Gears

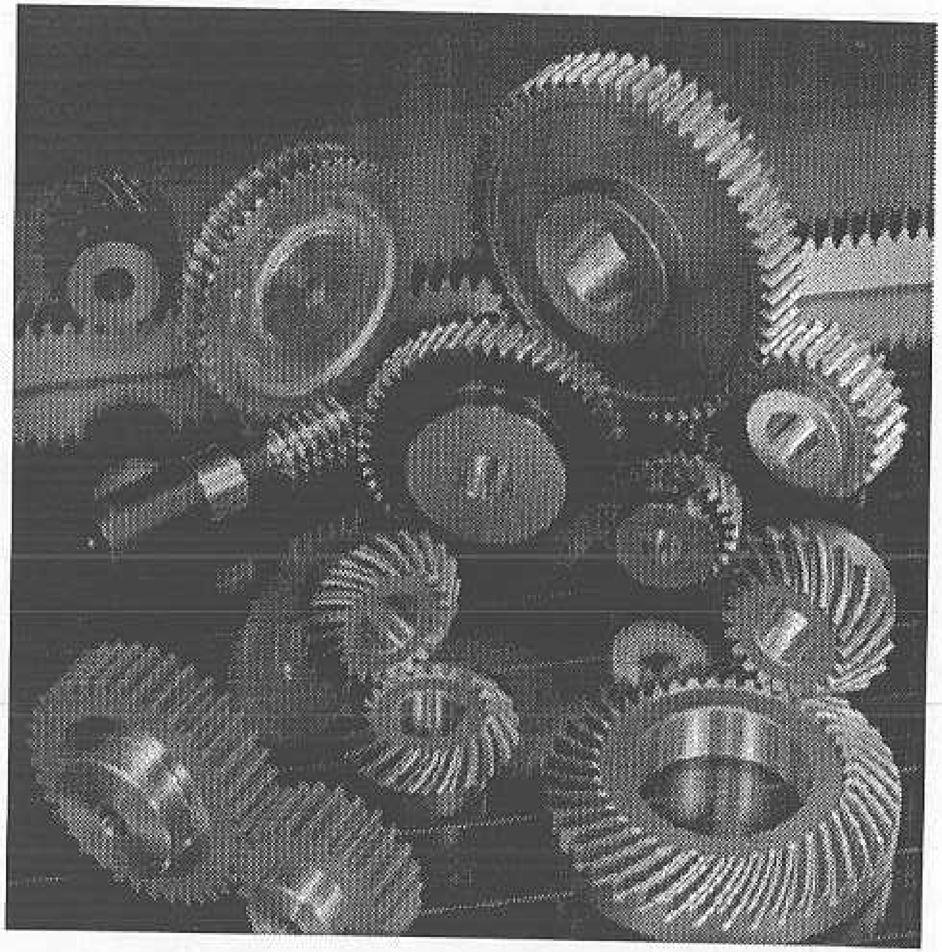


Image: An assortment of gears.

Just stare at the machine. There is nothing wrong with that. Just live with it for a while. Watch it the way you watch a line when fishing and before long, as sure as you live, you'll get a little nibble, a little fact asking in a timid, humble way if you're interested in it. That's the way the world keeps on happening. Be interested in it.

Robert Piersig, *Zen and the Art of Motorcycle Maintenance*

GEARS

- a very effective & efficient method of motion & power transfer
  - + efficient upto 98%
  - + strong (teeth)
  - + compact

  - − Cost ?
  - − Noise and vibration

I Types of Gears

II Tooth geometry, congugate action

III Gear trains ... Kinematics

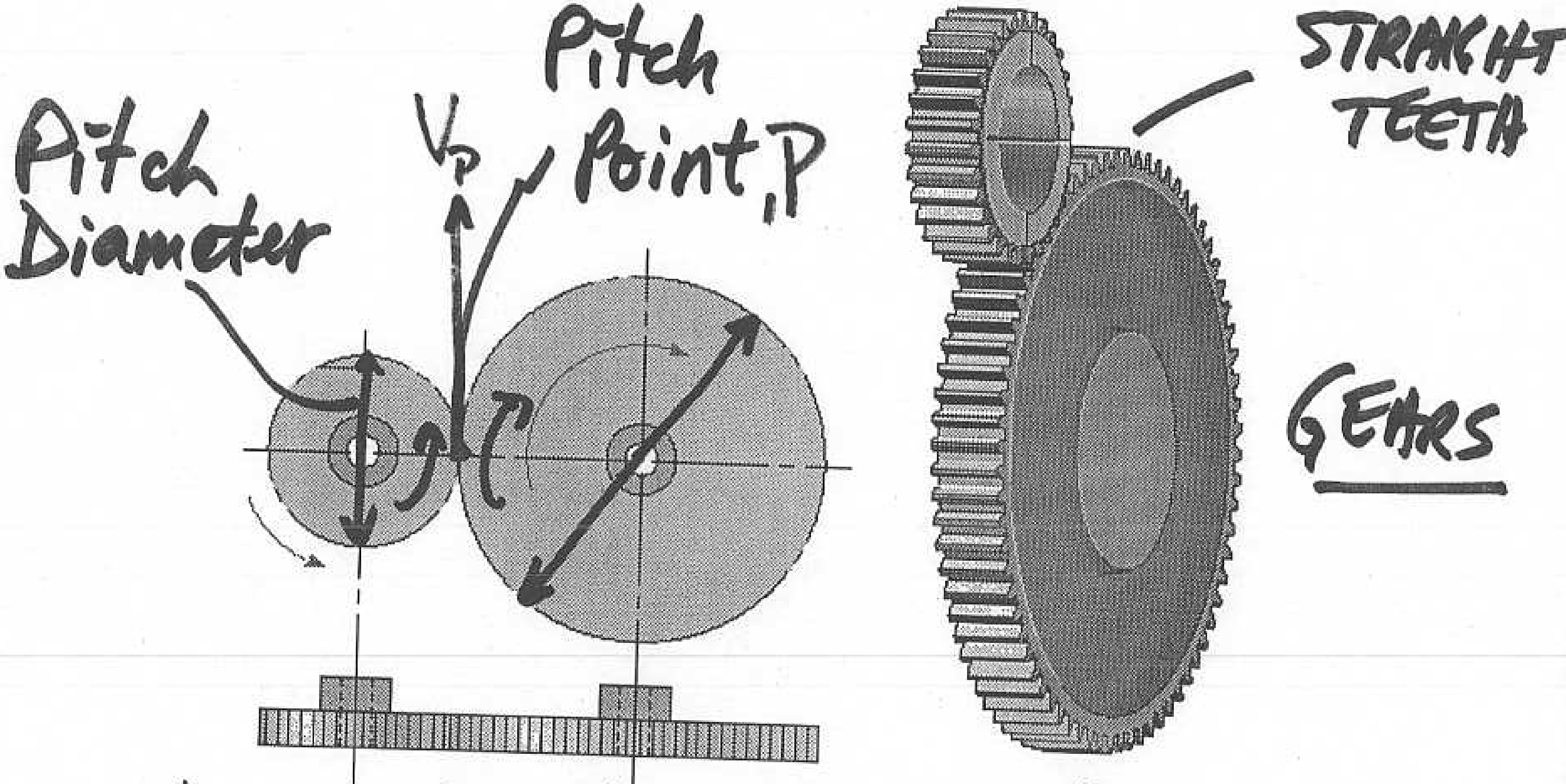
IV Design for Strength < bending teeth / surface fatigue

