

The Five-Minute Filter University, July Session

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By: Ed Ramsden

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Back in the late 1970s comedian Don Novello (a.k.a. Father Guido Sarducci) had a routine called the "Five-Minute University," which was supposed to impart to you, in the span of only five minutes, all the knowledge you would retain five years after graduating from a regular university. So, in the same spirit, I offer "Dr. Ed's Five-Minute Analog Filter Design University."

What Filters Do

The purpose of a filter is to pass along signals you are interested in, while blocking or attenuating (reducing) the ones you aren't. Traditional filter design focuses on discriminating between wanted and unwanted signals on the basis of their frequencies. Although there are potentially lots of ways to sort signals by frequency, the two most common are to pass along only those signals lower than some arbitrary cut-off frequency (low-pass filters), or pass along signals higher than the cutoff frequency (high-pass filters), as shown in Figure 1. Corner frequency is also referred to as the filter's -3 dB frequency because the response drops -3 dB (a factor of ~ 0.707) here.

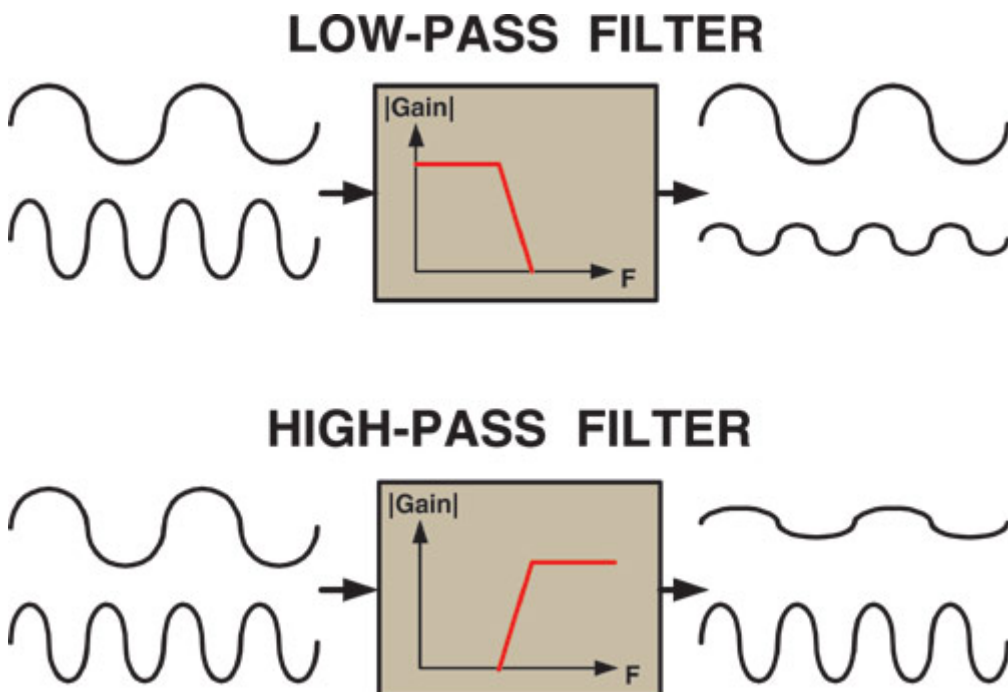


Figure 1. Low-pass filters pass along signals lower than some arbitrary cutoff frequency; high-pass filters pass signals higher than the cutoff frequency

For a real-world example of the difference between a low-pass and a high-pass filter, consider the bass and treble knobs on your stereo. The bass knob attenuates or enhances low-frequency sounds like drums or tubas, while the treble knob does the same for high-frequency sounds like flutes. The two main defining characteristics of a filter (among many) are the cutoff or corner frequency, and how well it discriminates against signals on the wrong side of that corner.

Now for Some Circuits

Two minutes in and we have already covered theory and applications! Now it's time to see some circuits. Many simple passive filters can be derived by starting from a resistive voltage divider (**Figure 2**). If you don't know how one of these works, I suggest you enroll in the "Five-Minute Circuit Theory" course and come back next semester. If you already took this course (and remember only $P = IV$) then I can tell you that a voltage divider's gain (ratio of output voltage to input voltage) is $R_2/(R_1+R_2)$.

