inertia and viscous forces. We can quantify the relative effects of these two forces by taking their ratio.

Inertia force $= \rho dV / dt$ which is the force per unit volume.

The velocity in a flow can vary from zero to some maximum value so a crude estimate would be $DV \sim V$ where V is some characteristic or reference velocity for the particular flow.

The time increment dt is taken to be the time for the flow to move a length L in the flow which gives dt~L/V.
Then the inertia force per volume is estimated

$$\rho \frac{Du}{Dt} \sim \rho \frac{V}{L/V} = \frac{\rho V^2}{L}$$

**Viscous force** per unit volume, $\mu\partial^2 u/\partial x^2$, can be estimated

$$\mu\frac{\partial^2 u}{\partial x^2} \sim \mu\frac{V}{L^2}$$

$$\text{Reynolds} = \text{Re} = \frac{\textit{inertia force}}{\textit{viscous force}} = \frac{\rho V^2/L}{\mu V/L^2} = \frac{\rho VL}{\mu}$$

For Flow of water in a 10cm diameter pipe at an average velocity of 12m/s

$$\text{Re} = \frac{(998kg/m^3)(12m/s)(0.1m)}{1.00x10^{-3}\,Ns/m^2} = 1.20x10^6\,\frac{kg\cdot m}{N\cdot s^2} = 1.20x10^6$$

which is a dimensionless number in that it has no units..

The Reynolds number characterizes the relative amount of viscous effects in a flow. For large viscous effects the Reynolds number is relatively small, i.e. $\sim 10^{-4}$ for small particles settling in oil. For relatively small viscous effects the Reynolds number is large, i.e. $\sim 10^8$ for a commercial airplane.

The transition process occurs over a range of Reynolds numbers. For viscous flow near the surface (boundary layer) instability occurs for the Reynolds number $\text{Re}_x = \rho Ux/\mu = 91{,}000$ down the surface where x is the distance from the leading edge and U is the velocity outside the boundary layer. Fully developed turbulence follows at a much higher Reynolds number and transition is taken to occur at $(\text{Re}_x)_t \cong 3.5x10^5$ for engineering purposes. In flows in pipes transition can occur over a range in Reynolds numbers, but for engineering purposes transition is taken to occur at $\text{Re} = \rho VD/\mu = 2300$ for circular pipes

**Average values** are used in turbulent flows

$$\overline{u} = \frac{1}{T}\int_0^T u\,dt$$

where the period T of integration is long enough to includes all sizes of the disturbances passing the point.

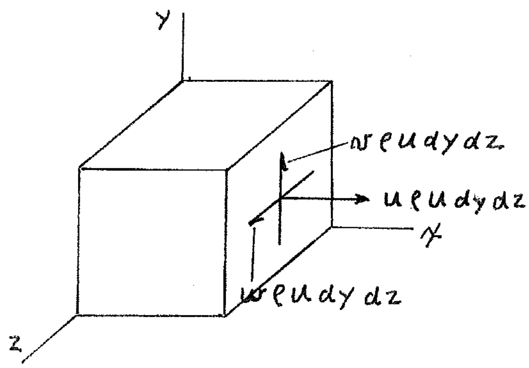
Decompose the velocity into mean and fluctuating parts

$$u = \overline{u} + u'$$

$$\overline{u} = \frac{1}{T}\int_0^T (\overline{u} + u')dt = \overline{u} + \overline{u'} \qquad \overline{u'} = 0$$

Equations for the mean turbulent flow look the same as those for laminar flow except there are extra terms arising from turbulent momentum exchanges. These momentum exchanges can be interpreted as turbulent stresses analogous to molecular exchanges between fluid layers causing viscous stresses.

$$\sum \vec{F} = \int_{CS} \vec{V}\rho\vec{V}\cdot\hat{n}dA$$




Flow of Momentum across the x-Surface

x-face mass flow rate is $\rho u dydz$ carries u,v and w components of momentum per unit mass.

$$\rho\overline{uv} = \rho\overline{(\overline{u}+u')(\overline{v}+v')} = \rho(\overline{\overline{u}\,\overline{v}} + \overline{\overline{u}v'} + \overline{u'\overline{v}} + \overline{u'v'}) = \rho(\overline{u}\,\overline{v} + \overline{u'v'})$$

Turbulent exchanges of momentum across the x-face of the control volume which can be interpreted as a stress.