- AGMA design codes
  - American
  - Gear
  - Manufacturers
  - Assiciation.

- GEARS usually lubricated & operate under elasto-hydrodynamic lubrication to avoid metal to metal contact

- Of course not all gears are metal .... plastic gear design is specialty area ... we won't consider.

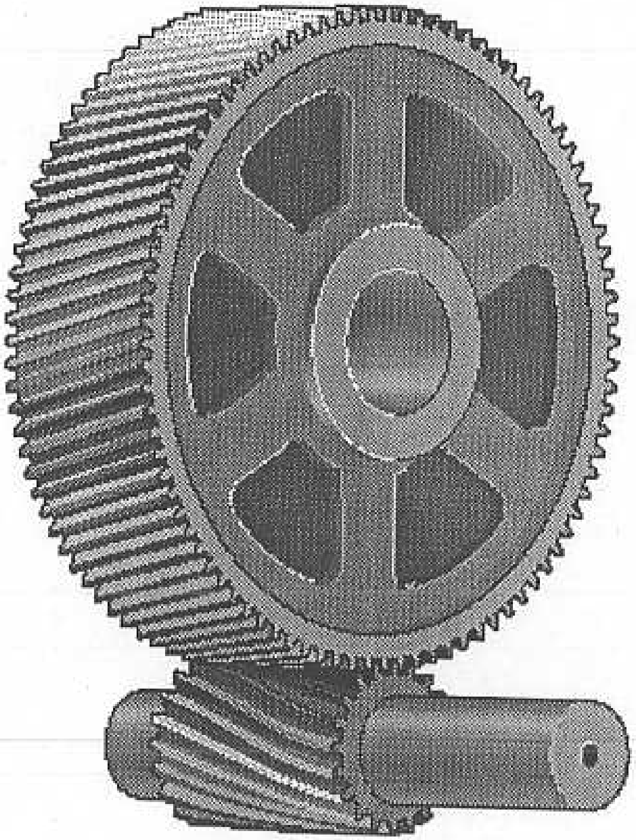
Spur Gear Drive

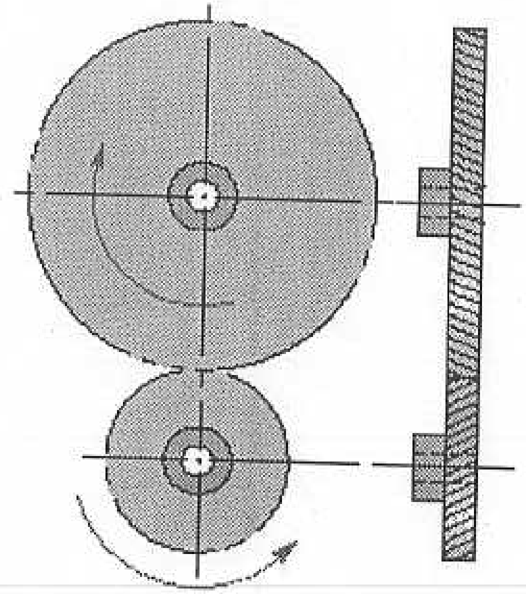


Kinematically → cylinders rolling without slip

Helical Gear Drive

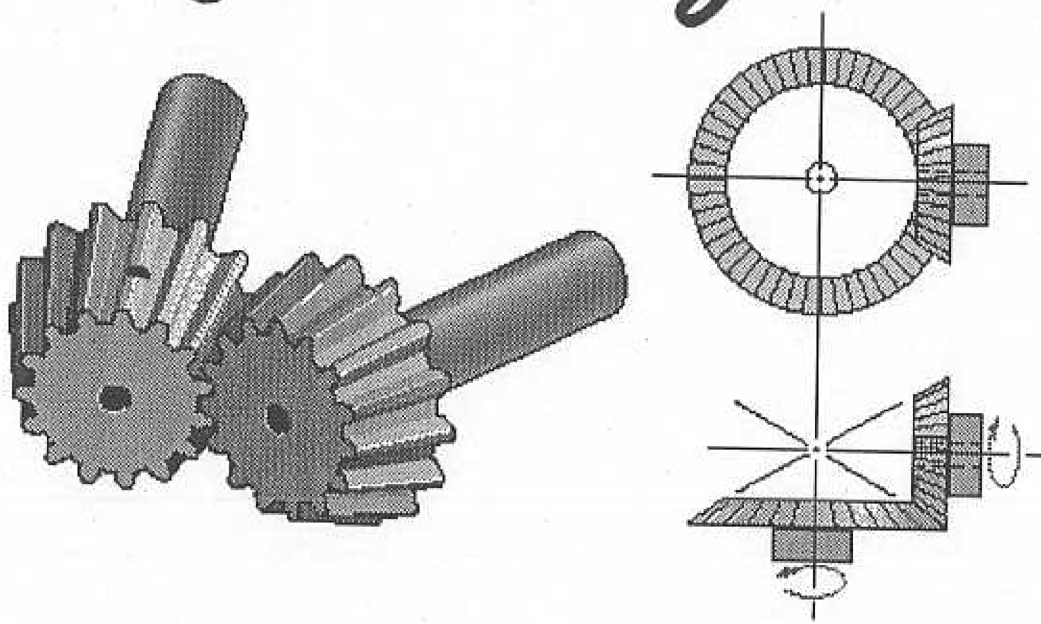
- "HELICAL" TEETH
- SMOOTHER ACTION.





