BEARINGS — allow relative motions in some direction

① Support load

② Reduce Friction

③ Avoid, reduce wear

④ Minimize heating

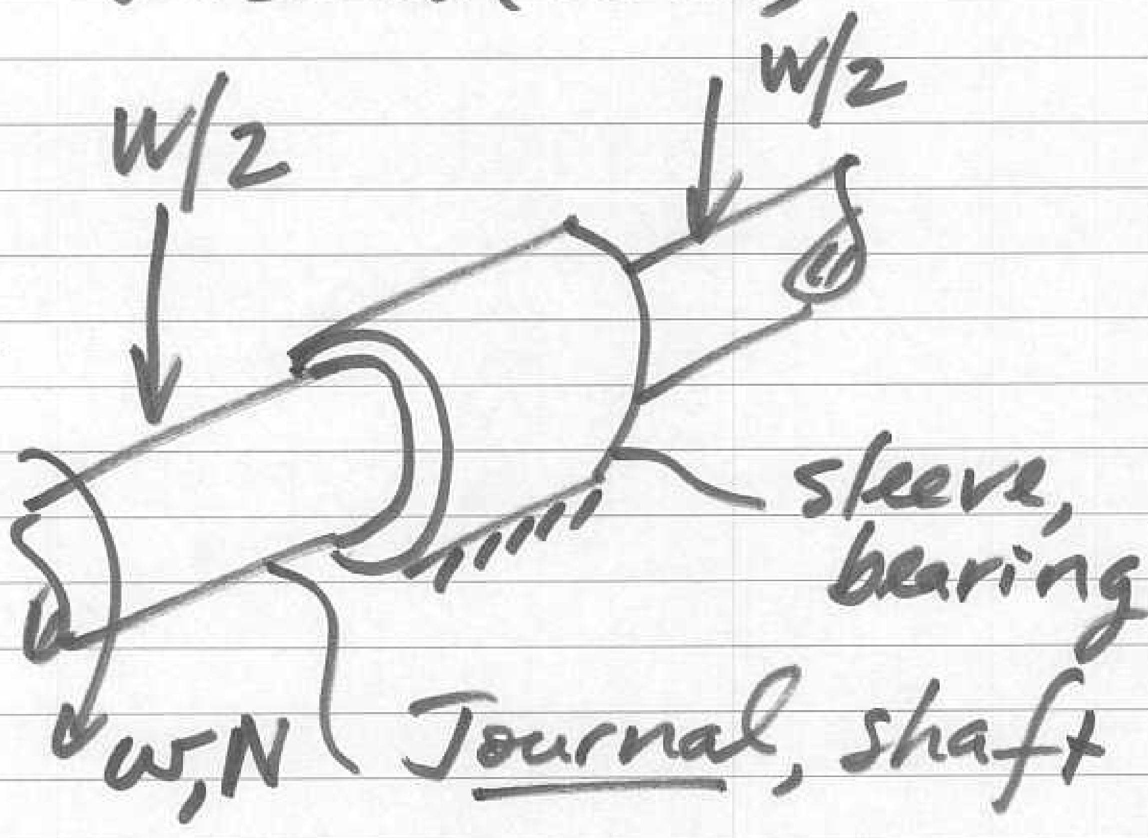
Sliding bearings Ch12

Rolling bearings Ch 11

Support radial or axial (thrust) loads

Journal bearing

- automotive crankshafts
- electric motors
- turbines (some)



TYPES OF LUBRICATION

HYDRODYNAMIC

- relatively Thick fluid films (0.0001 in $\rightarrow 0.010$ in)
  $10 < h <$ 100's μm
- pressure in film (to carry load) generated by relative motion between sleeve & shaft

Shaft offset in sleeve

Fluid wedge (exaggerated)

HYDROSTATIC

Pressure in bearing due to lubricant supplied at high pressure (100's or 1000's of psi, 10's of MPa)

(We won't look at these in detail).

ELASTOHYDRODYNAMIC

- shape of film/wedge determine by elastic deformation
- still hydrodynamic

ELASTOHYDRODYNAMIC, cont'd

- rolling bearings
- gears
- some pistonrings

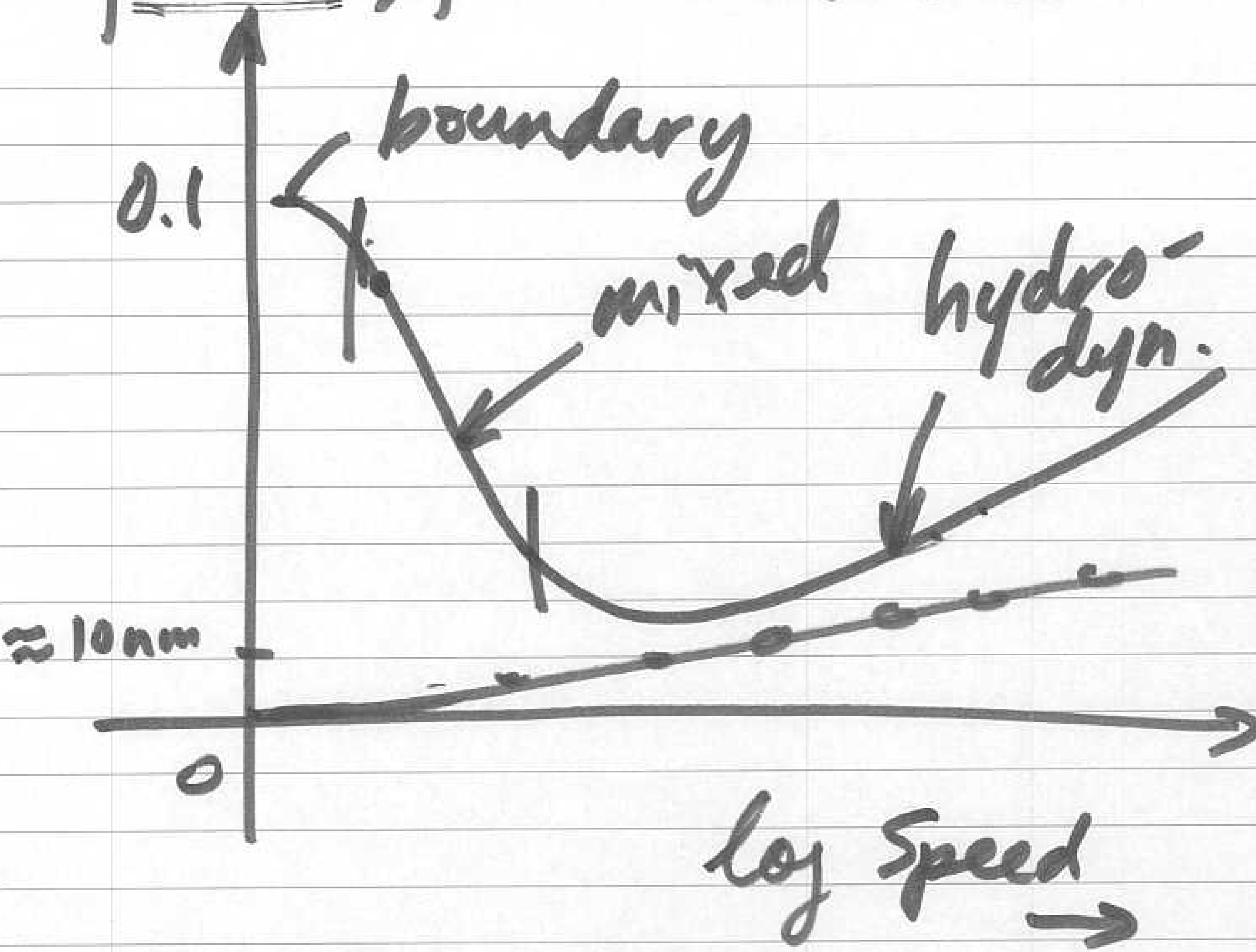
$0.1 < h < 10\ \mu m$

BOUNDARY, SOLID FILM

- solid to solid contact separated by additives, powders, solid films
- some hydrodynamic bearings at low speed