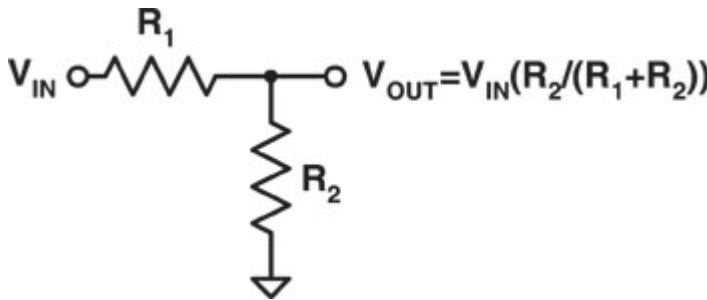


Figure 2. You can derive a passive filter by starting from a resistive voltage divider

To turn a voltage divider into a filter, you substitute a capacitor or inductor for one of the resistors. This results in a

voltage divider where the division ratio becomes frequency dependent. This is because the "resistance" of an inductor can be defined as $2\pi fLj$ and that of a capacitor can be defined as $1/2\pi fCj$. If this were a real filter design course we would now combine these expressions with the voltage divider equation to derive frequency-dependent expressions for various filters. Note that these expressions have imaginary values, however, which means complex math. Since this is the five-minute course (and we are three minutes in), let's just skip the math part and take a look at some rules of thumb to describe qualitative behavior.

Rules of Thumb—and Where It Gets Hard

- At DC (frequency = 0), capacitors are open circuit ($R = \text{infinity}$) and inductors are short circuits ($R = 0$)
- At infinitely high frequency, capacitors are short circuits ($R = 0$) and inductors are open circuits ($R = \text{infinity}$)
- Resistors are resistors at all frequencies

You can now consider the four cases shown in Figure 3 (page 34) and Figure 4.

Figure	R1	R2	F = 0 gain	F = infinity gain	Filter type?
XXXa	R	C	1	0	Low-pass
XXXb	R	L	0	1	High-pass
XXXc	C	R	0	1	High-pass
XXXd	L	R	1	0	Low-pass

Figure 3. Some quick-reference rules of thumb for filters

Note that we haven't considered the cases where you completely throw out the resistors and just use inductors and capacitors. If you use only inductors and only capacitors, then you get voltage dividers that work everywhere except at DC. If you use both, you wind up with a low-pass or high-pass filter with resonance, which vastly complicates the subject and is therefore taught in Five-Minute Grad School.

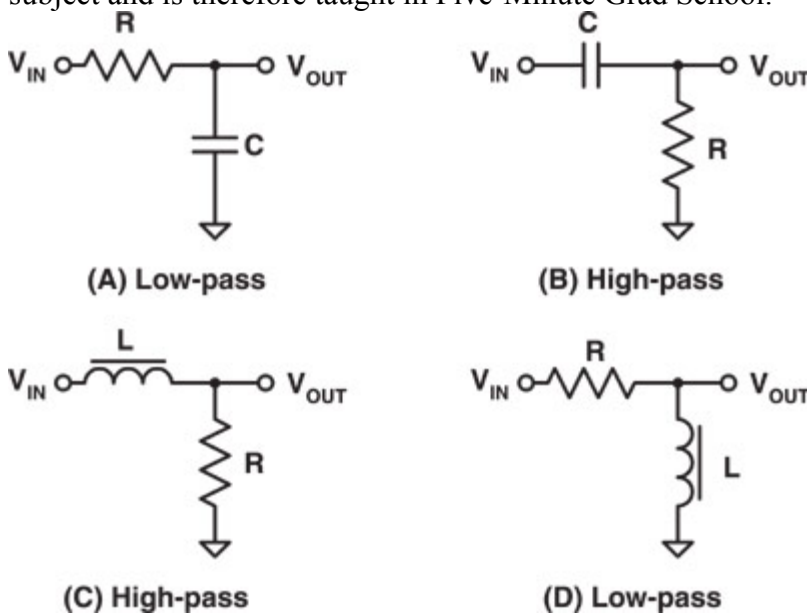


Figure 4. Schematics of the four examples given in Figure 3

Find the Corner Frequency

Now that we have low-pass and high-pass circuits, we need to be able to determine what the corner frequency is. For circuits with resistors and capacitors, the corner frequency is:

$$f_c = \frac{1}{2\pi RC}$$

For circuits with resistors and inductors: the corner frequency is given by:

$$f_c = \frac{1}{2\pi L/R}$$

Now that we know the corner frequencies of some filter designs, the other big question is how well they discriminate against unwanted signals. The answer is -20 dB/decade. This means that for every factor of 10 you vary the frequency outside the passband, the response also varies by a factor of 10. For example, if you have a low-pass filter that provides a gain of 0.1 to a 100 Hz signal, it will provide a gain of 0.01 to a 1000 Hz signal, attenuating it a factor of 10 more.