Straight Tooth Bevel Gear Drive

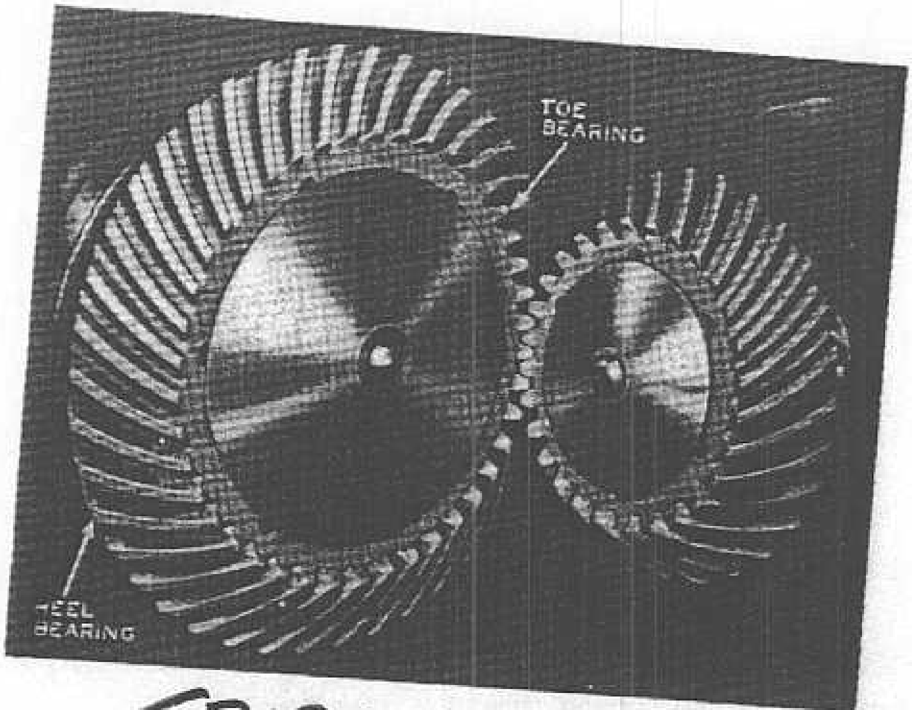
(Like rolling cones)



[redacted] Bevel gear drive with straight teeth.

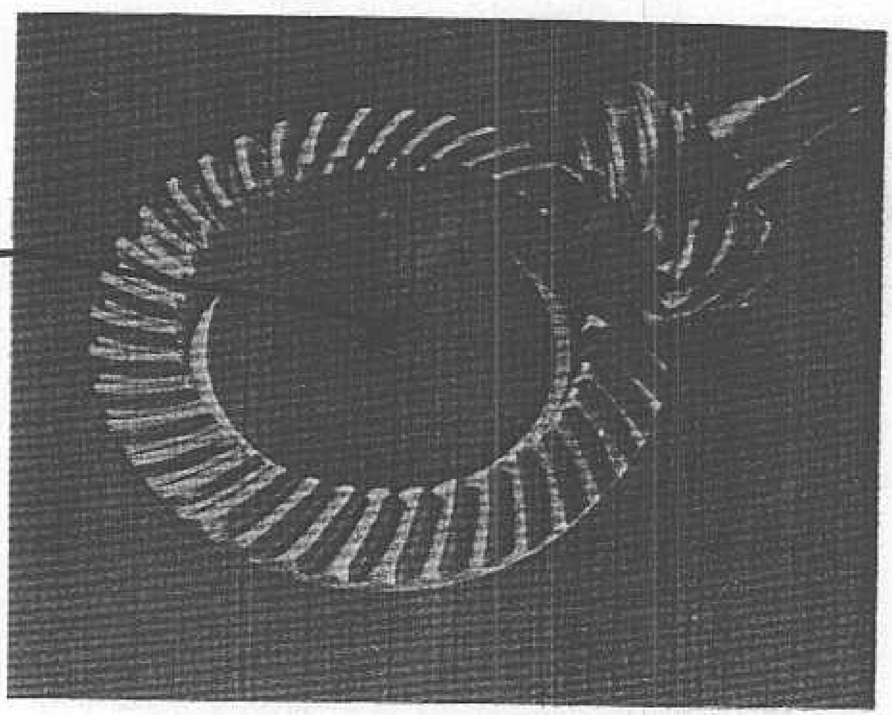
Non parallel shafts... can be 90° or other ∡

[redacted]

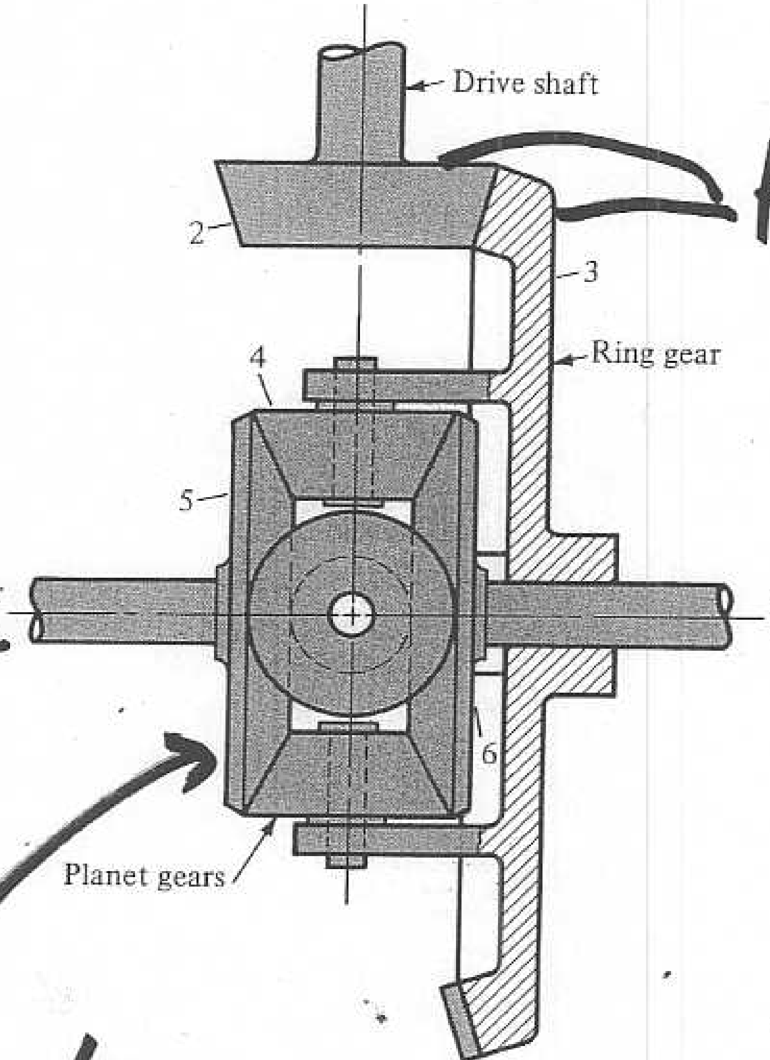


SPIRAL BEVEL
(bevel equiv. of helical)

center
of



HYPOID (AXES OFFSET)
(IN AUTO DIFFERETIALS)