$1 < h < 100\ nm$

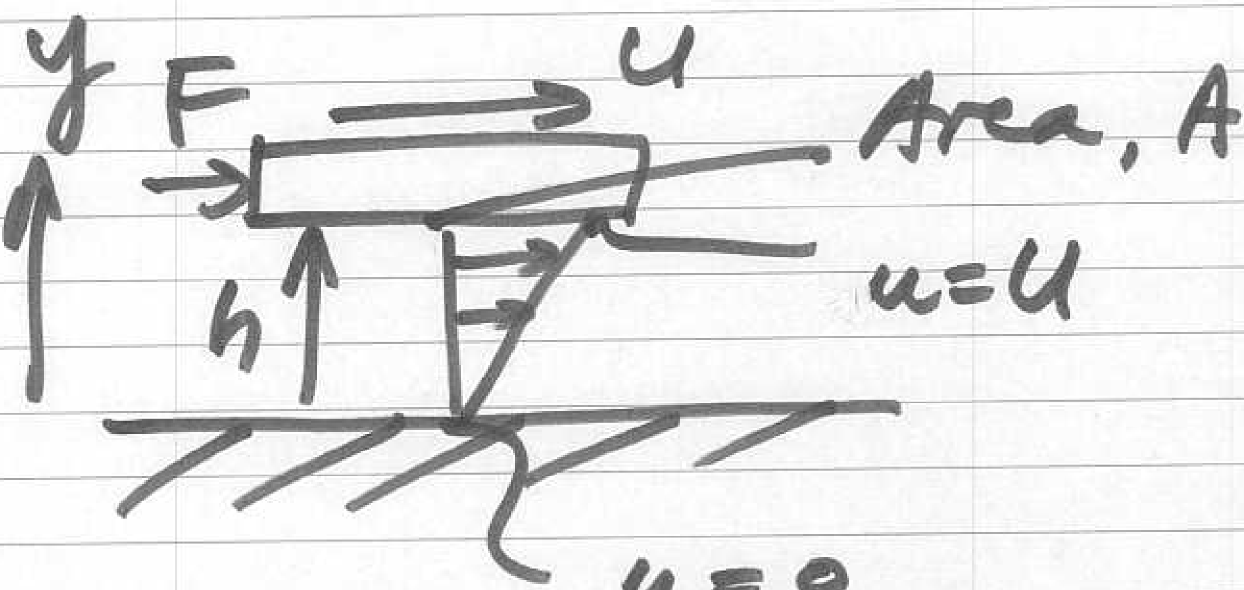
— friction, film thickness



- mixed → some solid to solid contact

- in mixed regime $h <$ surface roughness

Viscosity — key parameter



Newtonian fluid

$$\tau = \frac{F}{A} = \mu \frac{du}{dy}$$

Shear Stress

absolute viscosity

Units of viscosity

$$\mu = \frac{\tau h}{U} \qquad \frac{du}{dy} \approx \frac{U}{h}$$

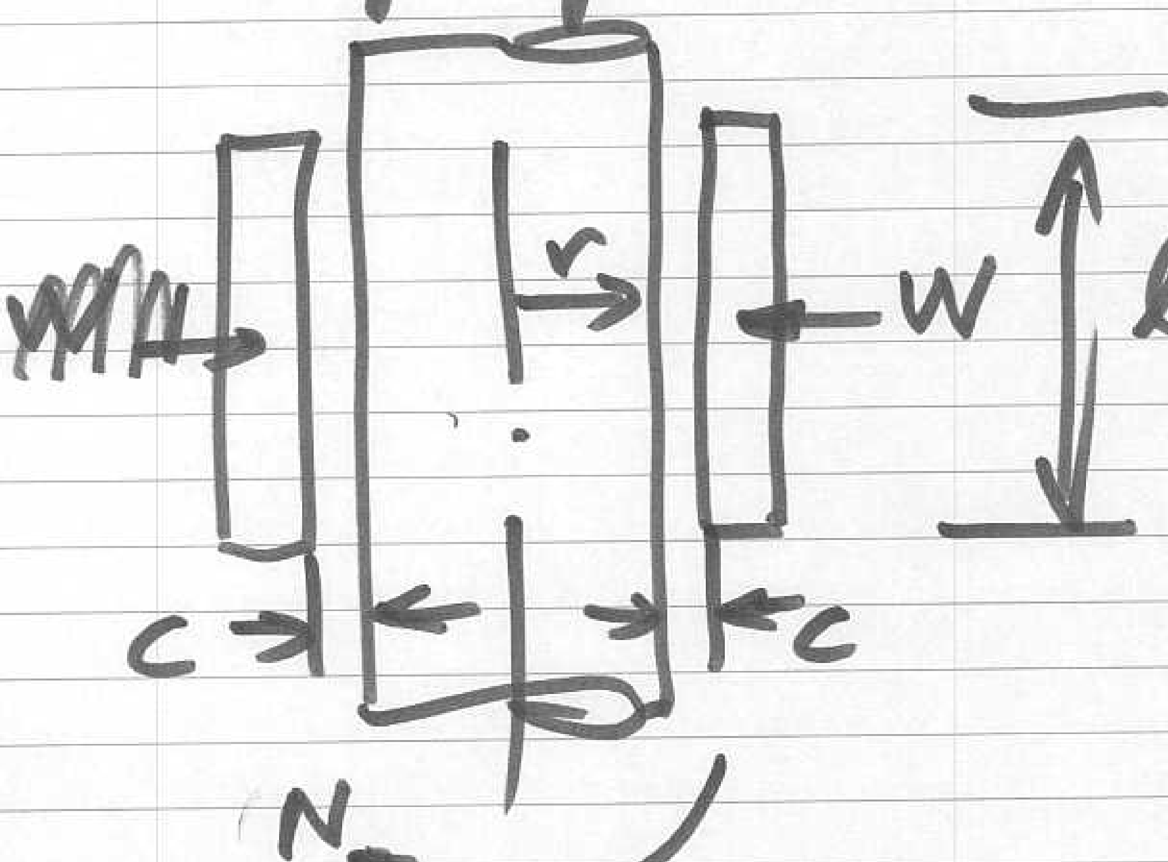
$$= \text{Pressure} \times \text{time}$$

IPS $\frac{lbf \cdot s}{in^2} \Rightarrow$ reyn

SI $\underbrace{m}_{10^{-3}}\,\underbrace{Pa}_{\text{Pascal}} \cdot s \Rightarrow$ centipoise (cP)

Petroff's Law

- just a point of reference
- helps define dimensionless groups.



c clearance   r radius

N revs/sec, rps

Assume W very small so shaft is centered in sleeve:

<u>Surface velocity</u>

$$U = 2\pi r N$$

<u>Shear Stress</u>

$$\tau = \mu \frac{U}{h} = \frac{2\pi r N \mu}{c}$$

<u>Shear Torque, friction</u>

$$T = A \tau r = 2\pi r l \left(\frac{2\pi r \mu N}{c}\right) r$$

$$= \frac{4\pi^2 r^3 l \mu N}{c}$$

Sommerfeld Number, S

$$S = \frac{\mu N}{P}\left(\frac{r}{c}\right)^2$$

(Most) Impt dimensionless # in bearing design

Example

$$f = 2\pi^2\left(\frac{\mu N}{P}\right)\left(\frac{r}{c}\right)$$

$\frac{r}{c} = 500$ $\mu = 10$ mPas

$N = 1000$ rps $P \rightarrow 1.0$ MPa, 10.0 MPa

$$f = \frac{2\pi^2 (10 \times 10^{-3})(1000)(500)}{P}$$

$$f \doteq \frac{10^5}{P} \qquad \left(\text{very lightly loaded}\right)$$

$$P = 1.0 \times 10^6 \qquad f \Rightarrow 0.1$$

$$P = 10 \times 10^6 \qquad f = 0.01$$

etc $\quad \left(0.001 \rightarrow 0.01 \text{ typical.} \right)$

$$f = \frac{\text{Friction Force}}{\text{Normal load}}$$

Beauchamp Tower's Expts

Railway axle bearings ca 1884

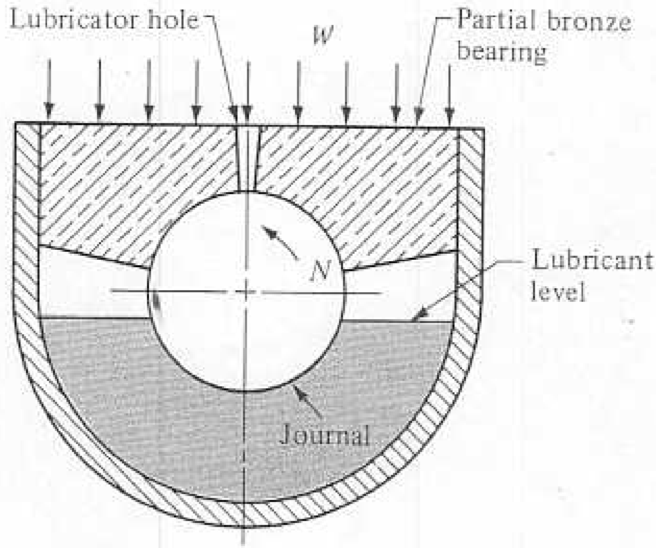
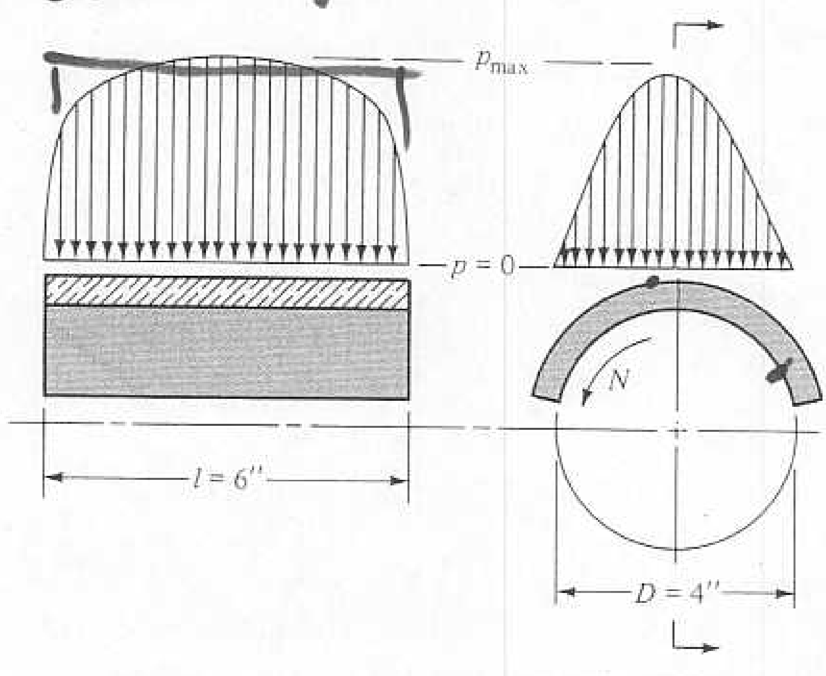


FIGURE 12-7

Schematic representation of the partial bearing used by Tower.

Discovered high pressures in oil films

Reynolds developed "~~solution~~ equation"



(1-D) Reynolds eq.

x is along surface of shaft .... i.e. circumference

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = -6U\frac{dh}{dx}$$

$h(x) \rightarrow$ film thickness

$U \rightarrow$ surface velocity

$p(x) \rightarrow$ variation in pressure in direction of motion.

$\Rightarrow$ relates $p(x)$ to $h(x)$ for any μ & U ... i.e. given film shape, μ & U, solve for $p(x)$

Journal bearing analysis

I DERIVE 1-D Reynolds Eq (infinitely wide bearing)

II USE Raimondi & Boyd solutions for finite (real) bearings

Basic Assumptions in bearing analysis:

1. Newtonian fluid $\left(\tau = \mu \frac{du}{dy}\right)$
2. No fluid inertia forces & laminar flow
3. Incompressible flow
4. Constant viscosity along film
5. Constant pressure across film

