Voltmeters

![Diagrams](image)

**Voltmeter Circuit**

- $I_c$ - F.S. current
- $V$ - F.S. voltage
- $R_m$ - multiplier resistance
- $R_c$ - D'Arsonval coil resistance

Apply Kirchhoff's voltage law:

$$ V - R_c I_c - R_m I_c = 0 $$

$$ V = (R_c + R_m) I_c $$

If $I_c$ is measured and $R_c$ and $R_m$ are known, $V$ can be calculated.
Example 6.04: A very good 10 volt, 1000 Ω/volt voltmeter and a cheap 10 volt, 10 Ω/volt voltmeter are available. In an attempt to evaluate the emf of a battery whose internal resistance is 18 Ω, the good voltmeter is connected and reads 9.00 volts. If the scale readings of both voltmeters are accurately calibrated (a rash assumption probably for the cheap voltmeter), what should be the reading on the cheap voltmeter? What is the loading error for the cheap voltmeter?

\[
\begin{align*}
\text{Good Voltmeter (10 v full scale)} \\
R_c + R_m = (1000 \, \Omega/v) \times 10 \, v = 10,000 \, \Omega
\end{align*}
\]
current through voltmeter

\[ I = \frac{V_{\text{meter}}}{R_{\text{meter}}} = \frac{9 \text{v}}{10000 \Omega} = 9 \times 10^{-4} \text{A} \]

Battery emf

\[ V_B = E - R_i I \]

\[ E = V_B + R_i I = 9 \text{v} + 18 \Omega \times (9 \times 10^{-4} \text{A}) \]

\[ E = 9.016 \text{v} \]

Cheap Voltmeter (10v full scale)

\[ R_c + R_m = \left( \frac{10 \text{v}}{0.2 \text{v}} \right) \times (10 \text{v}) = 100 \Omega \]

\[ I = \frac{E}{R_i + (R_c + R_m)} = \frac{9.016 \text{v}}{18 + 100} = 0.07641 \text{A} \]

Measured Voltage

\[ V_{\text{meter}} = (R_c + R_m) I = 100 \Omega \times (0.07641 \text{A}) \]

\[ = 7.641 \text{v} \]

Loading Error

\[ e_l = 9.016 \text{v} - 7.641 \text{v} = 1.375 \text{v} \]

\[ \% \text{ error} = 15.2 \% \]
Wheatstone Bridge

The Wheatstone bridge is used for accurate measurements of resistance.

![Wheatstone Bridge Circuit](image)

Figure 6.14 Basic Wheatstone bridge circuit
(G, galvanometer).

Wheatstone Bridge circuit

The circuit requires 3 known resistors, $R_1$, $R_2$, and $R_3$ at least one of which is variable, $R_3$. $R_4$ is the unknown resistance.

All 6 currents $I_1$, $I_2$, $I_3$, $I_4$, $I_5$ and $I_g$ can be non-zero for an unbalanced bridge, but for some value
of \( R_3 \), the adjustable resistor, the current \( I_q \) can be made zero; called a balanced bridge.

Then,

\[ I_1 = I_2 \]
\[ I_3 = I_4 \]

By Kirchhoff's law, when \( I_q = 0 \)

\[ -I_2 R_2 + I_4 R_4 = 0 \]
\[ -I_1 R_1 + I_3 R_3 = 0 \]

**Algebra**

\[ -I_1 R_2 + I_3 R_4 = 0 \]

\[ I_1 R_2 = I_3 R_4 \]

\[ I_1 R_1 = I_3 R_3 \]

**Divide**

\[ \frac{R_2}{R_1} = \frac{R_4}{R_3} \]

\[ R_4 = \left( \frac{R_2}{R_1} \right) R_3 \]

Adjust \( R_3 \) until \( I_q = 0 \).
of $R_3$, the adjustable resistor, the current $I_g$ can be made zero; called a balanced bridge. Then,

$$I_1 = I_2$$
$$I_3 = I_4$$

By Kirchhoff's law, when $I_g = 0$

$$-I_2 R_2 + I_4 R_4 = 0$$
$$-I_1 R_1 + I_3 R_3 = 0$$

Algebra

$$-I_1 R_2 + I_3 R_4 = 0$$

$$I_1 R_2 = I_3 R_4$$

$$I_1 R_1 = I_3 R_3$$

Divide

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$R_4 = \left(\frac{R_2}{R_1}\right) R_3$$

Adjust $R_3$ until $I_g = 0$
Deflection Method

Replace the galvanometer by a voltmeter having infinite internal impedance; \( R_m \rightarrow \infty \).

Measure the voltage drop from B to C, assuming an unbalance bridge; \( E_m = E_o + \Delta \). There is no current flow through the voltmeter.

Write Kirchhoff voltage equations

Loop 1: \( E_o + I_3 R_3 - I_1 R_1 = 0 \)

Loop 2: \( E_i - I_3 R_3 - I_4 R_4 = 0 \)

Since \( I_m = 0 \),

\[ I_1 = I_2 \]

\[ I_3 = I_4 \]

\[ E_o = \frac{I_1 R_1 - I_3 R_3}{I_3 R_3 + I_4 R_4} \]

\[ E_i = \frac{I_3 R_3}{I_3 R_3 + I_4 R_4} \]

\[ = \frac{I_1 R_1}{I_3 R_3 + I_4 R_4} - \frac{I_3 R_3}{I_3 R_3 + I_4 R_4} \]
But

\[ E_i = I_3 R_3 + I_4 R_4 = I_1 R_1 + I_2 R_2 \]

\[
\frac{E_0}{E_i} = \frac{I_1 R_1}{I_1 R_1 + I_2 R_2} - \frac{I_3 R_3}{I_3 R_3 + I_4 R_4}
\]

since \( I_1 = I_2 \) and \( I_3 = I_4 \)

\[ E_0 = E_i \left( \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) \]

Assume the bridge is initially balanced.

One of the resistances changes as a result of a change in a measured variable. For example, the resistance of an RTD changes because of a change in temperature. This will cause a deflection in the bridge voltage away from the balanced condition; i.e., \( E_0 = 0 \).

Suppose \( R_1 \) changes to a new value \( R'_1 \)

\[ R'_1 = R_1 + 8R \]
\[ E_0 + SE_0 = E_i \left( \frac{R_1'}{R_1' + R_2} - \frac{R_3}{R_3 + R_4} \right) \]

\[ = E_i \left( \frac{R_1'(R_3 + R_4) - R_3(R_1' + R_2)}{(R_1' + R_2)(R_3 + R_4)} \right) \]

\[ SE_0 = E_i \left( \frac{R_1'R_4 - R_3R_2}{(R_1' + R_2)(R_3 + R_4)} \right) \]

Take the initial resistances to be equal

\[ R_1 = R_2 = R_3 = R_4 = R \]

\[ \frac{SE_0}{E_i} = \frac{(R + SR)R - R^2}{(R + SR + R)(2R)} \]

\[ = \frac{(SR)R}{4R^2 + 2R(SR)} \]

\[ \frac{SE_0}{E_i} = \frac{SR/R}{4 + 2(SR/R)} \]

Measure \( SE_0 \), and calculate \( SR \).
Method requires an accurate voltmeter and a stable input voltage \( E_0 \).

Examples 6.2 & 6.3