Hypoid gearset

WHEEL

WHEEL

Planetary bevel gears

Worm Gear Drive

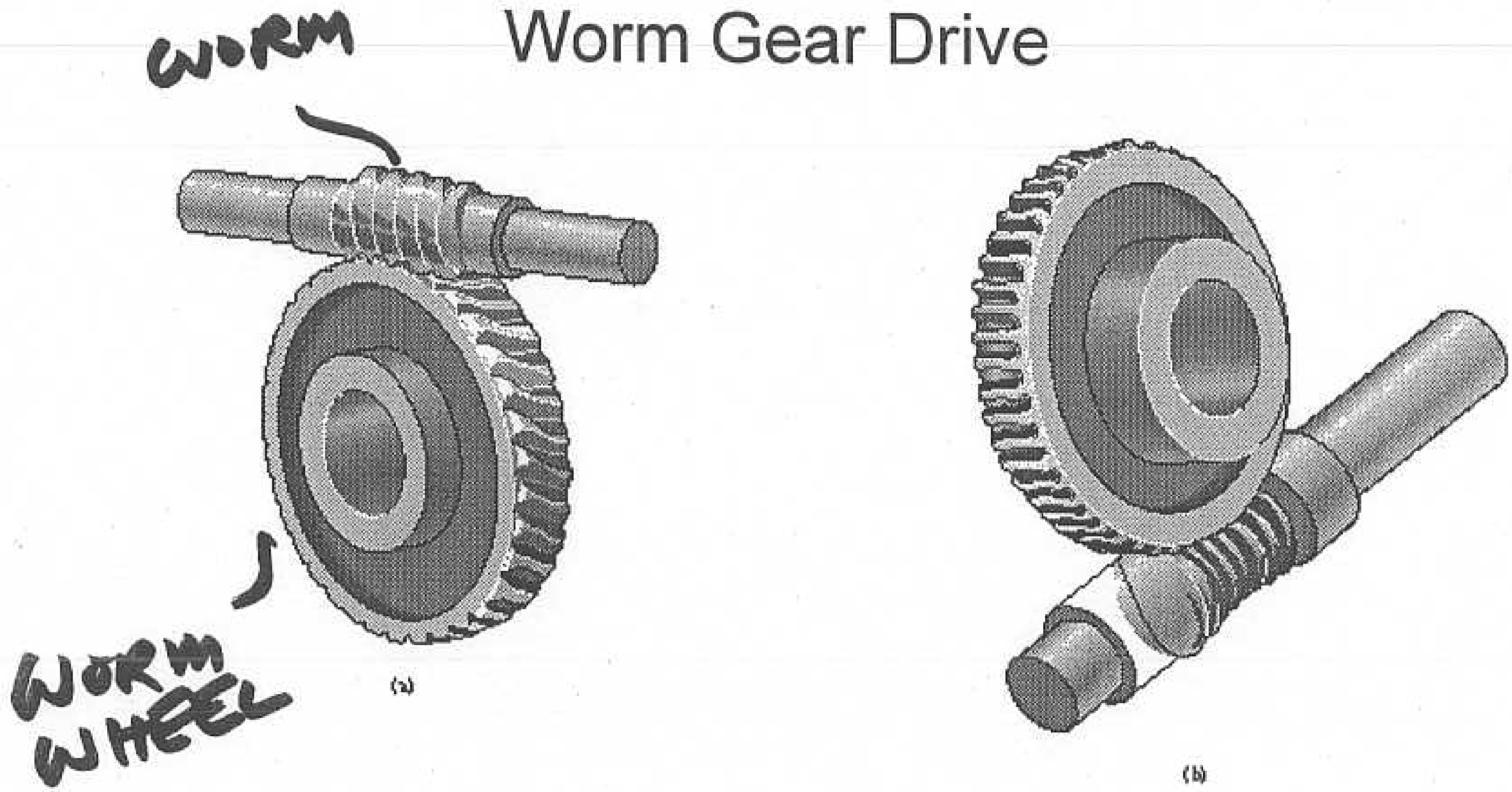


Figure 14.4 Worm gear drive. (a) Cylindrical teeth; (b) double enveloping.

"Cross between screw & gear"

WORM DRIVES WHEEL

(WHEEL CAN'T DRIVE WORM)

Hamrock, Jacobson and Schmid

Law of Gearing

- angular velocity ratio is constant
- pitch point velocity same on "gear" & "pinion" → smaller of two gears