Complete set of **turbulent stresses** for the three faces

$$\begin{array}{ccc} -\rho\overline{u'^2} & -\rho\overline{u'v'} & -\rho\overline{u'w'} \\ -\rho\overline{v'u'} & -\rho\overline{v'^2} & -\rho\overline{v'w'} \\ -\rho\overline{w'u'} & -\rho\overline{w'v'} & -\rho\overline{w'^2} \end{array}$$

x-momentum equation for the mean flow

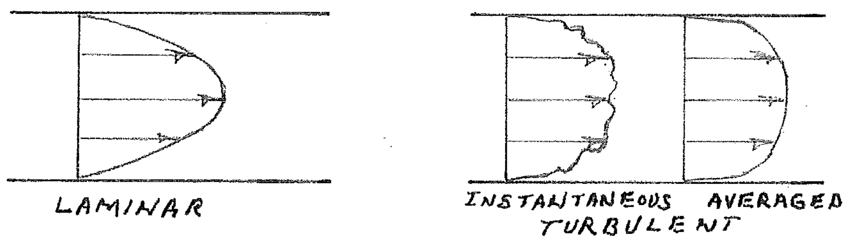
$$\rho\frac{\overline{D}\overline{u}}{Dt} = -\frac{\partial\overline{p}}{\partial x} + \frac{\partial\overline{\tau}_{xx}}{\partial x} + \frac{\partial\overline{\tau}_{yx}}{\partial y} + \frac{\partial\overline{\tau}_{zx}}{\partial z}$$

Stresses have a laminar part for the mean flow and a turbulent part $\quad \overline{\tau}_{yx} = \mu\left(\frac{\partial\overline{u}}{\partial y} + \frac{\partial\overline{v}}{\partial x}\right) - \rho\overline{v'u'}$

Turbulent stresses could be assumed proportional to the mean flow strain rate $\quad \overline{\tau}_{yx} = (\mu + \mu_t)\left(\frac{\partial\overline{u}}{\partial y} + \frac{\partial\overline{v}}{\partial x}\right)$

Works ok for a lot of flows depending what formulation is used for the turbulent viscosity $\mu_t$. The turbulent viscosity is not a property of the fluid and must be formulated from additional theories.

## Consequences if the flow is laminar or turbulent




Laminar and Turbulent Velocity Profiles in a channel

Laminar flow: $U = \frac{3}{2}\overline{U}(1 - \frac{4y^2}{h^2})$ $\quad \overline{U} = \frac{2}{h}\int_0^{h/2} u dy$

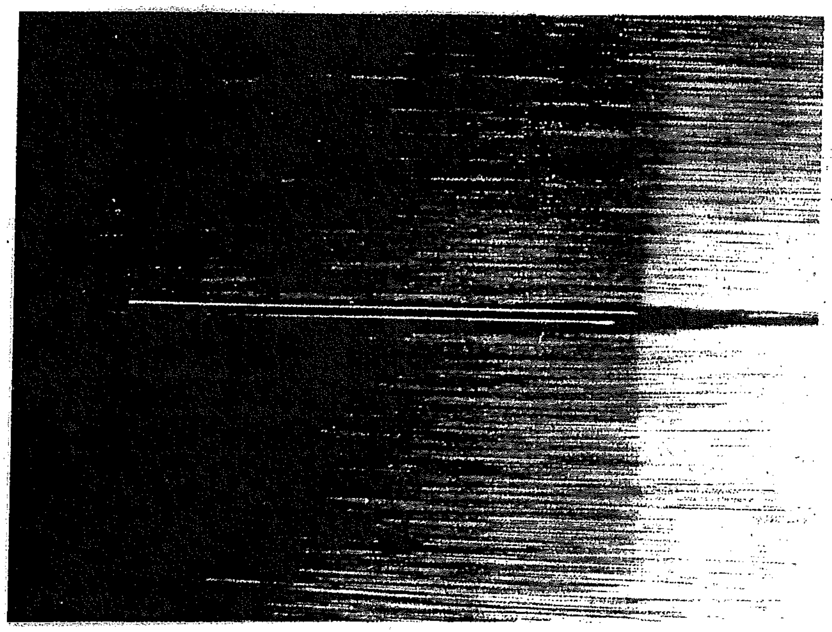
Mean turbulent flow: $U = \frac{n+1}{n}\overline{U}(1 - \frac{2y}{h})^{1/n}$

