Uncertainty Analysis

Now we will use what we learned in Chap. 4 to estimate the uncertainty of actual measurements.

Remember that errors can be divided into two categories, bias and precision errors. The true value of a quantity is related to the mean of several measurements by:

$$x' = \bar{x} \pm U_x (P\%)$$

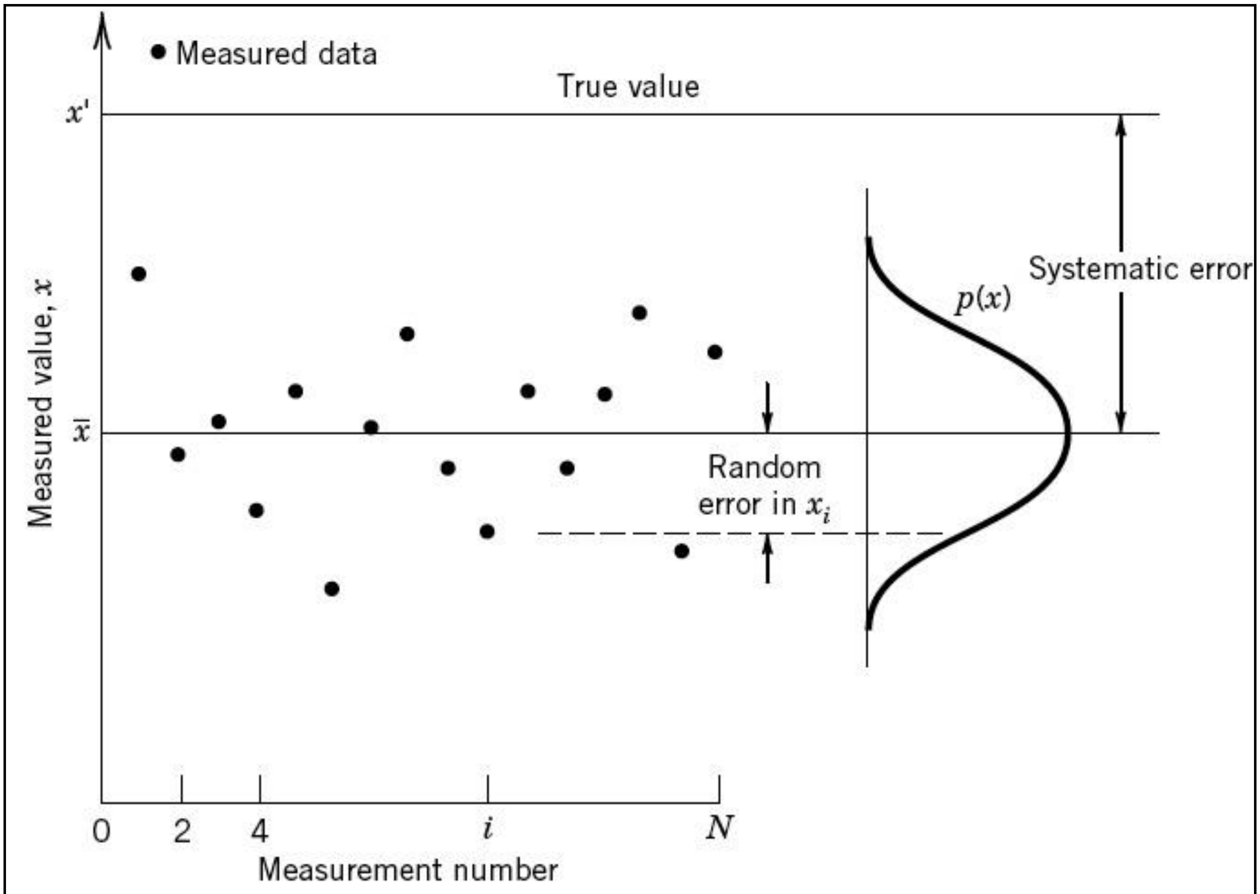


Figure 5.1. Distribution of errors upon repeated measurements.

Zero Order Uncertainty

All errors except instrument resolution are perfectly controlled.

$$u_0 \equiv \pm \frac{1}{2} \text{resolution } (95\%)$$

Instrument uncertainty, u_c, is an estimate of the systemic error.