pitch radius, ang velocity

$$r_g \omega_g = V_p = r_p \omega_p$$

$$\frac{\omega_g}{\omega_p} = \frac{r_p}{r_g}$$

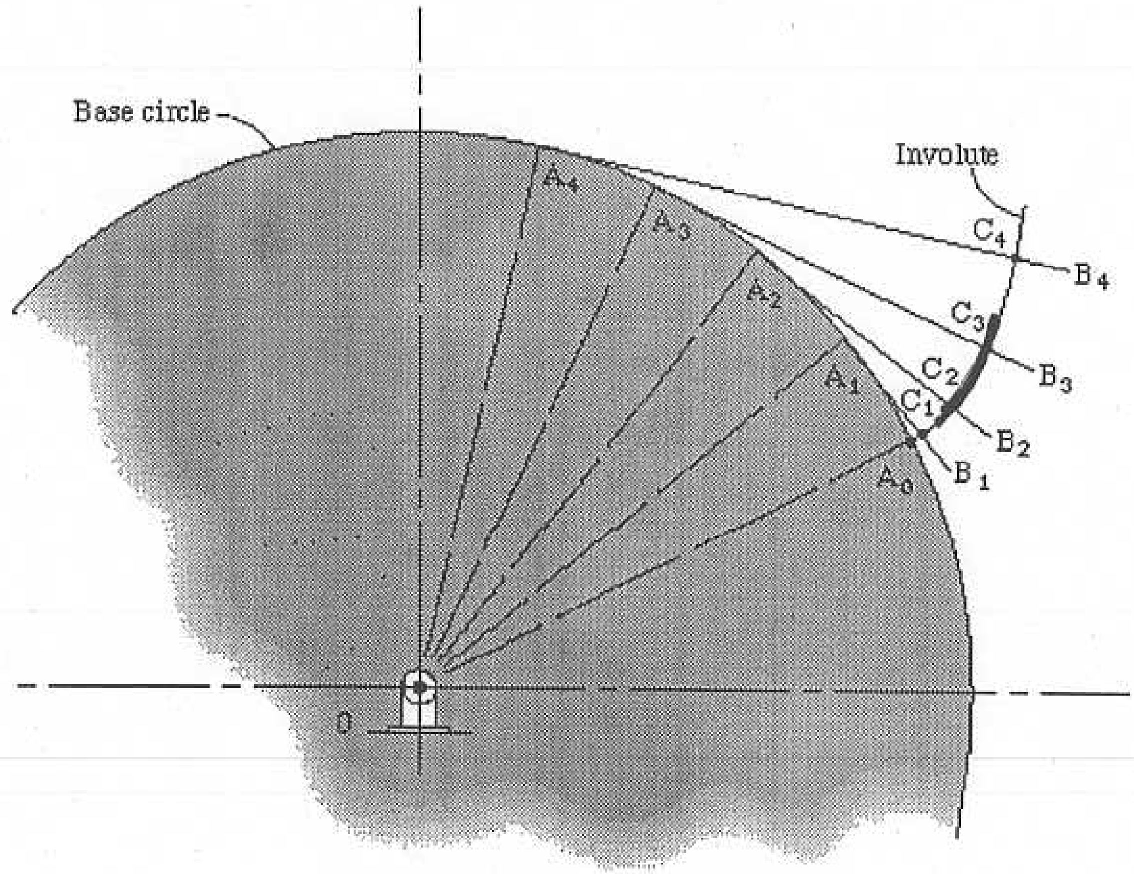
- pitch point "P" is fixed

- Various tooth profiles are possible to achieve

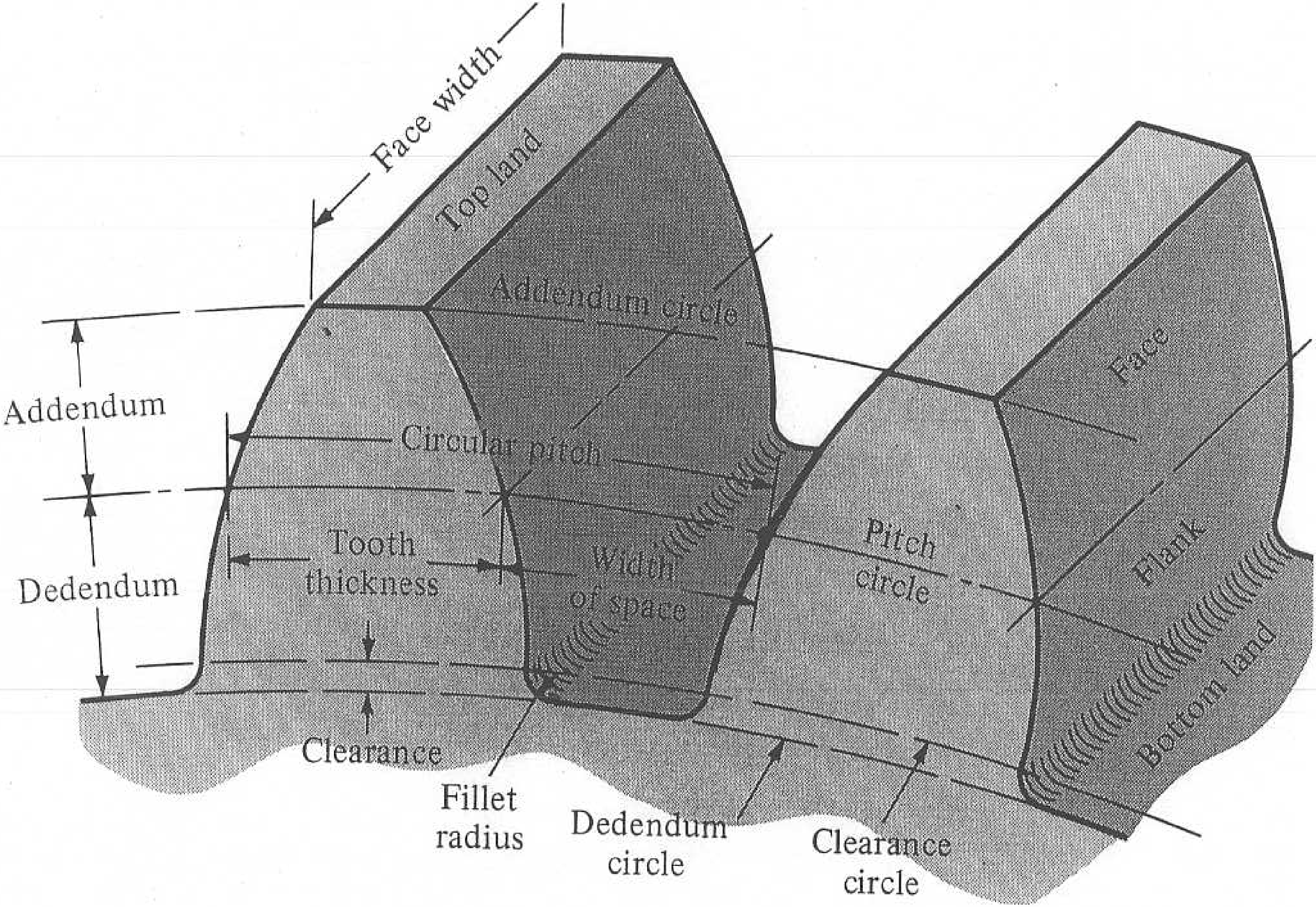
$$\frac{\omega_g}{\omega_p} = \text{Const.}$$

- Most common/std is the <u>involute</u> shape.
- another is cycloid

Involute Curve



Construction of involute curve.



