Loading Errors

Consider again the voltage divider circuit:

\[ R_{eq} = R_2 + R_L = R_2 + \frac{R_1R_m}{R_1 + R_m} \quad (6.36^1, 6.32^{2,3}) \]
The output voltage $E_o$ is related to the input voltage $E_i$ by:

$$\frac{E_o}{E_i} = \frac{1}{1 + \left(\frac{R_2}{R_1}\right)\left(\frac{R_1}{R_m} + 1\right)} \quad (6.37^1, 6.35^{2,3})$$

We can define a loading error as the normalized difference between the output voltage with an infinite meter resistance and that with a finite value:

$$\frac{e_i}{E_i} = \left[\left(\frac{E_o}{E_i}\right)' - \left(\frac{E_o}{E_i}\right)\right]$$

$$\frac{e}{E_i} = \frac{R_1 - R_T + \left(R_T - R_1\right)\left(\frac{R_1}{R_m} + 1\right)}{R_T + \left(\frac{R^2_T}{R_1 - R_T}\right)\left(\frac{R_1}{R_m} + 1\right)}$$

(6.38^1, 6.37^{2,3})

Here $(E_o/E_i)'$ is the ideal value and $(E_o/E_i)$ is the actual value. (Notice the correction of the equation in the First Edition of the text). The value of $R_T/R_m$ is constant. Let's plot the error as a function of $R_1/R_T$ for various meter resistances.
For a meter resistance $R_m$ equal to the total resistance $R_T$, the error is a significant fraction of the signal. To get accurate measurements, we must therefore keep $R_m \gg R_T$.

When we consider the effects of loading, we must distinguish between measurement of effort variables such as voltage and measurement of flow variables such as current.
Effort Variable | Flow Variable
---|---
Voltage | Current
Pressure | Flow
Temperature | Heat Flux
Force | Velocity

We can define the input impedance of the instrument and the output impedance of the system. For the voltage divider circuit, the output impedance is $R_T$ and the input impedance of the meter is $R_m$.

For effort variables, the requirement is that the input impedance of the instrument should be much greater than the output impedance of the system.

**Figure 6.17 in 2nd and 3rd Edition**

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**Figure 6.20** Equivalent circuit formed by interstage (parallel) connections.

(a) Device 1  Device 2

(b) Equivalent circuit

Figure 6.17 in 2nd and 3rd Edition
For flow variables, the input impedance of the instrument should be much less than the output impedance of the system.

\[
\frac{e_1}{E_1} = \left( 1 - \frac{l}{1 + Z_1/Z_m} \right) \quad (6.39)
\]

If our instrument has more than one component, the same rules are applied to each stage, treating the output of one stage as the input of the next.
In designing an instrument, we want impedance mismatching. If we wanted to transfer a maximum amount of power, say to the speakers of a sound system, we would want impedance matching.

Let us consider an example which is not electrical, the measurement of the temperature of a cup of water using a thermometer.

\[ \text{Ambient Temp} = T_a \]

\[ \text{Initial Temp} = T_i \]

\[ \text{Final Temp} = T_f \]
An energy balance gives:

\[ C_t(T_f - T_o) = C_w(T_i - T_f) \]

where \( C \) is the heat capacity, and \( t \) refers to the thermometer and \( w \) to the water.

Here we will define the impedances as:

\[ Z_l = 1/C_w, \quad Z_m = 1/C_t \]

The error in the measurement \( e = (T_i - T_f) \) can be written:

\[ \frac{e}{T_i - T_o} = \frac{Z_l/Z_m}{1 + Z_l/Z_m} = 1 - \frac{1}{1 + Z_l/Z_m} \]

which is the same as Equation (6.39)