Table 1.1 Manufacturer's Specifications: Typical Pressure Transducer

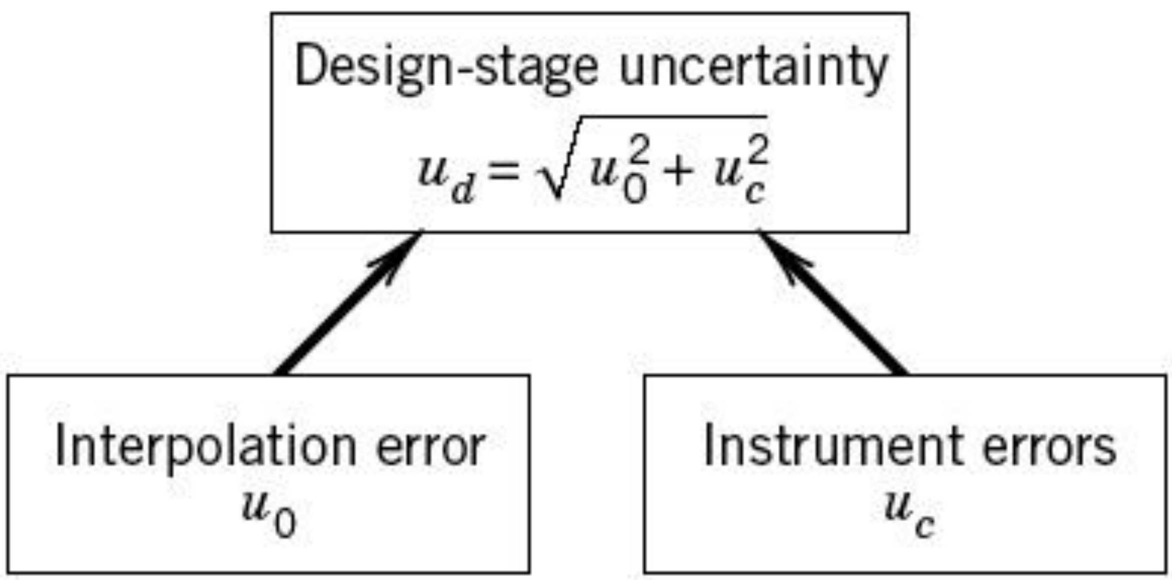
Operation	
Input range	0–1000 cm H_2O
Excitation	±15 V dc
Output range	0–5 V
Performance	
Linearity error	±0.5% FSO
Hysteresis error	Less than ±0.15% FSO
Sensitivity error	±0.25% of reading
Thermal sensitivity error	±0.02% /°C of reading
Thermal zero drift	±0.02% /°C FSO
Temperature range	0–50 °C

Combining Elemental Errors: RSS Method

$$u_x = \pm\sqrt{e_1^2 + e_2^2 + \ldots + e_k^2}$$
$$= \pm\sqrt{\sum_{j=1}^{K} e_j^2}\ (P\%) \qquad (5.2)$$

As a general rule *P* = 95% is used throughout all uncertainty calculations. Remember ±2δ accounts for about 95% of a normally distributed data set!

Design-Stage Analysis



Ex: Spa temperature regulation using a 3 digit voltmeter and thermocouple.

Ex: What is the smallest zero-order uncertainty, u_0, obtainable with the ADC used in our lab?

Error Sources

Errors can arise from three sources:

- Calibration
- Data Acquisition
- Data Reduction

TABLE 5.1 Calibration Error Source	
Element (j)	Error Source[a]
1	Primary to interlab standard
2	Interlab to transfer standard
3	Transfer to lab standard
4	Lab standard to measurement system
5	Calibration technique
Etc.	

[a]Bias and/or precision in each element.