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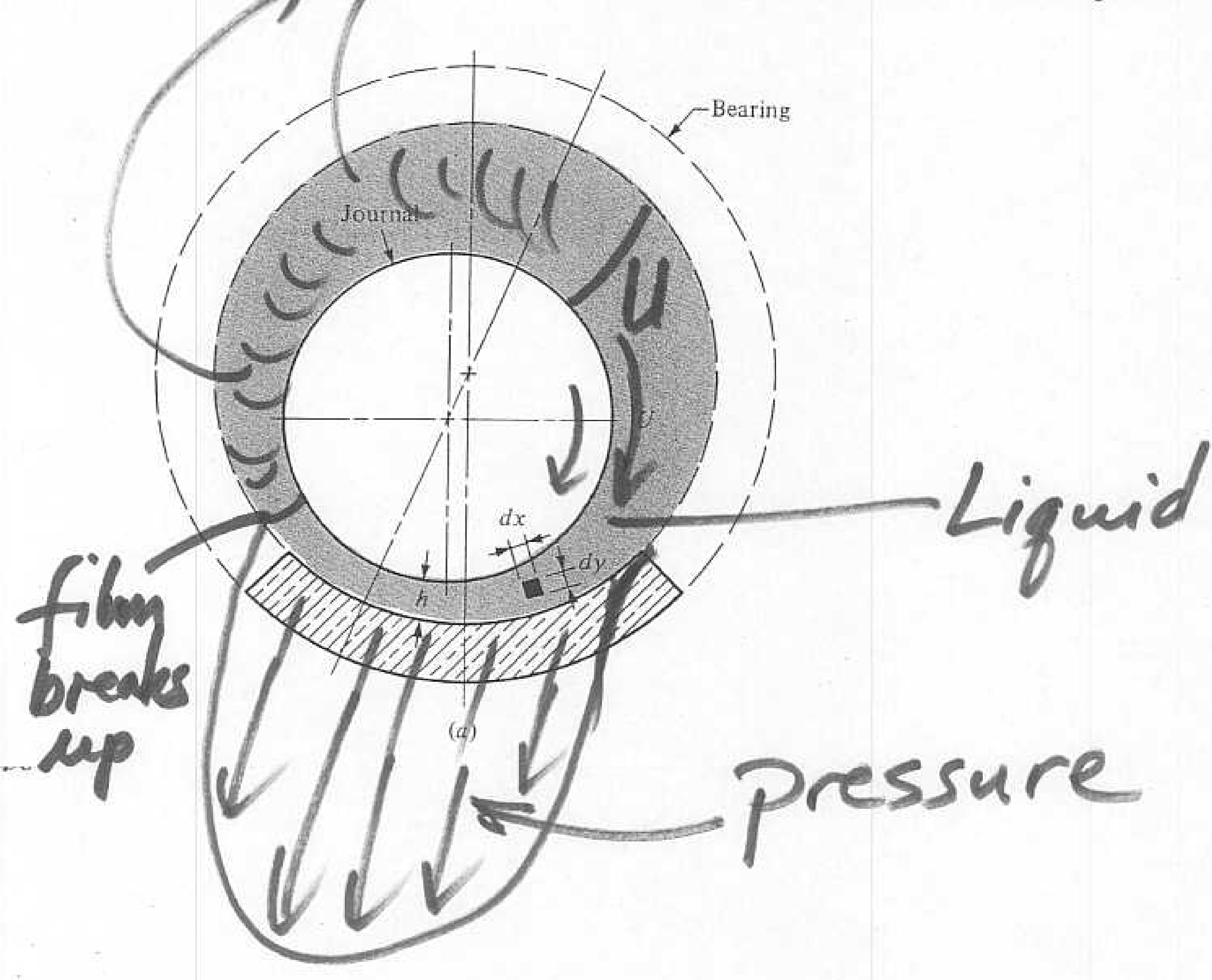
Air



(a)

To analyse bearing we invoke:

1. Equilibrium

$$\sum F_x = 0$$ on a fluid element

2. Constitutive relation, Newtonian viscosity

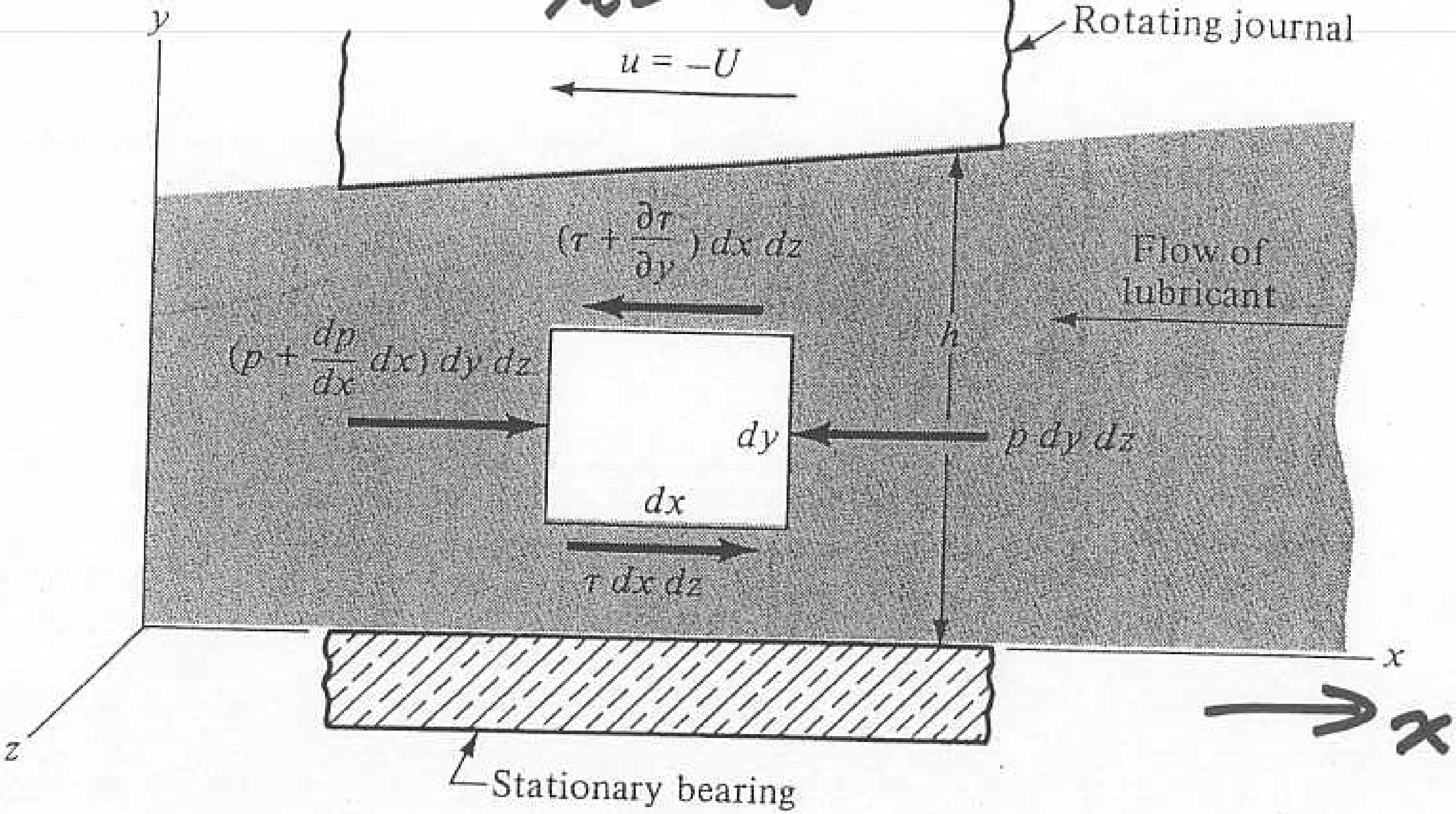
$$\tau = \mu \frac{du}{dy}$$

3. Conservation of mass, volume flow ⟹ continuity

4. Boundary conditions

I-D Reynold's Equation:

$u = -U$



(b)

$$\sum F_x = 0 = \underbrace{\left[\frac{dp}{dx}\,dx\right]}_{\substack{\text{Pressure difference} \\ (x\text{ direction})}}\underbrace{dy\,dz}_{\text{Area}} - \underbrace{\left[\frac{\partial \tau}{\partial y}\,\partial y\right]}_{\substack{\text{Shear stress} \\ \text{difference (y direction}}}\underbrace{dx\,dz}_{\text{Area}}$$

Leads to:

$$\boxed{\frac{dp}{dx} = \frac{\partial \tau}{\partial y}} \quad \ldots (1)$$

From Newtonian Viscosity:

$$\tau = \mu \frac{du}{dy}$$

(local velocity $u \Rightarrow u(x,y)$)

substitute into (1)

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$

integrate @ $x = \text{constant}$.
with respect to y (twice)

<u>First:</u>

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\left[\frac{dp}{dx}\right] y + C_1 \quad \text{...(a)}$$

<u>Second</u>

$$u = \frac{1}{2\mu}\frac{dp}{dx} y^2 + C_1 y + C_2$$

...(b)

Apply boundary conditions at walls:

$$y = 0 \Rightarrow u = 0$$

$$y = h(x) \Rightarrow u = -U$$

from (a) + (b) above

$$C_1 = -\frac{U}{h} - \frac{h}{2\mu}\frac{dp}{dx} \qquad \underline{C_2 = 0}$$

$$\therefore$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - hy\right) + \frac{U}{h}y$$

Anywhere:

$$Q = \int_0^h u\,dy \quad \ldots \frac{m^2}{s}$$

Volume
flow per
unit width (in x direction)

Performing the integration
of u, get

$$Q = -\frac{Uh}{2} + \frac{h^3}{12\mu}\frac{dp}{dx}$$

But, due to incompressible flow, <u>conservation of volume</u>

<u>continuity</u>,

$$\frac{dQ}{dx} = 0$$

$$\therefore \frac{dQ}{dx} = -\frac{U}{2}\frac{dh}{dx} - \frac{d}{dx}\left(\frac{h^3}{12\mu}\frac{dp}{dx}\right)$$

$$= 0$$

and

$$\boxed{\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = -6U\frac{dh}{dx}}$$

- which can be solved with appropriate b.c.'s of p @ $x_?$

& $\frac{dp}{dx}$ @ $x_?$

- Quite simple to do for some situations

e.g. $h(x) = ax + b$

linear wedge

(not applicable to journal bearings)

— With side "leakage"
... i.e. flows in z direction.

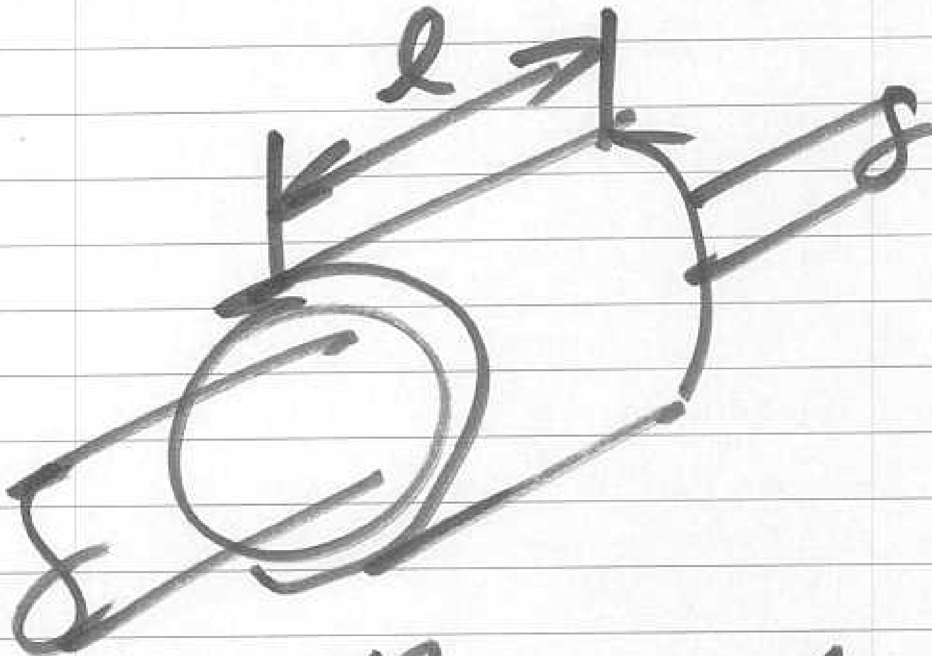
— Get 2-D Reynolds eq.