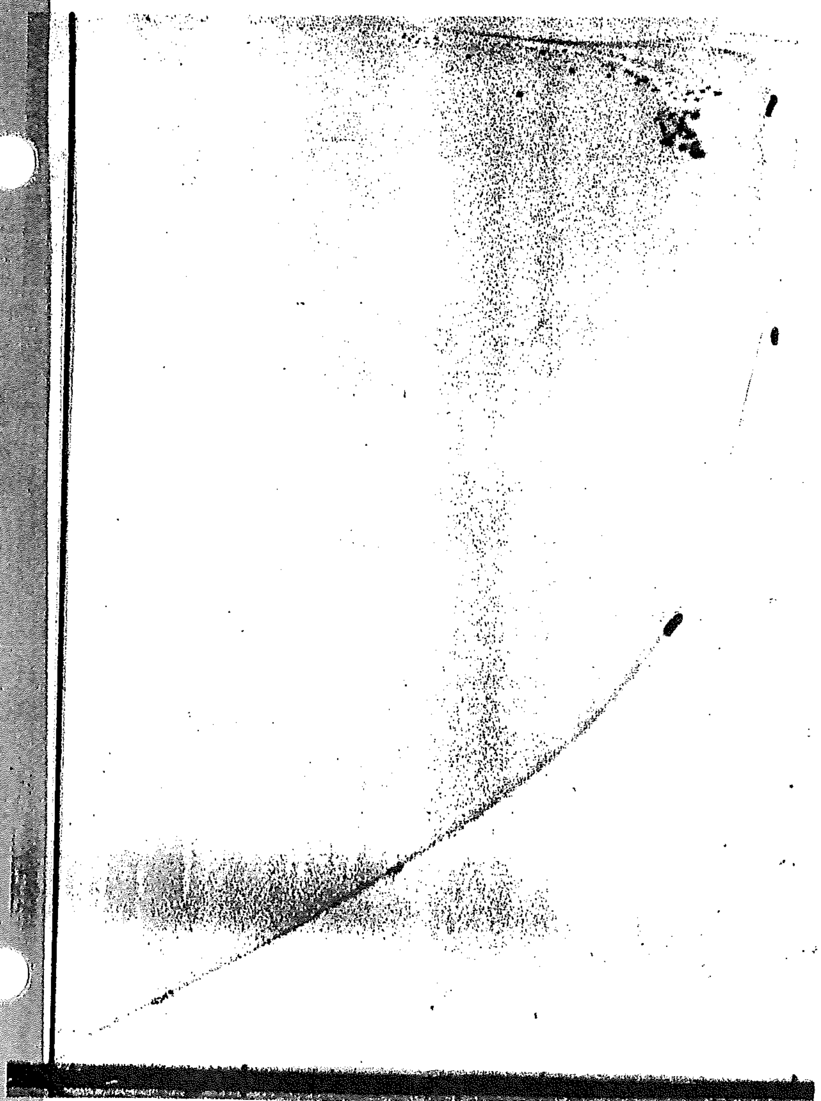
n=7 for $\mathrm{Re} = \rho \overline{U} h / \mu = 500{,}000$.

Slope $(\partial U / \partial y)_{wall}$ much bigger for turbulent flow as compared to the laminar profile.
Wall shear stress $\tau_w = \mu(\partial U / \partial y)_{y=0}$ much bigger for turbulent flow as compared to laminar.
Friction portion of drag is bigger for turbulent flow