TABLE 5.2 Data Acquisition Error Source Group	
Element	**Error Source[a]**
1	Measurement system operating conditions
2	Sensor-transducer stage (instrument error)
3	Signal conditioning state (instrument error)
4	Output stage (instrument error)
5	Process operating conditions
6	Sensor installation effects
7	Environmental effects
8	Spatial variation error
9	Temporal variation errer
Etc.	
[a]Bias and/or precision in each element	

TABLE 5.3 Data Reduction Error Source Group	
Element (j)	**Error Source[a]**
1	Calibration curve fit
2	Truncation error
Etc.	
[a]Bias and/or precision in each element.	

Error Propagation

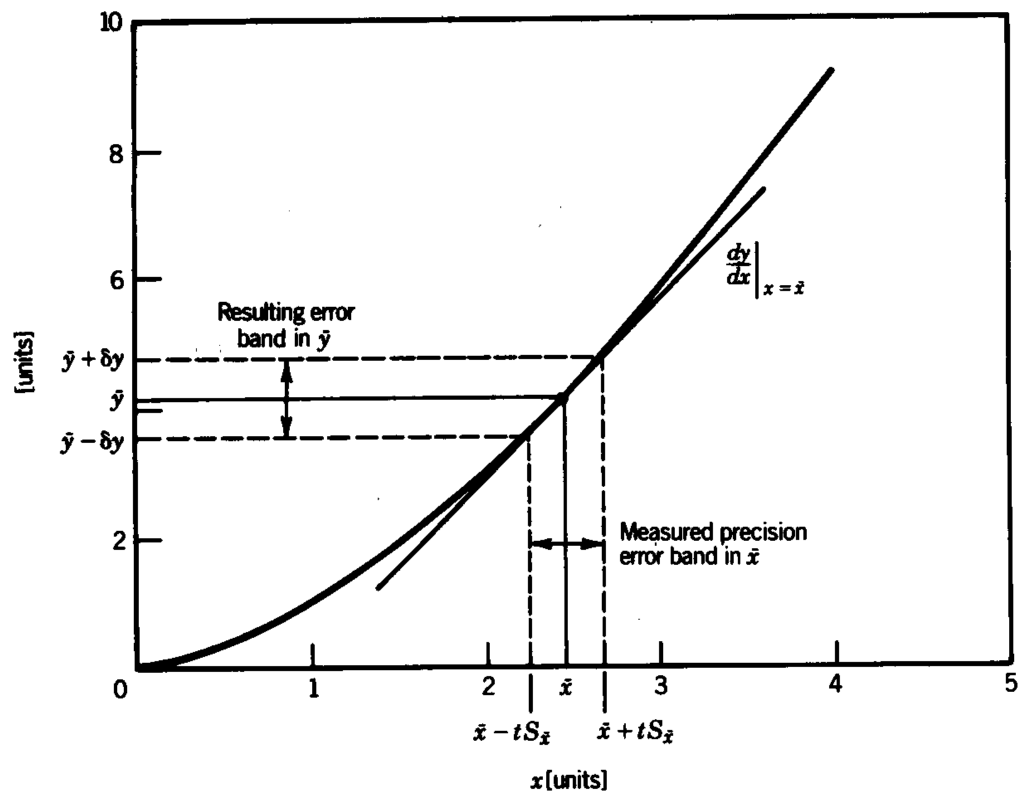
Most measurements are subject to more than one type of error. We need to estimate the cumulative effect of these errors. It is unlikely that all of the errors will be in one direction - more likely there will be some cancellation. The root-sum-squares (RSS) approximation is a good estimate:

$$U_x = \pm \sqrt{e_1^2 + e_2^2 + \ldots + e_k^2} = \pm \sqrt{\sum_{j=1}^{K} e_j^2} \; (P\%) \qquad (5.1)$$

Since the overall result may be more sensitive to some errors than to others, we need to consider the functional relationships between the output and the various inputs.

Error Propagation Continued

FIGURE 5.2 Relationship between a measured variable and a resultant.