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{dp}{\partial x}\right) - \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)$$

$$= -6U\frac{\partial h}{\partial x}$$

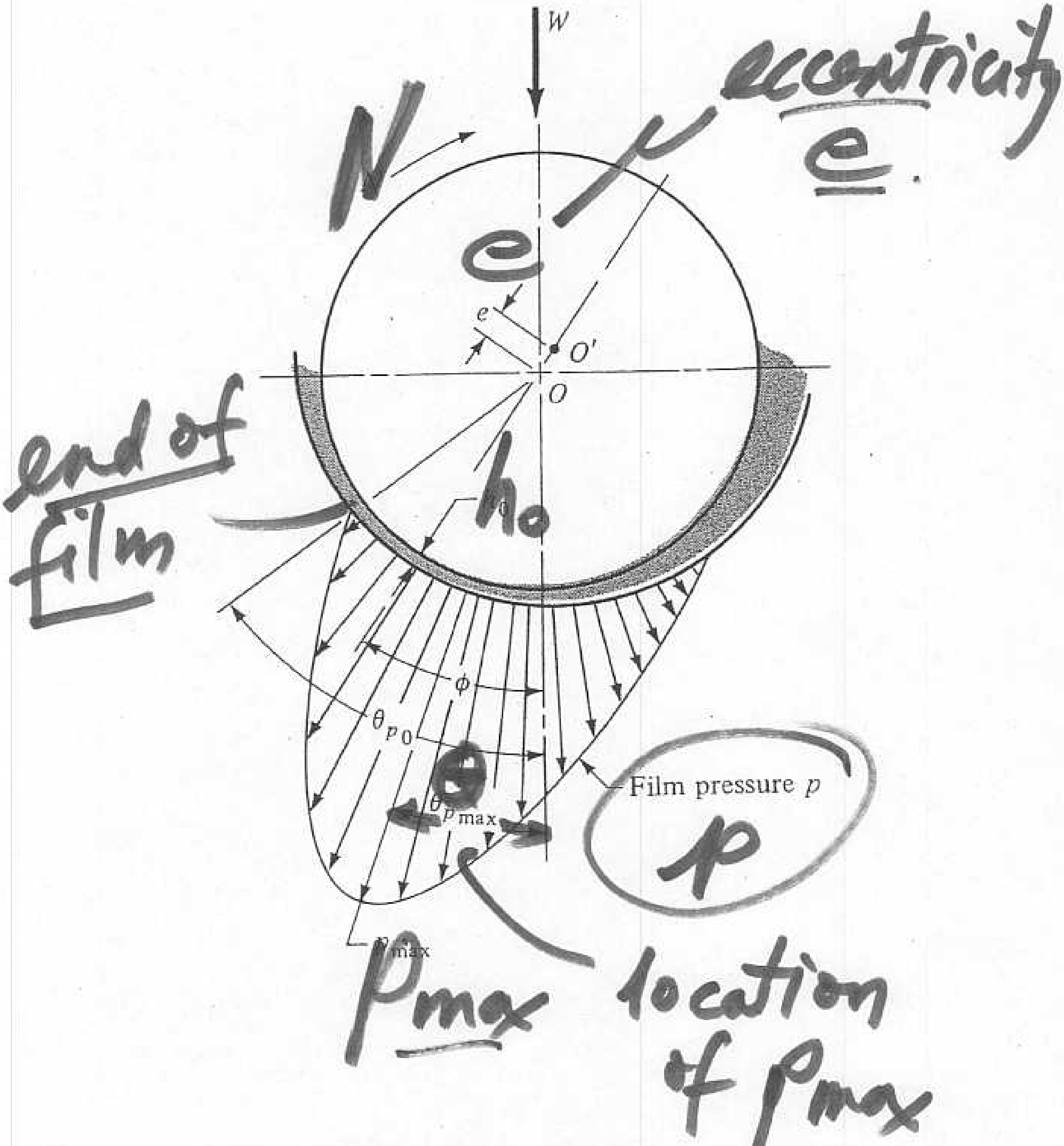
$$\left.\begin{array}{l} h \Rightarrow h(x) \\ p \Rightarrow p(x,z) \end{array}\right\} \text{NOTE}$$

- The 2-D Reynolds eq. was solved for journal bearing geometries

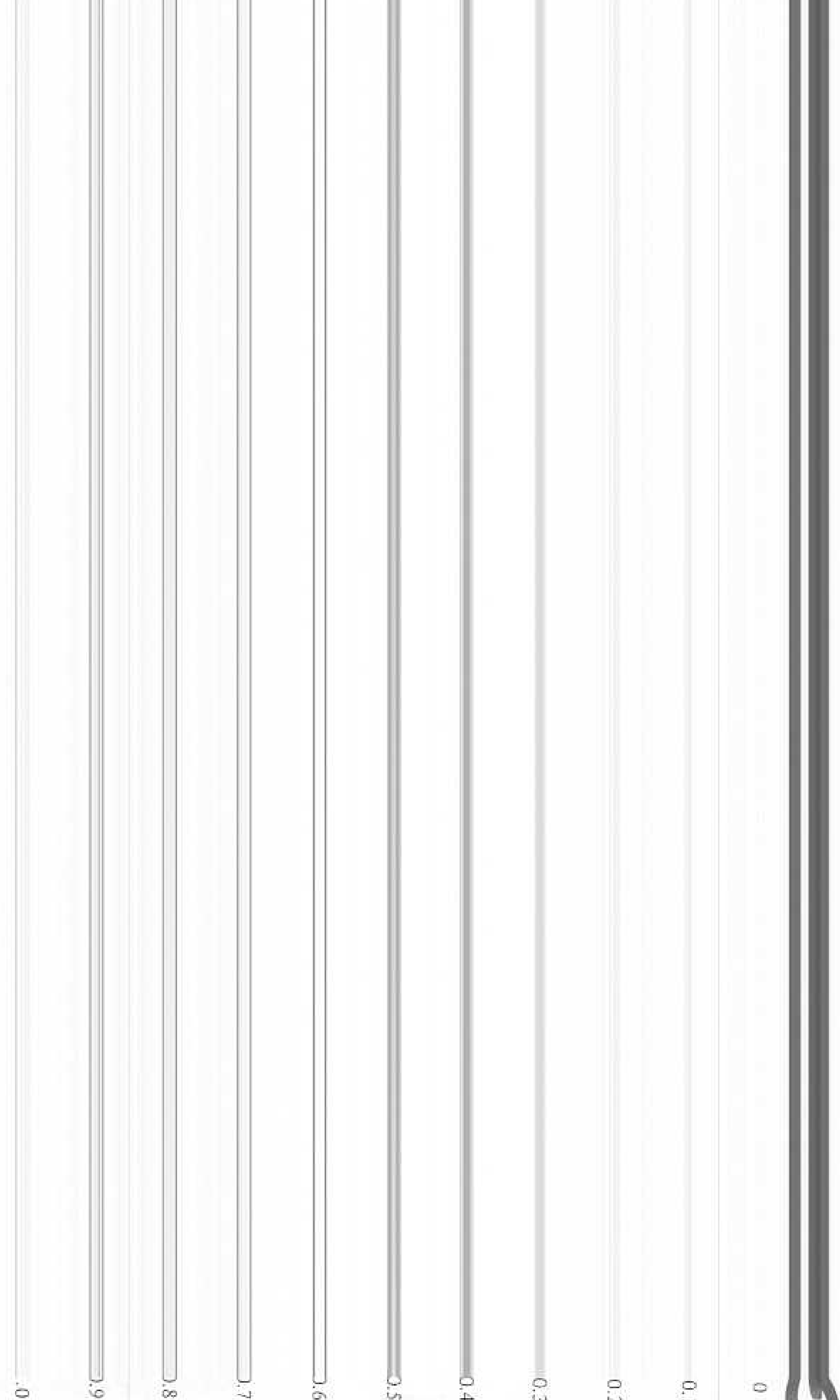


by Raimondi & Boyd in 1950's

- Results presented in graphical form.