Quiz

Extra Final Exam Question:

What do you think the circuit in **Figure 5** does?

Answer: This is a low-pass filter, specifically called a pi filter. At DC, the capacitors appear as

open circuits and the inductor connects input to output. At high frequencies, the inductor opens up and the capacitors short to ground, preventing high-frequency signals from passing.

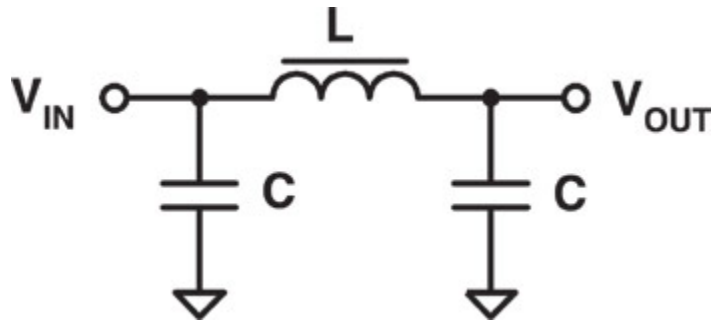


Figure 5. Quiz: What does this circuit do?

The Five-Minute Filter University—August Session

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Last month we discussed a number of simple passive filters in both low-pass and high-pass configurations. Although these filters could reject out-of-band signals, this capability was relatively limited because they all had an attenuation roll-off rate of -20 dB/decade. You will find that many applications require a much greater ability to reject out-of-band signals than that provided by the passive low-pass filters we looked at.

Try Cascading

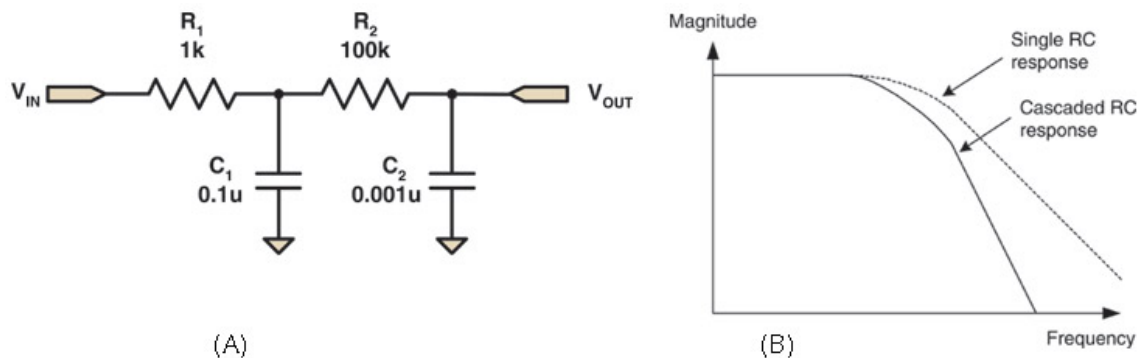


Figure 6. Cascading several passive filters together gives you better out-of-band signal rejection (A); adding a second stage increases the rate at which the response drops off (B)

One obvious way to get better out-of-band rejection is to cascade a number of passive filters together (**Figure 1A**). The addition of a second stage dramatically increases the rate at which the response drops off (**Figure 1B**) compared to that of a single RC filter. One drawback of this circuit, though, is that the second filter stage loads down the first, changing its frequency response. One solution is to increase the impedance of the second

stage so as to minimize these loading effects. In the example shown, the way to do this is to increase R_2 by a factor of 100 to 100 k and correspondingly reduce C_2 to $0.001 \mu\text{F}$.

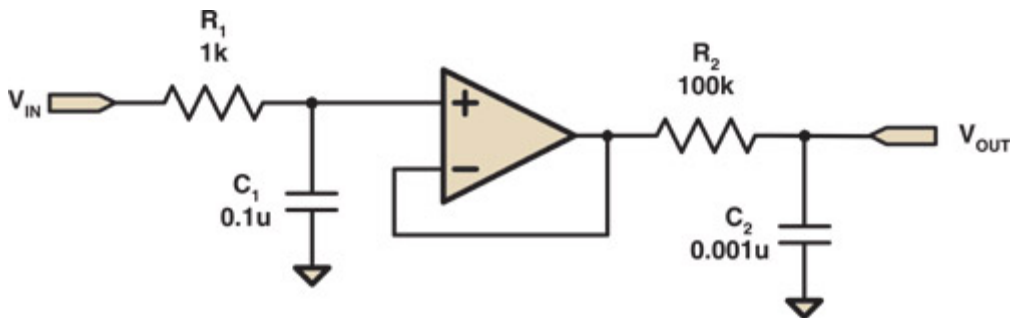


Figure 7. To keep the later filter stages from loading down the earlier ones, try putting a unity gain amplifier between them