The uncertainty in the dependent variable will be related to the uncertainty in the independent variable by the slope of the curve.

$$u_y = \left(\frac{dy}{dx}\right)_{x=\bar{x}} u_x \qquad (5.2)$$

(5.5 in 2nd Edition)

If we have more than one independent variable

$$R = f_1\{x_1, x_2, ..., x_L\} \qquad (5.3)$$

(5.6 in 2nd Edition)

The true mean R' can be obtained from the sample mean $\bar{R}$ with a precision ± u_R

$$R' = \bar{R} \pm u_R (P\%) \qquad (5.4)$$

(5.7 in 2nd Edition)

where

$$\bar{R} = f_1\{\bar{x}_1, \bar{x}_2, ... \bar{x}_L\} \qquad (5.5)$$

(5.8 in 2nd Edition)

and

$$u_R = f_2\{u_{x_1}, u_{x_2}, ... u_{x_L}\} \qquad (5.6)$$

(5.9 in 2nd Edition)

In order to account for the different sensitivities of the measurement to different inputs, we define a sensitivity index:

$$\theta_i = \frac{\partial R}{\partial x_i}\Big|_{x=\bar{x}_i} \quad i = 1, 2, ..., L \qquad (5.7)$$

(5.10 in 2nd Edition)

and thus

$$u_R = \pm \sqrt{\sum_{i=1}^{L} (\theta_i u_{x_i})^2} \; (P\%) \qquad (5.8)$$

(5.11 in_ 2nd Edition)

Design Stage Example

Example 5.3

A voltmeter is to be used to measure the output from a pressure transducer that outputs an electrical signal. The nominal pressure expected will be about 3 psi (3 lb/in.^{2}). Estimate the design-stage uncertainty in this combination. The following information is available:

Voltmeter	
Resolution:	**10 µV**
Accuracy:	**within 0.001% of reading**
Transducer	
Range:	**± 5 psi**
Sensitivity:	**1 V/psi**
Input power:	**10 VDC ± 1%**
Output:	**± 5 V**
Linearity:	**within 2.5 mV/psi over range**
Repeatability:	**within 2 mV/psi over range**
Resolution:	**negligible**

KNOWN

Instrument specification

Assumptions

Values representative of instrument at 95% probability

Find

u_o for each device and u_d for the measurement system

Design Stage Example

Solution

The procedure in Figure 5.3 will be used for both instruments to estimate the design stage uncertainty in each. The resulting uncertainties will then be combined using the RSS approximation in estimate the system u_d.

The uncertainty in the voltmeter at the design stage is given by equation 5.10 as (5.17 in the second edition)

$$(u_d)_E = \pm\sqrt{(u_o)_E^2 + (u_c)_E^2}$$

From the information available,

$$(u_o)_E = \pm 5\,\mu V\ (95\%)$$

For a nominal pressure of 3 psi, we expect to measure an output of 3V. Then

$$(u_c)_E = \pm\ (3V\ X\ 0.001\%) = \pm\ 30\mu V\ \ (95\%\ assumed)$$

so that the design-stage uncertainty in the voltmeter is

$$(u_d)_E = \pm\sqrt{5^2 + 30^2} = 30.4\ \mu V \quad (95\%)$$

The uncertainty in the pressure transducer output at the design stage is also given by (5.10). Assuming that we operate within the input power range specified, the instrument output uncertainty can be estimated by considering each of the instrument elemental errors of linearity, e_1, and repeatability, e_2:

$$(u_c)_p = \sqrt{e_1^2 + e_2^2}\ (95\% assumed)$$

$$= \pm\sqrt{(2.5mV/psi\ \ x\,3psi)^2 + (2mV/psi\ \ x\,3psi)^2}$$

$$= \pm 9.61mV(95\%)$$

Since (U_o) negligable, ≈(0) V/psi, then the design-stage uncertainty in pressure in terms of indicated voltage is $(u_d)_p$ = ±9.61 mV (95%). But since the sensitivity is 1 V/psi, this uncertainty can be stated as $(u_d)_p$ = ±0.0096 psi (95%).

Finally, u_d for the combined system is found by use of the RSS method using the design-stage uncertainties of the two devices. The design-stage uncertainty in pressure as indicated by this measurement system is estimated to be

$$U_d = \sqrt{(U_d)_E^2 + (U_d)_P^2}$$
$$= \pm\sqrt{0.030mV)^2 + (9.61mV)^2} \quad (95\%)$$
$$= \pm 10.06mV(95\%) \quad or \quad \pm\ 0.010\ psi\ (95\%)$$