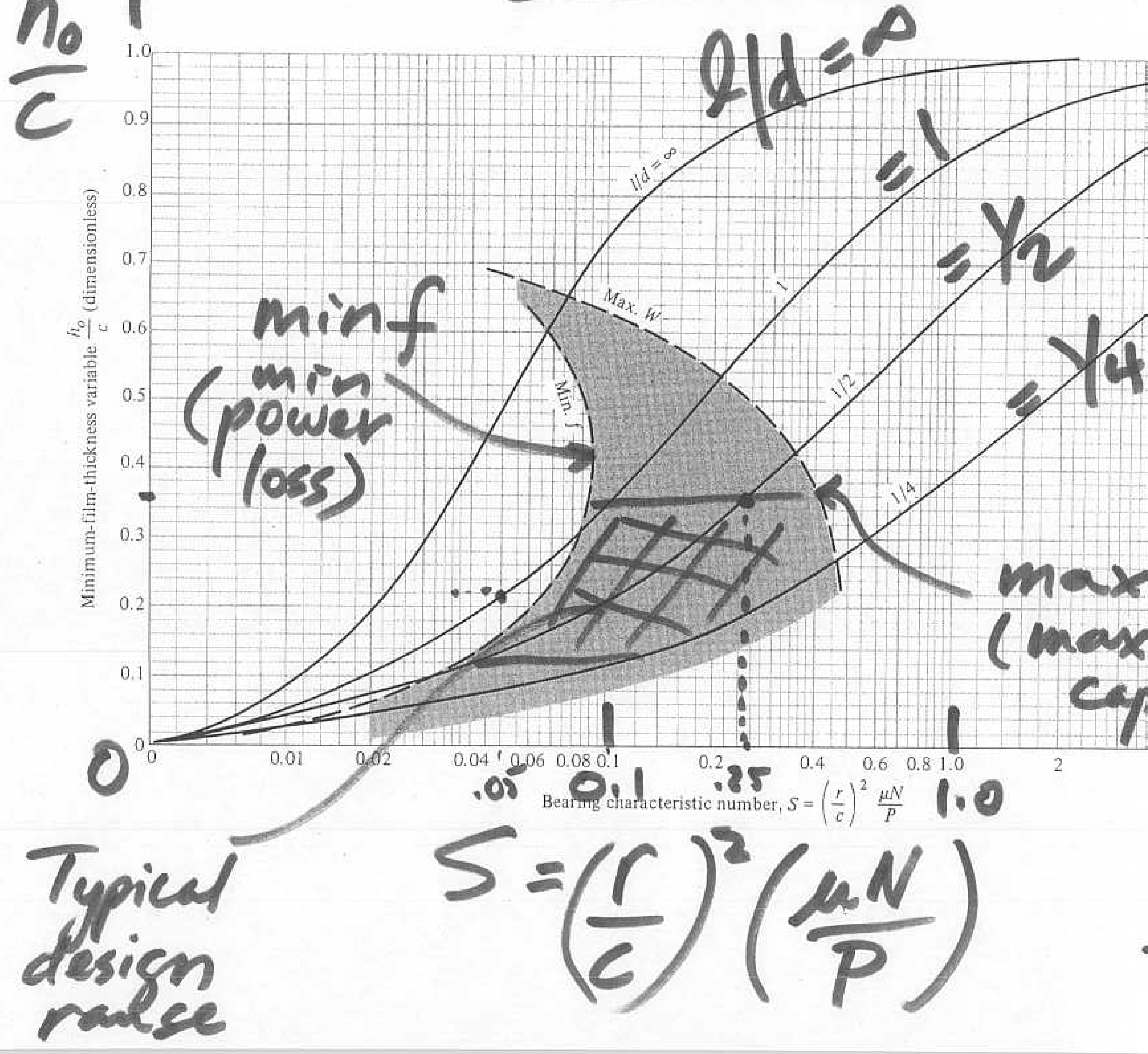


eccentricity ratio $\epsilon = \dfrac{e}{c}$

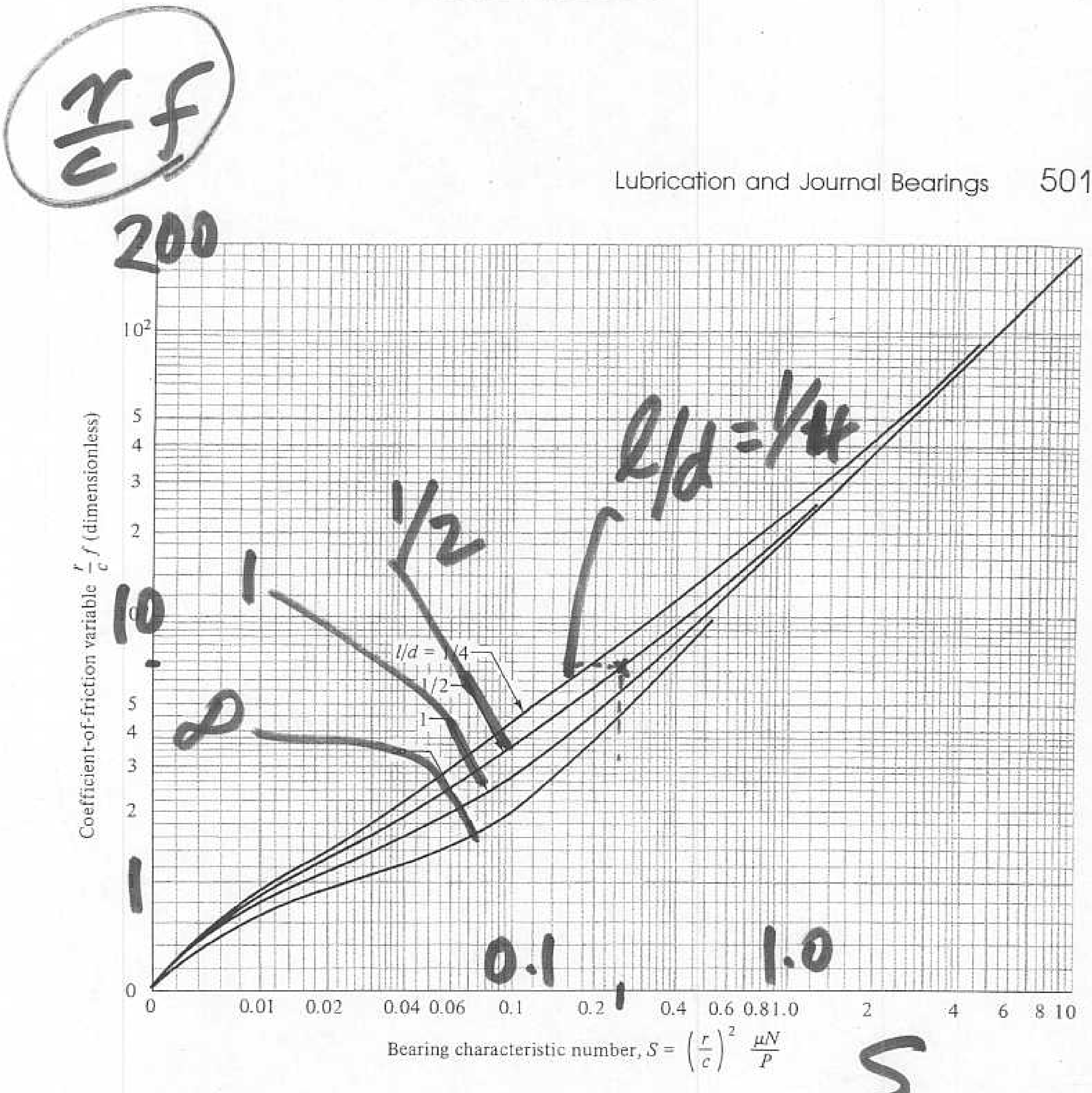

0
0.1
0.2
0.3
0.4
.5
.6
.7
.8
.9
.0
Eccentricity ratio ϵ (dimensionless)

284

FILM THICKNESS



FRICTION



12-17

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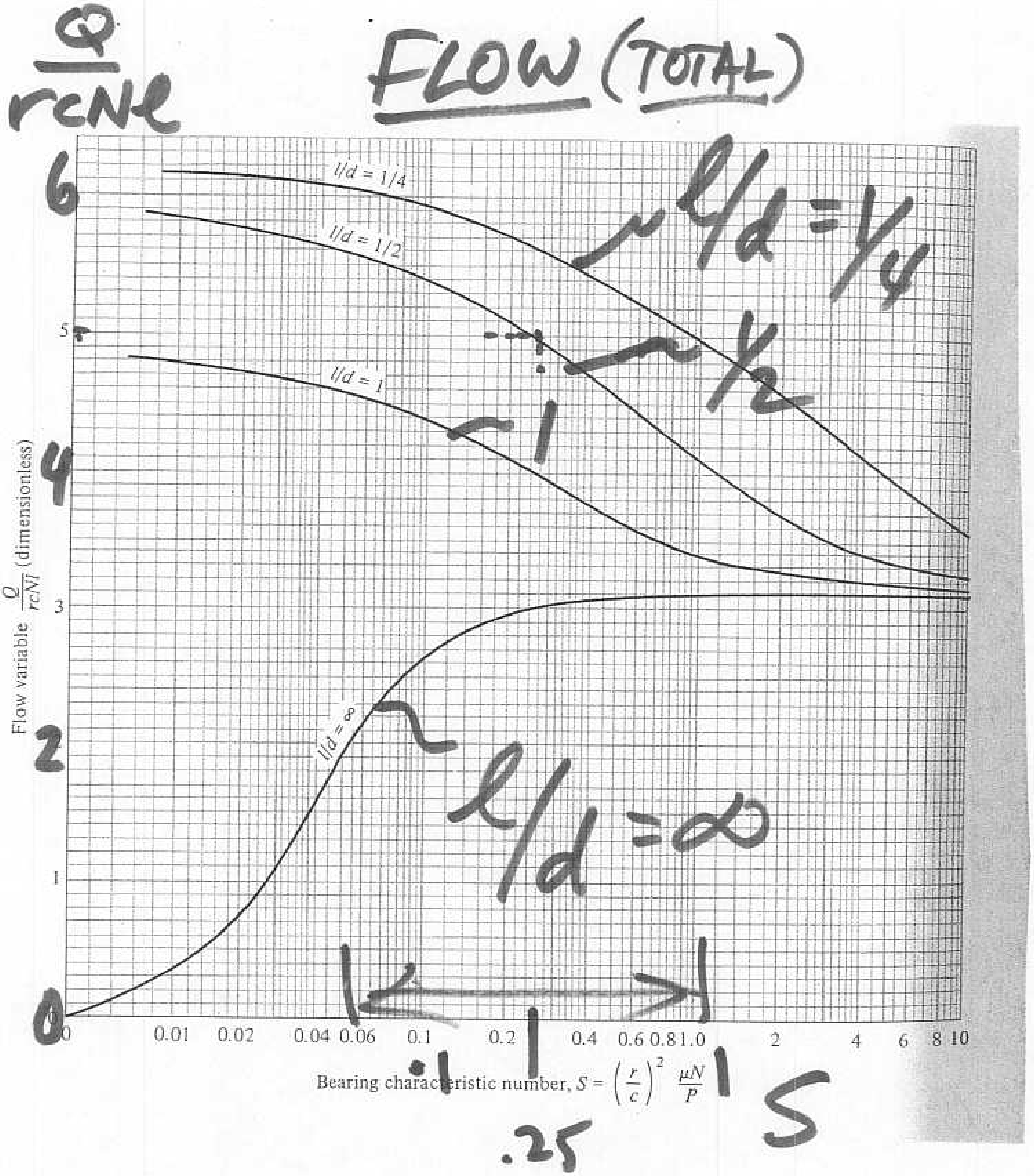
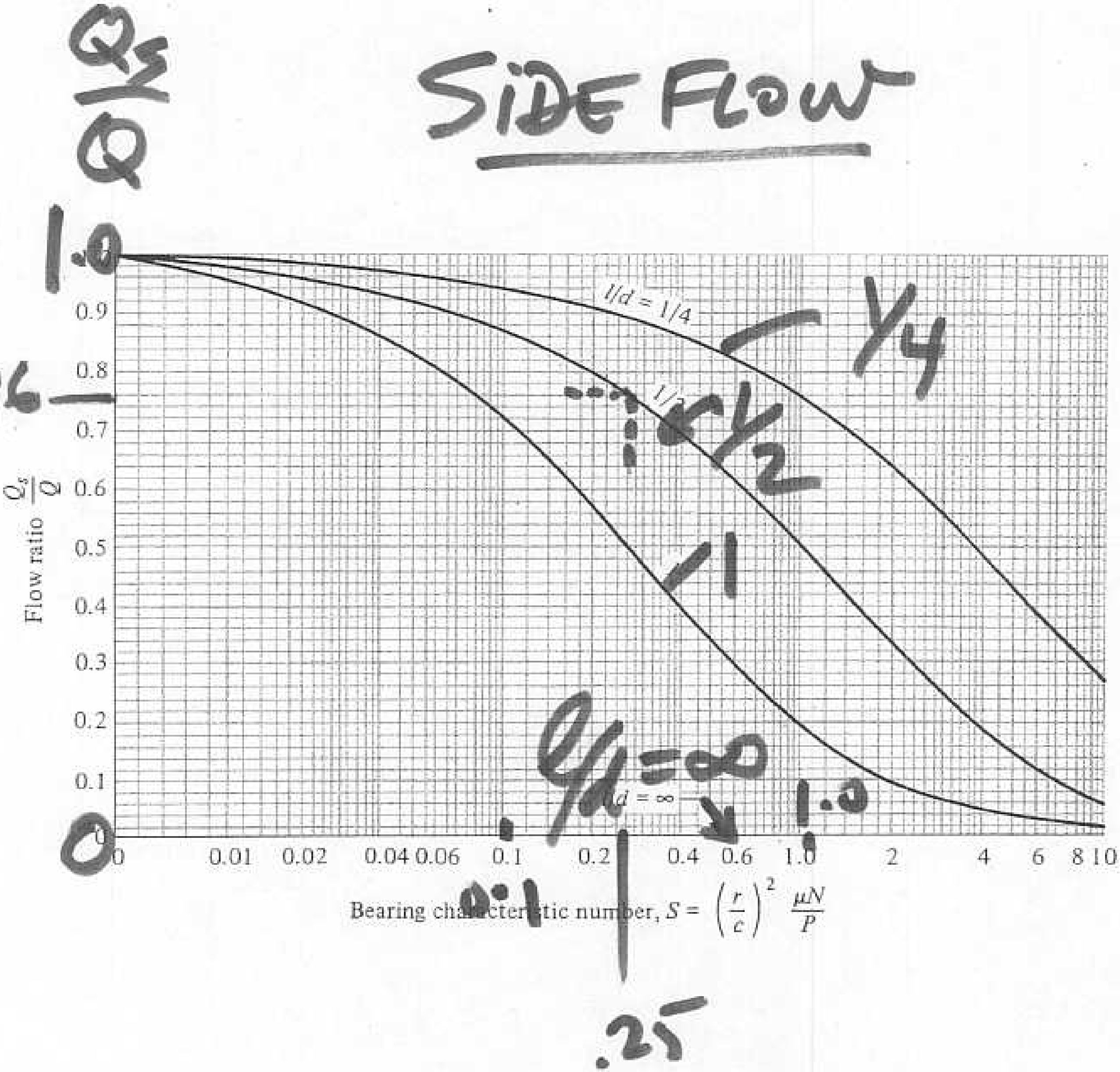