Interposing a unity gain amplifier between filter stages (**Figure 2**) is another way to keep the later ones from loading the earlier ones. This scheme has the advantage of almost completely eliminating loading effects on the first stage. You can also see that it allows you to use comparable impedance levels. A final benefit is that it provides the ability to cascade an unlimited number of stages.

Why You Need an Amplifier

One of the major disadvantages of both the buffered and unbuffered filters described above is that as you add stages, the transition from the pass band (where signals are transmitted) to the stop band (where signals are blocked) gets less and less distinct. This reduces a filter's effectiveness at discriminating among signals with slightly different frequencies. If you take a look back at Figure 1A, you will notice that this transition region, the "knee" of the response curve, is softer in the case of two filter sections compared to the case of the single RC section. While the ultimate attenuation rate of the more complex filters will increase as you get further and further into the stop band, their ability to distinguish among signals of similar frequency near the corner frequency may not improve very much at all.

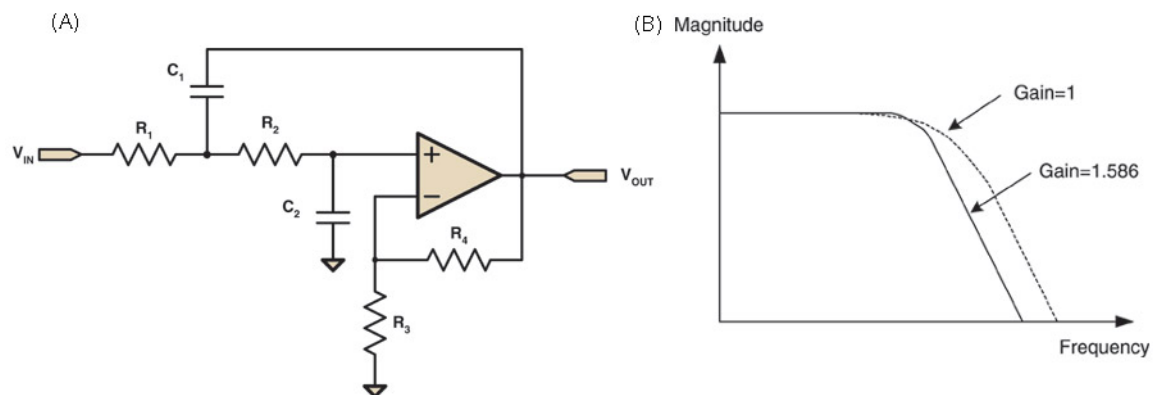


Figure 8. Using an amplifier to provide a gain of >1 for feedback sharpens up the filter response (A); if you set the gain set to larger values, the feedback loop will tend to make the response sharper (B)

By using an amplifier to provide feedback, as opposed to merely buffering signals, you can significantly sharpen up the filter's response. The first change is to set up the amplifier so that it provides a gain of >1 (**Figure 3A**). You do this by the addition of R3 and R4, and the resulting gain is $G = 1 + R4/R3$. The next step is to use one of the capacitors (C1) to provide feedback, as opposed to just tying it to ground.

Where It Gets Interesting

At DC, the capacitors effectively drop out of the circuit, and the output is simply the gained-up version of the input. Note that the DC gain of this circuit is not unity, but is set by R3 and R4. At very high frequencies the capacitors short out, and the output is zero. The interesting part occurs near the corner frequency. One terminal of C1 is driven by the amplifier output. Because of the amplifier's gain, and because R2 and C2 can provide phase shift, the voltage across C1 can be different, both in terms of phase and magnitude, from the input voltage. If you set the amplifier gain to equal 1 (leave out R3), the overall response of this circuit will be similar to that of the circuit of Figure 2. When you set the gain to larger values, however, the feedback loop will tend to make the response sharper (**Figure 3B**). While you can pick any gain value you care to (really large ones will make the filter unstable), certain values result in filters with well-understood characteristics. For example, using a gain of 1.586 results in a Butterworth filter with a maximally flat pass band and a corner frequency of $\frac{1}{2} RC$.

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