Face width
Top land
Addendum circle
Face
Addendum
Circular pitch
Dedendum
Tooth thickness
Width of space
Pitch circle
Flank
Bottom land
Clearance
Fillet radius
Dedendum circle
Clearance circle

Some terms

Circular pitch, p

... distance along pitch circle between same pts on adjacent teeth

Module, m (in SI)

... $\dfrac{\text{pitch diameter, } d \text{ (mm)}}{\text{Number of teeth, } N}$

Diametral Pitch, P

... $\dfrac{\text{Number of Teeth}}{\text{pitch diameter,}} \quad \dfrac{N}{d \text{ (in)}}$

Diametral Pitches

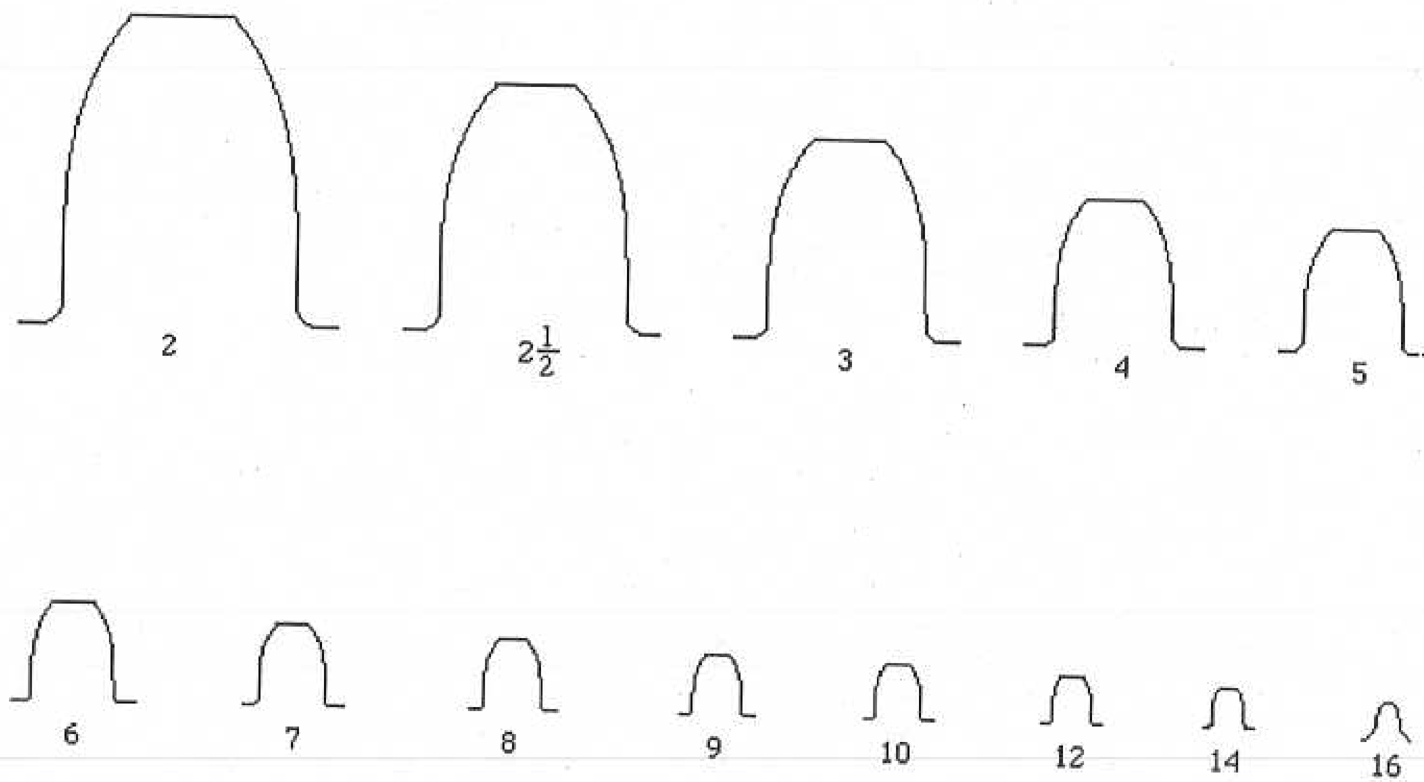


Figure 14.7 Standard diametral pitches compared with tooth size. Full size is assumed.

Preferred Diametral pitches & modules

DIAMETRAL PITCHES

Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

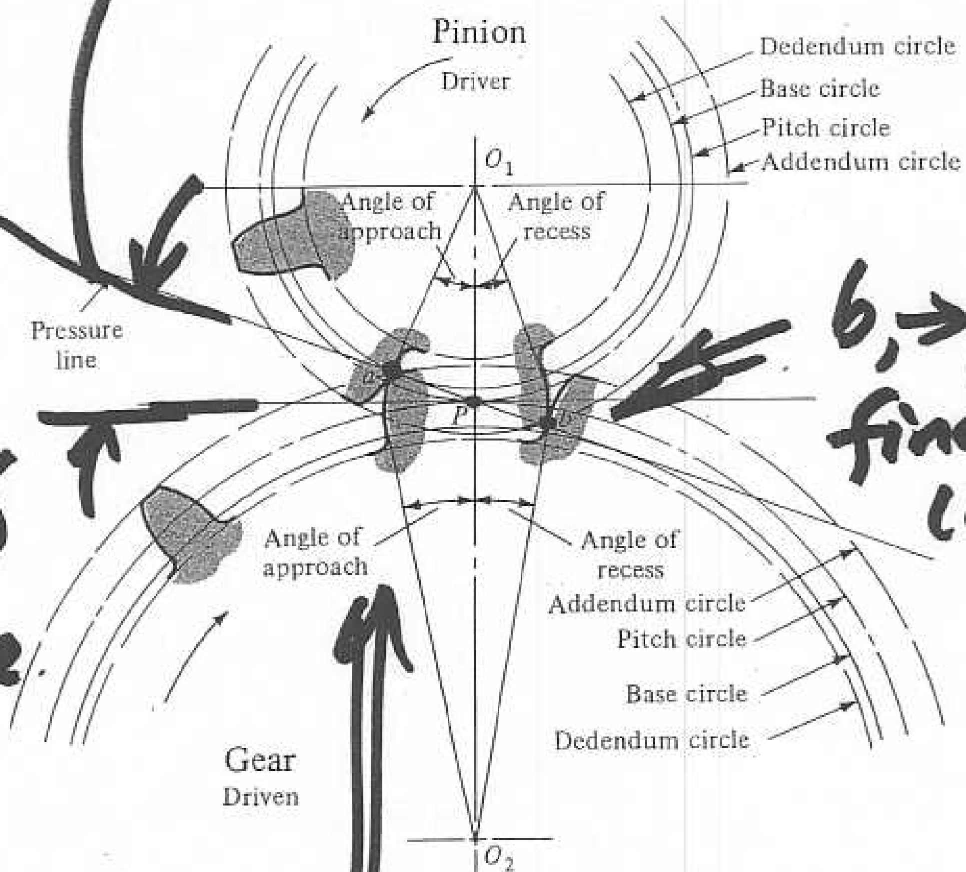
MODULES

Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Tooth action

- Pitch point is fixed
- Pressure line, Line of action followed as teeth interact along <u>common normal</u>
- Theoretically there is pure rolling between teeth @ pitch point... elsewhere slide + roll action.

