Signal Characteristics

Analog Signals

Analog signals are always continuous (there are no time gaps). The signal is of infinite resolution.

Discrete Time Signals

Figure 2.2 Analog signal concepts.

Figure 2.3 Discrete time signal concepts.
**Discrete Time Signals:** Information about the signal magnitude is available only at discrete points in time

**Sampling:** The process of obtaining discretized information from a continuous variable at finite time intervals.
Table 2.1  Classification of Waveforms

I. Static  \[ y(t) = A_0 \]

II. Dynamic  
- Periodic waveforms
  - Simple periodic waveform  \[ y(t) = A_0 + C \sin(\omega t + \phi) \]
  - Complex periodic waveform  \[ y(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n) \]
- Aperiodic waveforms
  - Step\(^a\)  \[ y(t) = A_0 U(t) \]
    \[ = A_0 \quad \text{for } t > 0 \]
  - Ramp  \[ y(t) = Kt \quad \text{for } 0 < t < t_f \]
  - Pulse\(^b\)  \[ y(t) = A_0 U(t) - A_0 U(t - t_1) \]

III. Nondeterminisitic waveform  \[ y(t) \approx A_0 + \sum_{n=1}^{\infty} C_n \sin(\omega_n t + \phi_n) \]

---

![Figure 2.5 Examples of dynamic signals.](image_url)

---
Characterization of Signals

A - Deterministic

1. Static

2. Dynamic

a. Simple periodic waveform

b. Complex periodic waveform

B – Random (stochastic)
A – Deterministic (cont)

2. Dynamic (cont)

c. Aperiodic waveform

Step

Ramp

Pulse
Characterization of Signals

B – Random (stochastic)
Signal Characteristics: Definitions

Magnitude - generally refers to the maximum value of a signal

Range - difference between maximum and minimum values of a signal

Amplitude - indicative of signal fluctuations relative to the mean

Frequency - describes the time variation of a signal

Dynamic - signal is time varying

Static - signal does not change over the time period of interest

Deterministic - signal can be described by an equation (other than a Fourier series or integral approximation)

Non-deterministic - describes a signal which has no discernible pattern of repetition and cannot be described by a simple equation.

Mean - average or static portion of a signal over the time of interest. Sometimes call the dc component or the dc offset of the signal [Excel tip: Mean = AVERAGE(numbers...)]

RMS - root-mean-square - average value of the square of the signal over the time of interest. [Excel tip: RMS = SQRT(SUMSQ(numbers1 to n)/n)]
Signal Analysis

Average or Mean Value

Provides a measure of the static portion of a signal over a period of time.

\[
y = \frac{\int_{t_1}^{t_2} y(t) dt}{\int_{t_1}^{t_2} dt}
\]

This is often referred to as the \textit{dc component} or \textit{dc offset}.

Fluctuating or AC Component

The dynamic portion of a signal, \( y \), is characterized by the various measures of the magnitude and the amount of fluctuation. One such characterization is the \textit{rms value}, or root mean square.

\[
y_{rms} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} y^2 dt}
\]

The fluctuating portion alone is often characterized by the a term called the \textit{variance}, \( \sigma^2 \), or the square root of the variance the \textit{standard deviation}, \( \sigma \).

\[
\sigma^2 = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \left[ y(t) - y' \right]^2 dt
\]

Where, \( y' \), is the true mean value of the signal.
Figure 2.7 Effect of time period on mean value for a nondeterministic signal.

Figure 2.8 Effect of subtracting dc offset for a dynamic signal.
Digital Sampling

Quantization: The truncation of an analog signal to a finite resolution.

A/D Resolution: is defined as the smallest voltage increment that will cause the digital output to change (a change in the least significant bit of the ADC output value)

A/D Resolution is a function of the ADC digital output word size and the analog input voltage range.

The input voltage range, $E_{FSR}$, is divided into $2^M$ equal increments where $M$ is the number of bits in the ADC output word.

$$Q = \frac{E_{FSR}}{2^M}$$

Lab ADC:
- 12 bit digital output word size
- 20 Volt analog input range
- Variable input signal gain

Gain: an signal amplification factor. If an input signal is amplified by a factor the effective $E_{FSR}$ is changed.

$$Q = \frac{E_{FSR}}{\text{Gain}} \times 2^M$$

Lab ADC:

$$Q = \frac{20V}{\text{Gain}} \times 2^{12}$$
Lab ADC Gain Values:
1, 2, 5, 10, 100 & 200

Lab ADC Example, gain=200: \( Q = \frac{20V}{200/4096} = 0.0000244 \text{ V} \)

Example: \( E_{\text{FSR}} = 4 \text{ V}, M = 2, Q = \frac{4}{2^2} = 1 \text{ V} \)

Figure 7.7 Binary quantization and saturation.
**Quantization Error**

Error = (ADC Output - True Value)/True Value

Example: From figure 7.7 if the input value is 1.5 what is the percent error?

Error = (1 - 1.5)/1.5 = -1/3 V or -33%

Error can range from 0 to 1 volt or the \( e_Q = Q \)

**Input voltage shifting** is used to minimize the error by adding a bias of \( Q/2 \). Think of this as a change from truncation to rounding off. The error then becomes

\[
e_Q = \pm \frac{1}{2}Q
\]

**ADC Signal-to-noise ratio**: (SNR) relates the power of the signal (Ohm's Law \( E^2/R \)) to the power of the smallest signal change, \( Q \), that can be detected \( E^2/R2^M \)

The recording industry expresses the ADC SNR in decibels (dB):

\[
\text{SNR}[\text{dB}] = 20 \log 2^M
\]
**Aliasing**

How rapidly do we need to sample a signal in order to accurately describe it? The answer depends on the frequency content of the signal.

![Signal Characteristics Diagram](image)

(a) Original 10-Hz sine wave analog signal  
(b) $f_s = 100$ Hz  
(c) $f_s = 27$ Hz  
(d) $f_s = 12$ Hz

**Figure 7.2** The effect of sample rate on signal frequency and amplitude interpretation.
The sampling theorem provides the guideline. The sample rate must be high enough to sample at least two points per period of the highest frequency (sine wave) contained in the measured signal.
If the sampling rate is too low, aliasing will occur. That is, signals will be distorted (Fig. 6.32 b,c) and may appear to have frequencies lower than their actual value, as shown by Figure 6.32d, 7.2²,³. Notice that in the figure above it is impossible to distinguish between the two waveforms by sampling at the points shown.

As shown on p. 228¹, 273²,³ the maximum frequency component which can be accurately measured is the Nyquist frequency:

\[ f_N = \frac{f_s}{2} = \frac{1}{2 \delta t} \quad (6.63¹, 7.8²,³) \]

where \( \delta t \) is the sampling interval.

The A/D converters in the lab have a maximum sampling rate of approximately 100,000 Hz with the software we are using. Thus we can reproduce signals with frequencies to about 50,000 Hz. The converters have a resolution of 12 bits, so the quantization error is 1 part in \( 2^{12} \) or 1 part in 4096 of full scale. We will talk later about the electronic circuits which are used.
Figure 7.3 The folding diagram for alias frequencies.