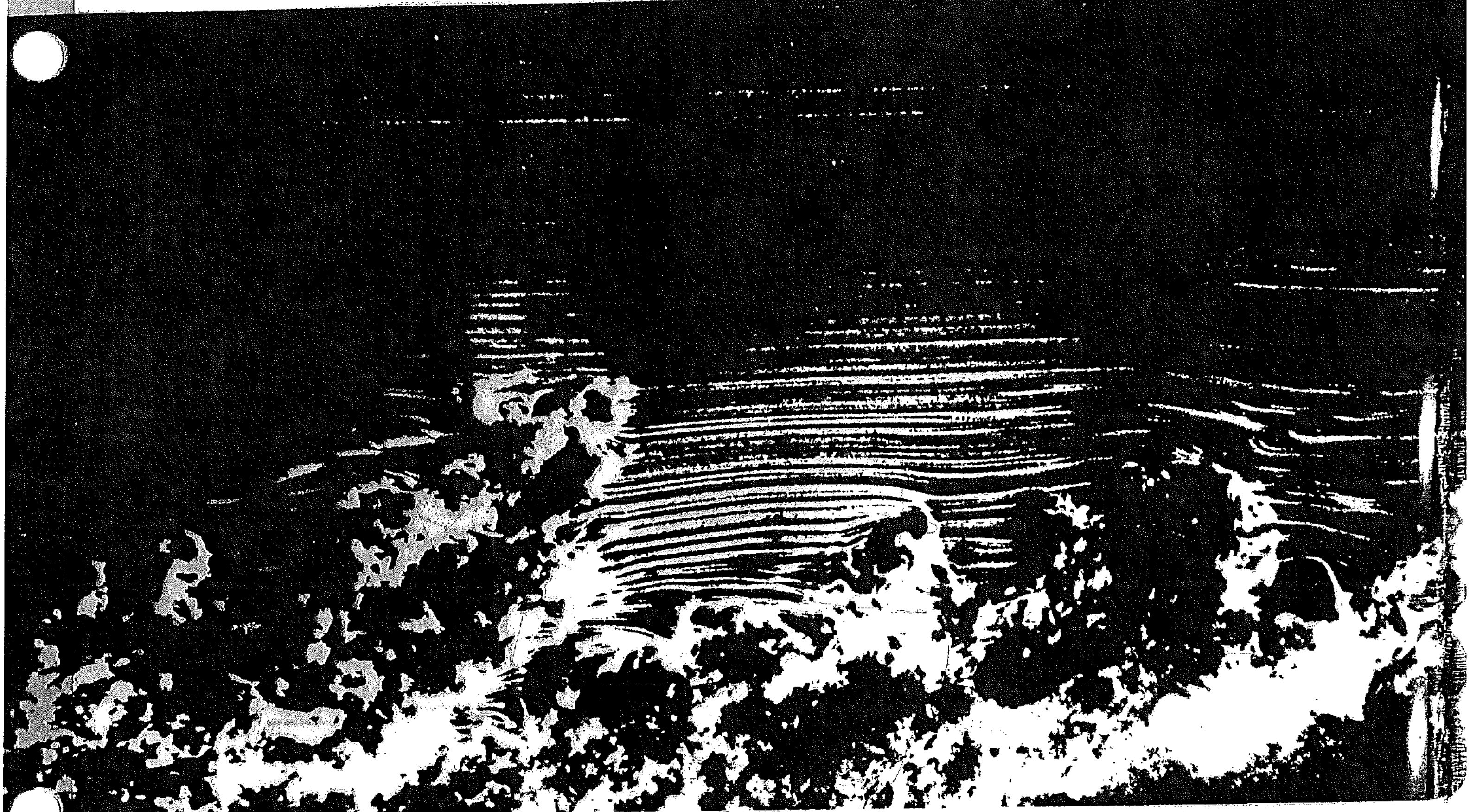
Velocity profile for turbulent flow is flatter that for laminar flow. The turbulent motions spread the momentum much more across the flow.
More momentum near the surface in turbulent flows.
Turbulent flows resist separation better than laminar flow

**29. Flat plate at zero incidence.** The plate is 2 per cent thick, with beveled edges. At this Reynolds number of 10,000 based on the length of the plate, the uniform stream is only slightly disturbed by the thin laminar boundary layer and subsequent laminar wake. Their thickness is only a few per cent of the plate length, in agreement with the result from Prandtl's theory that the boundary-layer thickness varies as the square root of the Reynolds number. Visualization is by air bubbles in water. *ONERA photograph, Werlé 1974*