Q / rcNl
FLOW (TOTAL)
6
5
4
3
2
1
0
l/d = 1/4
l/d = 1/2
l/d = 1
l/d = ∞
l/d = 1/4
1/2
1
l/d = ∞
Flow variable $\frac{Q}{rcNl}$ (dimensionless)
0.01 0.02 0.04 0.06 0.1 0.2 0.4 0.6 0.8 1.0 2 4 6 8 10
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$
.1
.25
1
S

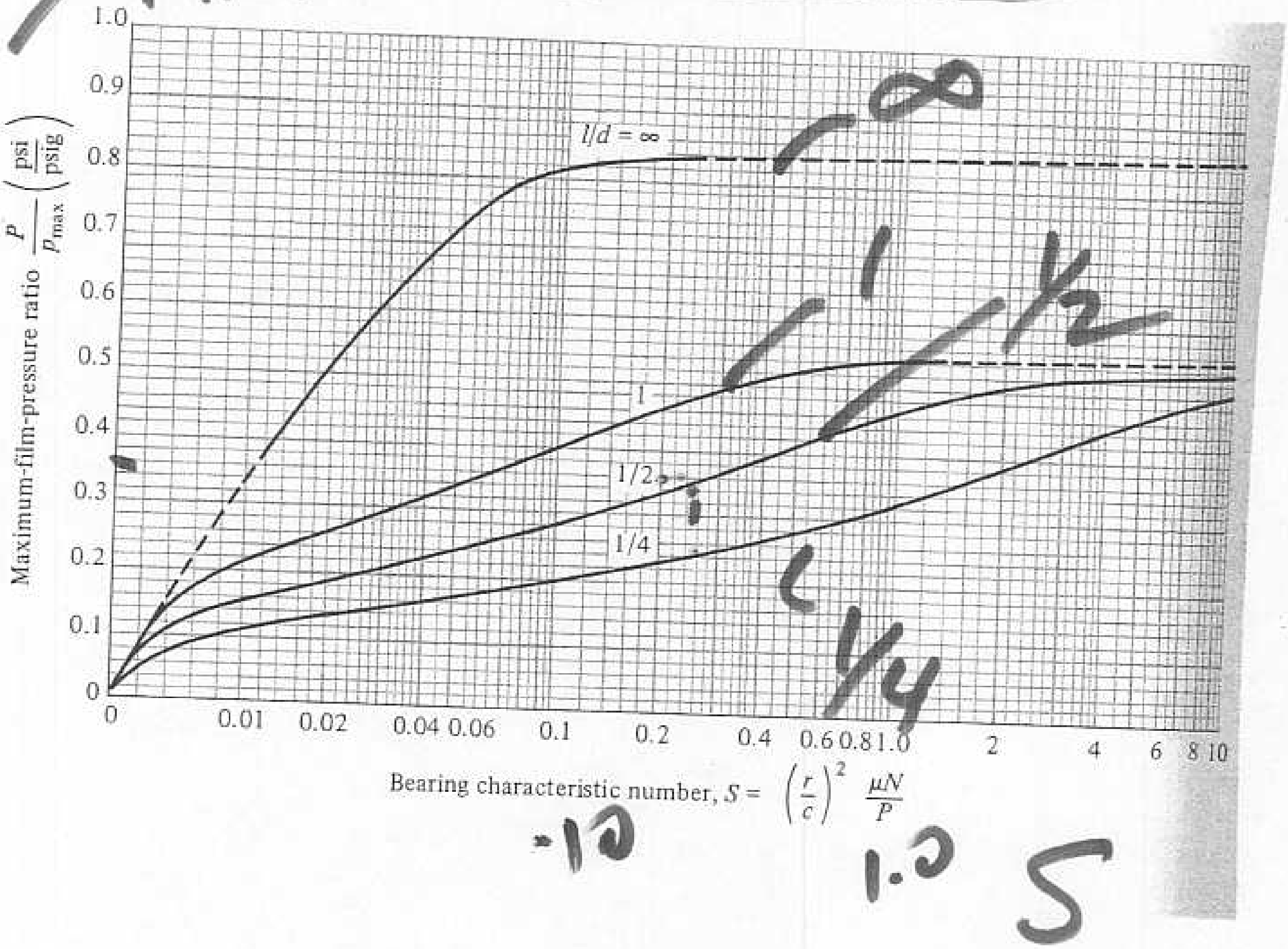
12-18

SIDE FLOW



12-19

P/Pmax MAX PRESSURE



Example:

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu N}{P}\right)$$

e.g. $\frac{r}{c} = 500$

$\mu = 100 \text{ mPas}$

$N = 100 \text{ rps}$

$P = 10 \text{ MPa}$

$$S = (500)^2 \frac{(100 \times 10^{-3})(100)}{10 \times 10^6}$$

$$\boxed{S = 0.25}$$

Obtain bearing info if:

$r = 12.5\text{ mm}$ $(d = 25\text{ mm})$

and $\frac{l}{d} = \frac{1}{2}$

1. Film thickness (minimum)

$\frac{r}{c} = 500$ $r = 12.5\text{ mm}$

$\therefore c = 0.025\text{ mm}.$

$c = 25\ \mu\text{m}.$ (0.001 in)

From Fig 12-14

$\frac{h_0}{c} = 0.36$ $\boxed{h_0 = 9\ \mu\text{m}}$

$\left(\epsilon = 0.64 = \frac{e}{c} \quad e = 16\ \mu\text{m} \right)$

2. Coefficient of friction

$$\frac{r}{c} f = 6.75 \quad \text{Fig 12-17}$$

$$\boxed{f = \frac{6.75}{500} = \underline{0.0135}}$$

3 Flow through bearing

$Q \Rightarrow in^3/sec, \; mm^3/sec, \; l/sec$ etc.

$$\frac{Q}{rcNl} = 5.0$$

$$Q = (5.0)(\overset{mm}{12.5})(\overset{rps}{100})(\overset{mm}{0.025})(\overset{mm}{12.5})$$

$$\boxed{Q = 1953 \; mm^3/sec}$$

4. Side flow

$$\boxed{\frac{Q_s}{Q} = 0.76}$$ Fig 12-19

∴ 76% of flow through bearing goes to side flow

$$\boxed{Q_s = 1484\ mm^3/sec}$$

∴ Must be "resupplied" to bearing via oil pump.