Pressure line, line of action along which contact occurs



b, → pt of final contact

ϕ

Pressure Angle

a → pt of first contact

356

20° Pressure angle is std & most common

TOOTH SYSTEM	PRESSURE ANGLE ϕ, deg	ADDENDUM a	DEDENDUM b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$ $1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$ $1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$ $1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

SHORTER TEETH

see proportions

Contact ratio, m_c

- Avg # of teeth in contact
- Should be greater than 1.2

Interference

- to be avoided ... when gear tooth & pinion tooth overlap. ... reason for clearance

Back lash

- amt by which tooth space exceeds thickness of mating gear

Backlash

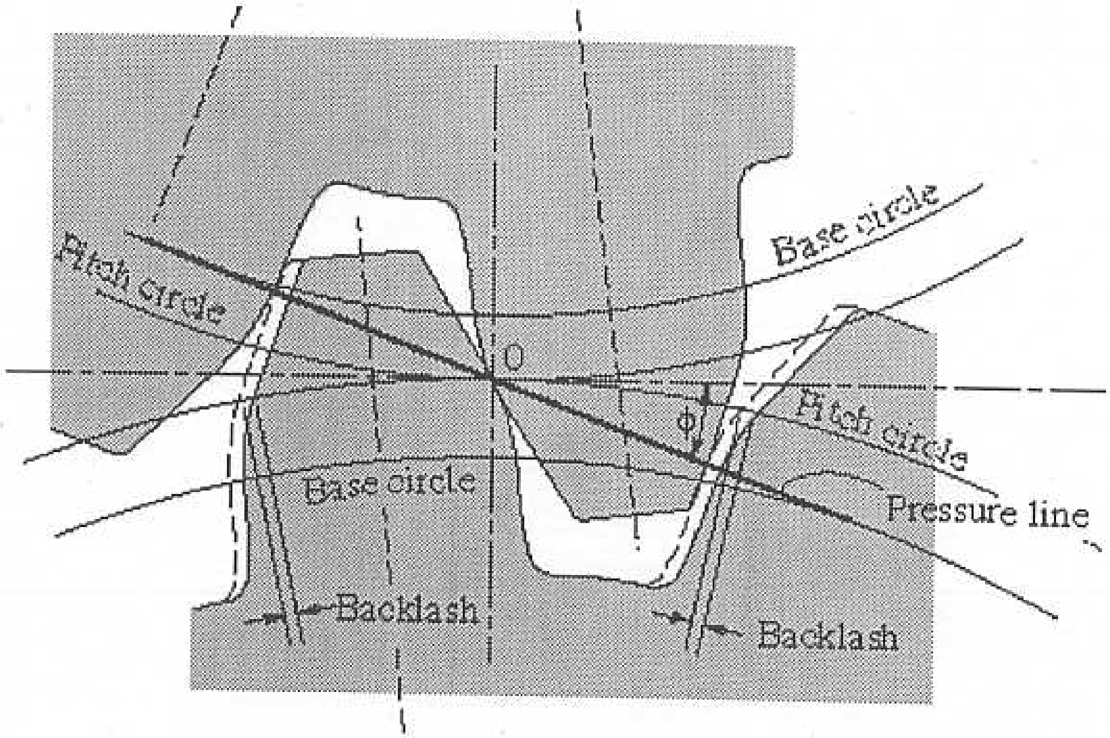


Illustration of backlash in gears.

Recommended Minimum Backlash

Diametral pitch p_d, in.$^{-1}$	Center distance, in.				
	2	4	8	16	32
	Backlash, in.				
18	0.005	0.006	-	-	-
12	0.006	0.007	0.009	-	-
8	0.007	0.008	0.010	0.014	-
5	-	0.010	0.012	0.016	-
3	-	0.014	0.016	0.020	0.028
2	-	-	0.021	0.025	0.033
1.25	-	-	-	0.034	0.042

Recommended minimum backlash for coarse-pitch gears.

GEAR TRAINS

- External mesh
- Internal mesh
- Rack & pinion
- Simple gear train
- Compound gear train
- Planetary gear sets

From Law of Gearing

$$\frac{\omega_g}{\omega_p} = \frac{r_p}{r_g} = \frac{d_p}{d_g}$$

number of teeth

but $d = \frac{N}{P}$ & $\boxed{\left|\frac{\omega_g}{\omega_p}\right| = \frac{N_p}{N_g}}$