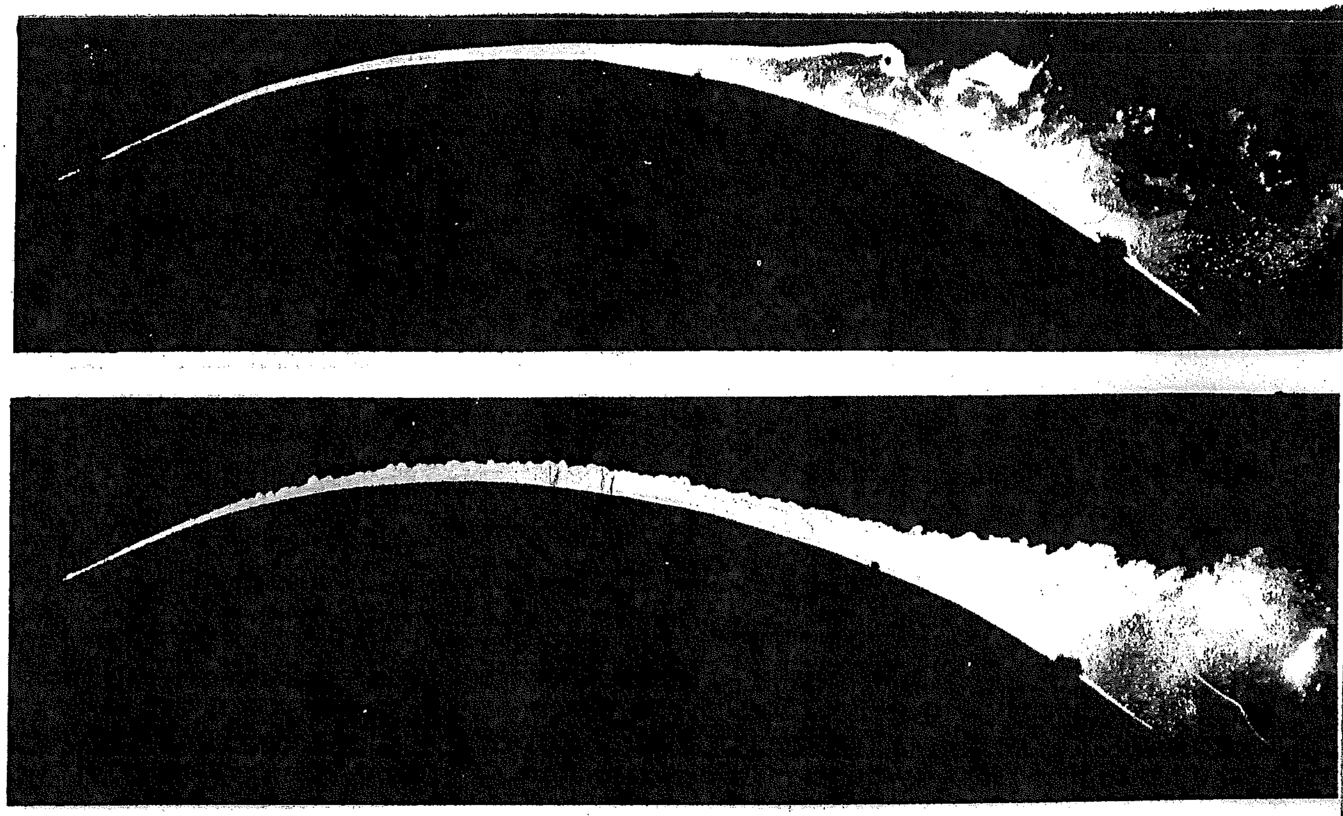


**30. Blasius boundary-layer profile on a flat plate.** The tangential velocity profile in the laminar boundary layer on a flat plate, discovered by Prandtl and calculated accurately by Blasius, is made visible by tellurium. Water is flowing at 9 cm/s. The Reynolds number is 500 based on distance from the leading edge, and the displacement thickness is about 5 mm. A fine tellurium wire perpendicular to the plate at the left is subjected to an electrical impulse of a few milliseconds duration. A chemical reaction produces a slender colloidal cloud, which drifts with the stream and is photographed a moment later to define the velocity profile. *Photograph by F. X. Wortmann*



**157. Side view of a turbulent boundary layer.** Here a turbulent boundary layer develops naturally on a flat plate 3.3 m long suspended in a wind tunnel. Streaklines from a smoke wire near the sharp leading edge are illuminated by a vertical slice of light. The Reynolds number is 3500 based on the momentum thickness. The intermittent nature of the outer part of the layer is evident. *Photograph by Thomas Corke, Y. Guezennec, and Hassan Nagib.*

**158. Turbulent boundary layer on a wall.** A fog of tiny oil droplets is introduced into the laminar boundary layer on the test-section floor of a wind tunnel, and the layer then tripped to become turbulent. A vertical sheet of light shows the flow pattern 5.8 m downstream, where the Reynolds number based on momentum thickness is about 4000. *Falco 1977*



**156. Comparison of laminar and turbulent boundary layers.** The laminar boundary layer in the upper photograph separates from the crest of a convex surface (cf. figure 38), whereas the turbulent layer in the second photograph remains attached; similar behavior is shown below for a sharp corner. (Cf. figures 55-58 for a sphere.) Titanium tetrachloride is painted on the forepart of the model in a wind tunnel. *Head 1982*