5. Max pressure

$$\frac{P}{p_{max}} = 0.35 \qquad p_{max} = \frac{P}{0.35}$$

Fig 12-20

$$\boxed{p_{max} = 28.6\ MPa}$$

μ VS T for SAE OILS

μ

Viscosity

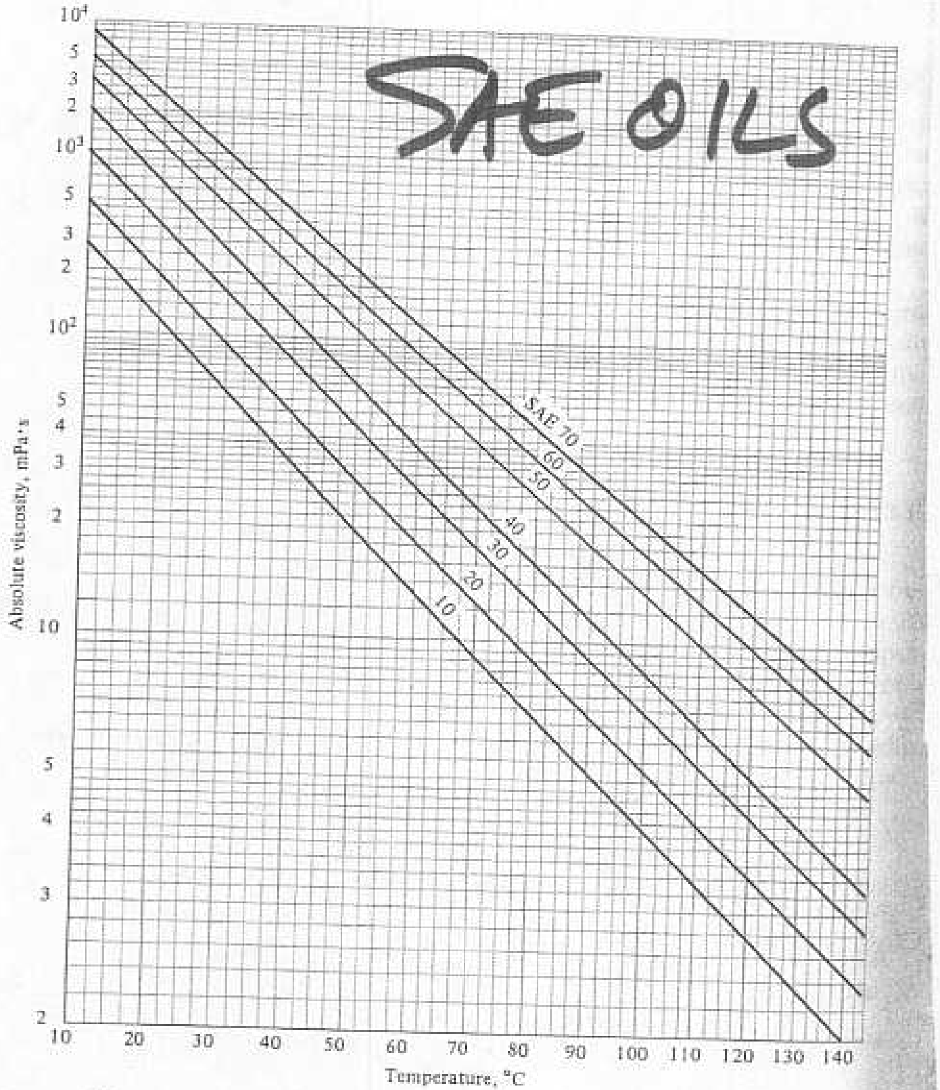


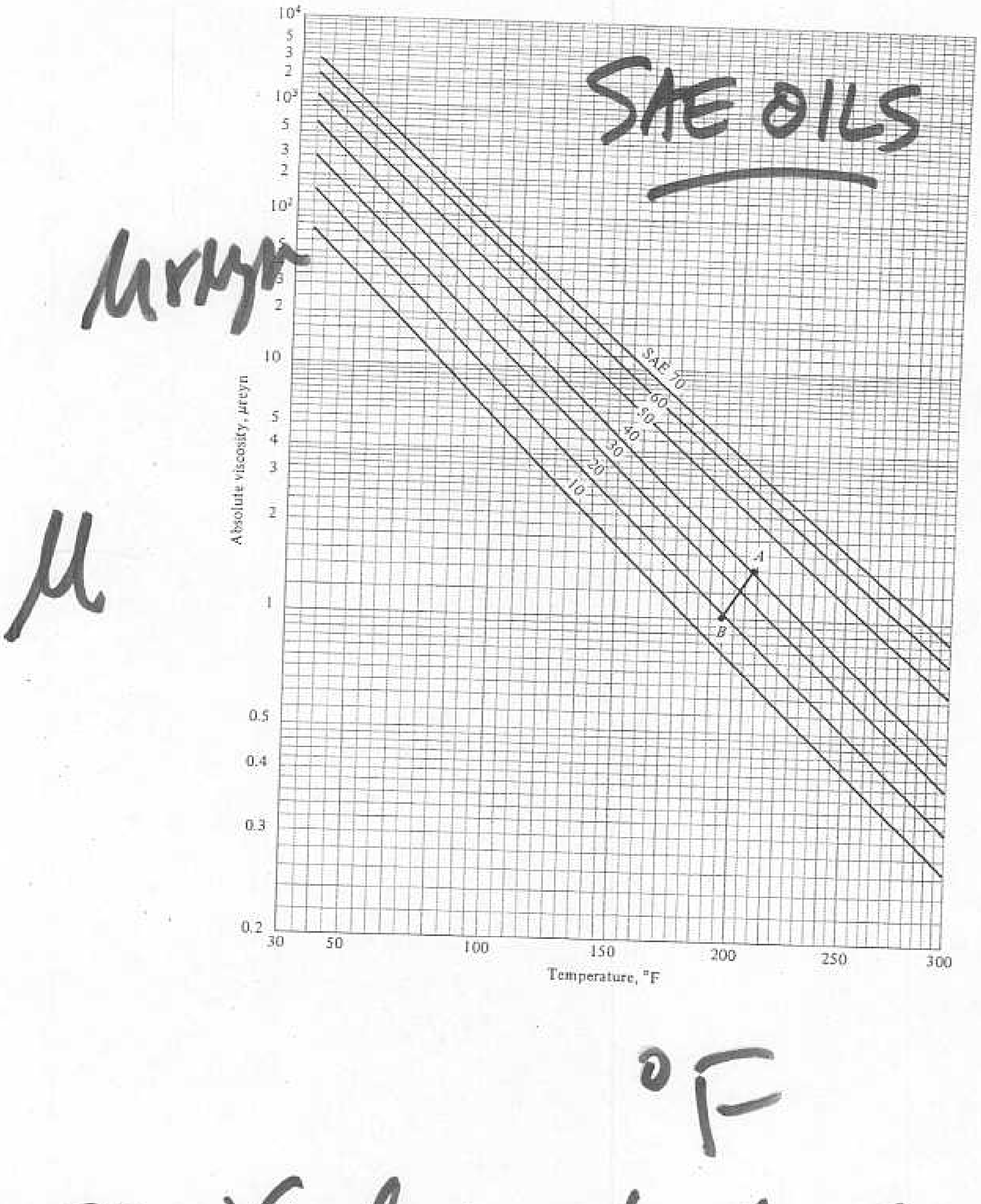
FIGURE 12-12
Viscosity-temperature chart in SI units. *(Adapted from Fig. 12-11.)*

10°C T_c 140°C

Temperature

$\Delta\mu > 90\%$ when

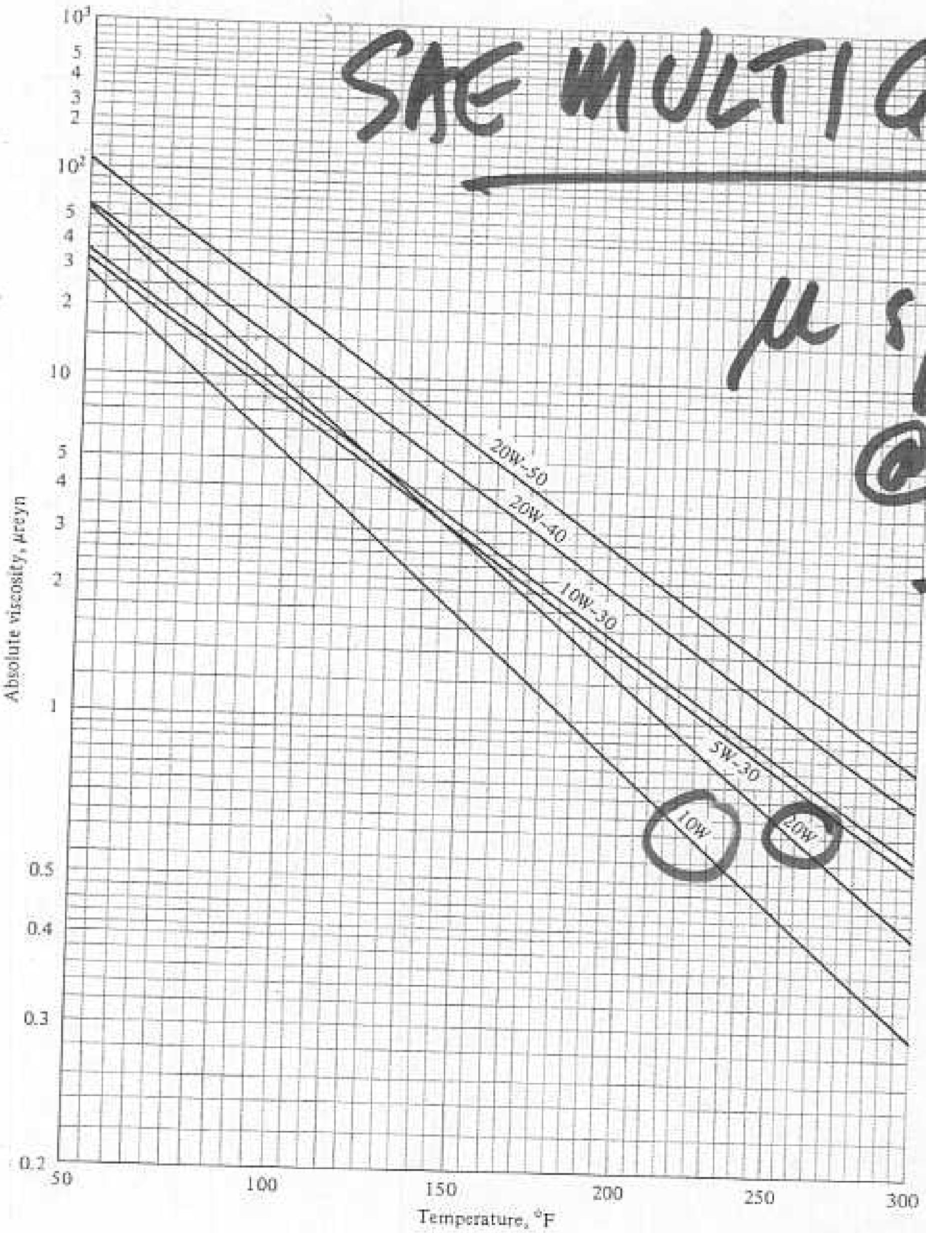
$\Delta T_c \simeq 50°C$



μ specified in standard
@ one temp FOR SAE 20+
20W, 10W, 5W are specified
↳ 'winter' @ two temps

μ

SAE MULTIGRADE OILS



μ specified
@ two temps
$-18°C$ & $100°C$
20W-50
meets $-18°C$
spec for 20W
oil & 100° spec
for 50 oil

Multi-grade oils have lesser
dependence of μ on T vs. single grade

Temperatare Rise of Oil in Bearing

- typically a few 10's of degrees F (30-50% decrease in μ within bearing)
- Use **avg oil** temp for design.
- **assume** all heat goes into lubricant

$$H = \frac{2\pi \overline{TN}}{J} \quad \text{N-m} \cdot 1/s$$

heat generated

mechanical equivalent of heat

also,

$$H = \frac{2\pi f W r N}{J} \quad \frac{BTU}{s} \left(\text{or WATTS in SI}\right)$$

H ⇒ heat to fluid

also, from Heat Transfer

$$\Delta T_{{}^\circ F} = \frac{H}{\gamma C_H Q}$$

Temp. Rise in oil.

weight per unit volume

Specific heat

(NO SIDE FLOW)

$$\therefore \ \Delta T_F = \frac{2\pi f W r N}{J \gamma C_H Q}$$

Substituting for J, γ, C_H we obtain:

Typical values for oil (see p 504)

$$\Delta T_F = \frac{2\pi f W_r N}{Q\underbrace{(9336)}_{\frac{lb_f\text{-}in}{Btu}}\underbrace{(0.031)}_{\frac{lb_f}{in^3}}\underbrace{(.42)}_{\frac{Btu}{lb_f\,^\circ F}}}$$

and $$\Delta T_F = \frac{0.103 P\boxed{(r/c) f}}{[Q/rcNl]}$$

Accounting for side flow (leakage):

<u>SIDE FLOW IS AT</u>

<u>MEAN OIL TEMP IN BRG</u>

$$\therefore \Delta T_F = \frac{0.103P\left[\left(\frac{r}{c}\right)f\right]}{\left(1-\frac{1}{2}\left[Q_s/Q\right]\right)\left[Q/rcNl\right]} \quad \ldots (a)$$

ITEMS IN [ ] can be obtained from Raimondi and Boyd sol'n Figs 12-17, 12-18, 12-19

$$\left[\left(\frac{r}{c}\right)f\right] \quad \left[\frac{Q}{rcNl}\right] \quad \left[Q_s/Q\right]$$

In SI units, MPa

$$\Delta T_c = \frac{8.30\,P\,\left[\left(\overset{\text{MPa}}{r}/c\right)f\right]}{\left(1-\frac{1}{2}\left[Q_s/Q\right]\right)\left[Q/rcNl\right]}$$

Continuing Previous Example:

$P = 10\,\text{MPa}$ $\quad (r/c)f = 6.75$

$\frac{Q}{rcNl} = 5.0$ $\quad \frac{Q_s}{Q} = 0.76$ (TOO BIG)

$$\therefore \Delta T_c = \frac{8.30(10)[6.75]}{\left[1-\frac{0.76}{2}\right][5]} = 180.7\,^\circ C$$

HUGE!!

÷ Problem in example was That an unrealistically high viscosity (100 mPas) was assumed .... normally operating viscosity for This type of bearing would be 10 mPas or less, resulting in an order of magnitude lower ΔT

BEARING OPERATING (DESIGN) TEMPERATURE

$$T_{AV} = T_1 + \frac{\Delta T}{2} \qquad (T_2 - T_1)$$

AVG OIL TEMP — INLET TEMP — AVG TEMP RISE

<u>TWO TYPES OF (ACADEMIC) BEARING DESIGN PROBLEMS:</u>

I <u>OPERATING TEMP</u>, T_{AV} <u>IS SPECIFIED</u>

... μ is known if oil specified

.... with values of P, N

$r, c,$ and l (rules of thumb design)

ALL KEY BEARING PARAMETERS

$f, \Delta T, Q, h_o, Q_s/Q$ etc

CAN BE OBTAINED.

II <u>INLET TEMP, T_1 is GIVEN</u>

... need to estimate/guess μ

... iterate to sol'n

P

TABLE 12-4

Range of Unit Loads in Current Use for Sleeve Bearings

APPLICATION	UNIT LOAD psi	UNIT LOAD MPa
Diesel engines:		
Main bearings	900–1700	6–12
Crankpin	1150–2300	8–15
Wristpin	2000–2300	14–15
Electric motors	120–250	0.8–1.5
Steam turbines	120–250	0.8–1.5
Gear reducers	120–250	0.8–1.5
Automotive engines:		
Main bearings	600–750	4–5
Crankpin	1700–2300	10–15
Air compressors:		
Main bearings	140–280	1–2
Crankpin	280–500	2–4
Centrifugal pumps	100–180	0.6–1.2

Typical loads (pressures) on journal bearings for different applications

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TABLE 12-5

Some Characteristics of Bearing Alloys

ALLOY NAME	THICKNESS, in	SAE NUMBER	CLEARANCE RATIO r/c	LOAD CAPACITY	CORROSION RESISTANCE
Tin-base babbitt	0.022	12	600–1000	1.0	Excellent
Lead-base babbitt	0.022	15	600–1000	1.2	Very good
Tin-base babbitt	0.004	12	600–1000	1.5	Excellent
Lead-base babbitt	0.004	15	600–1000	1.5	Very good
Leaded bronze	Solid	792	500–1000	3.3	Very good
Copper-lead	0.022	480	500–1000	1.9	Good
Aluminum alloy	Solid		400–500	3.0	Excellent
Silver plus overlay	0.013	17P	600–1000	4.1	Excellent
Cadmium (1.5% Ni)	0.022	18	400–500	1.3	Good
Trimetal 88*				4.1	Excellent
Trimetal 77†				4.1	Very good

*This is a 0.008-in layer of copper-lead on a steel back plus 0.001 in of tin-base babbitt.

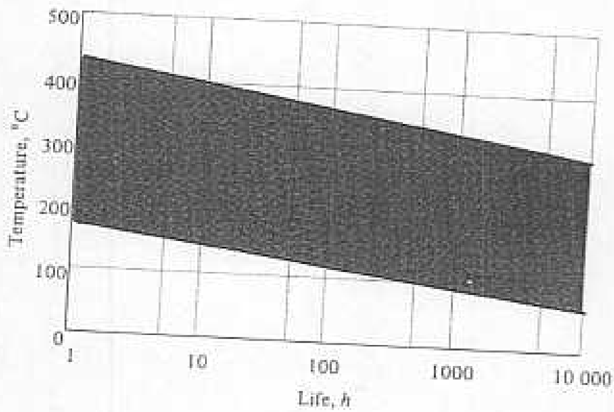
†This is a 0.013-in layer of copper-lead on a steel back plus 0.001 in of lead-base babbitt.

Soft metal coatings to avoid steel against steel contact at low speeds (e.g. start-up)

Typical clearances

FIGURE 12-23

Temperature limits for mineral oils. The lower limit is for oils containing antioxidants and applies when oxygen supply is unlimited. The upper limit applies when insignificant oxygen is present. The life in the colored zone depends on the amount of oxygen and catalysts present. *[Source: M. J. Neale (ed.), Tribology Handbook, Section B1, Newnes-Butterworth, London, 1975.]*



÷ TEMP RANGES

÷ Recommend design guide-lines next page

TABLE 12-2

Performance of 1.5-in-Diameter Journal Bearing with Various Clearances. (SAE 20 Lubricant, $T_1 = 100$°F, $N = 30$ rev/s, $W = 500$ lb, $L = 1.5$ in)

c, in	T_2, °F	h_0, in	f	Q, m³/s	H, Btu/s
0.000 5	226	0.000 38	0.011 3	0.061	0.086
0.001 125	142	0.000 65	0.009 0	0.153	0.068
0.001 55	133	0.000 77	0.008 7	0.218	0.066
0.001 75	128	0.000 76	0.008 4	0.252	0.064
0.002 75	118	0.000 73	0.007 9	0.419	0.060
0.003 95	113	0.000 69	0.007 7	0.617	0.059

Performance @ different clearances (see also Fig 12-22)

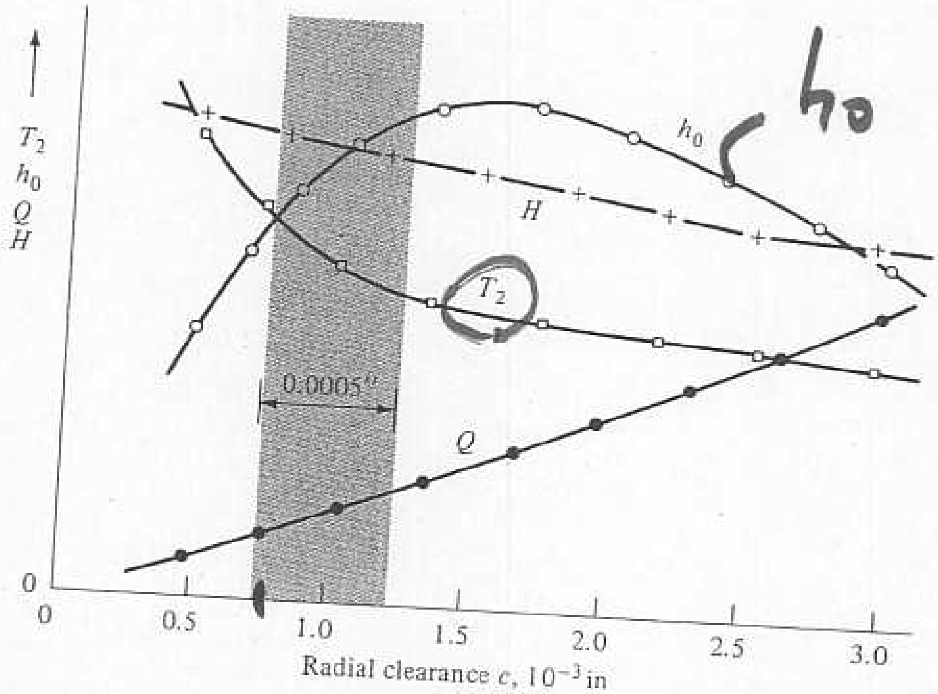


FIGURE 12-22

A plot of some performance characteristics of the bearing of Example 12-1 for radial clearances of 0.0005 to 0.003 in. The bearing outlet temperature is designated T_2. New bearings should be designed for the shaded zone, because wear will move the operating point to the right.

BEARING DESIGN GUIDELINES

$T_{OUTLET} = T_1 + \Delta T$

÷ TEMP LIMITS ON OIL

< 120°C mineral oils

< 150°C synthetics

÷ MIN FILM THICKNESS

$$h_0 \geq (5 + 40d) \times 10^{-6} \text{ m.}$$

diam in m

CLEARANCE

÷ $500 < \frac{r}{c} < 1000$

(Sometimes apply safety factor e.g. 2 to load. WE WON'T)

Returning to previous Example

(1) Change μ to 10 m Pas (Operating) ...(Need to reduce ΔT)

(2) Reduce P to 5 MPa $\Rightarrow l/d = 1$

(3) Specify oil inlet temp to be $60°C = T_1$

$$S = \left(\frac{r}{c}\right)^2\left(\frac{\mu N}{P}\right)$$

$$= (500)^2\left(\frac{(10\times10^{-3})(100)}{5\times10^6}\right)$$

$$\boxed{S = 0.05}$$

I FILM THICKNESS

$\frac{h_0}{c} = 0.24$ & $h_0 = 0.24 \times 25\,\mu m$

$\boxed{h_0 = 6\ \mu m.}$ $\epsilon = \underline{\underline{0.76}}$

by design guideline

$h_0 < \left[5 + 40(0.025)\right]\mu m$

$= 5 + 1 = 6\ \underline{\underline{\mu m}}$

[$0.7 < \epsilon < 0.9$ desirable, calcul. to be 0.76]

BARELY OK

$\boxed{\text{BUT OK}}$

OP MAX TEMP

$$\Delta T_c = \frac{(8.3)(5)(1.8)}{\left(1 - \frac{0.82}{2}\right)(4.5)} = \underline{28.1^\circ C}$$

II

OPERATING TEMP

$$60^\circ + \frac{\Delta T}{2} = \underline{74^\circ C}$$

MAX TEMP

$$T_1 + \Delta T_c = \underline{\underline{88.1^\circ C}}$$

This is less Than $120^\circ C$

So O.K.