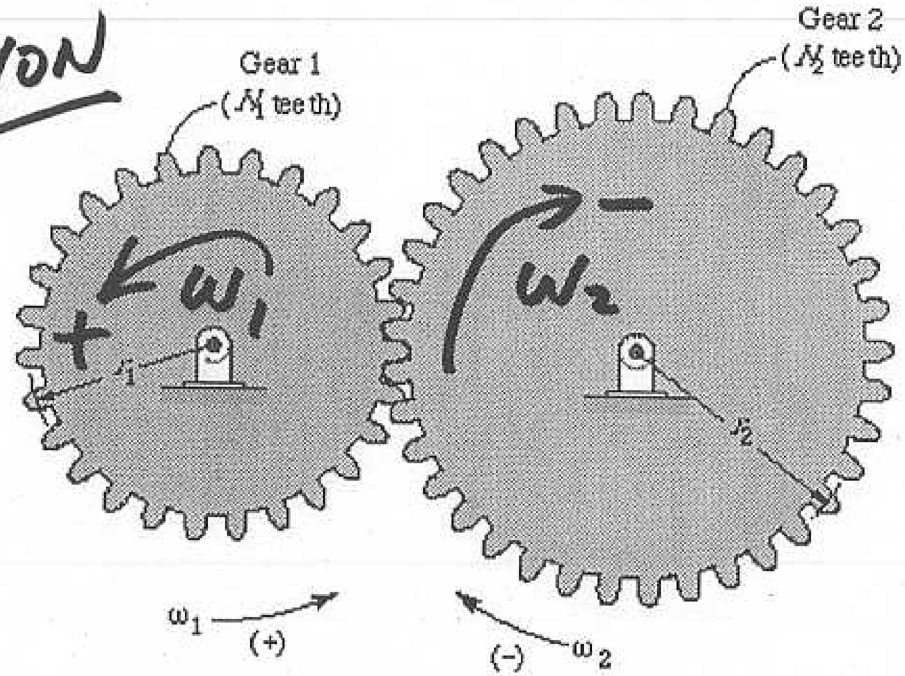
diametral pitch

ABS VALUE

Externally Meshing Spur Gears

PINION

N_1



GEAR

N_2

+ ccw

− cw

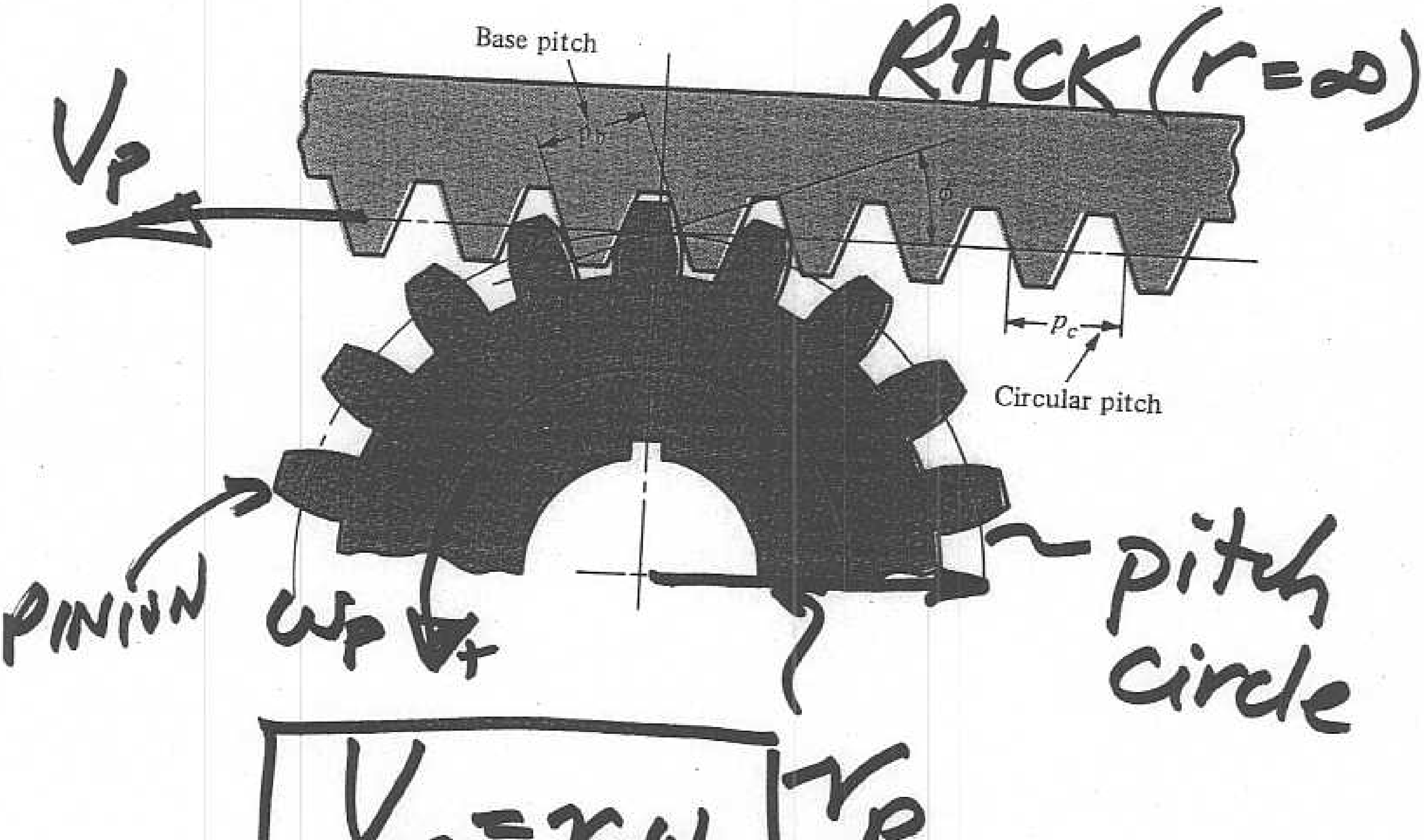
Externally meshing spur gears.

$$\therefore \boxed{\frac{\omega_2}{\omega_1} = -\frac{N_1}{N_2}}$$

ω_2 & ω_2 are in opposite directions

Hamrock, Jacobson and Schmid

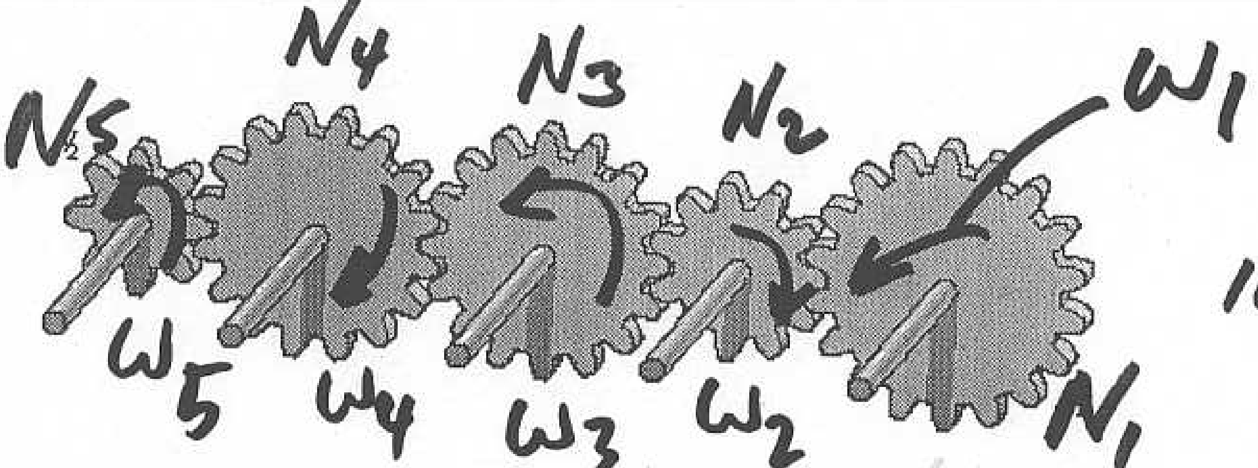
(INVOLUTE) RACK & PINION



$$\boxed{V_p = r_p \omega_p}$$

(NOTE STRAIGHT LINE
TOOTH SHAPE
ON RACK)

Simple Gear Train



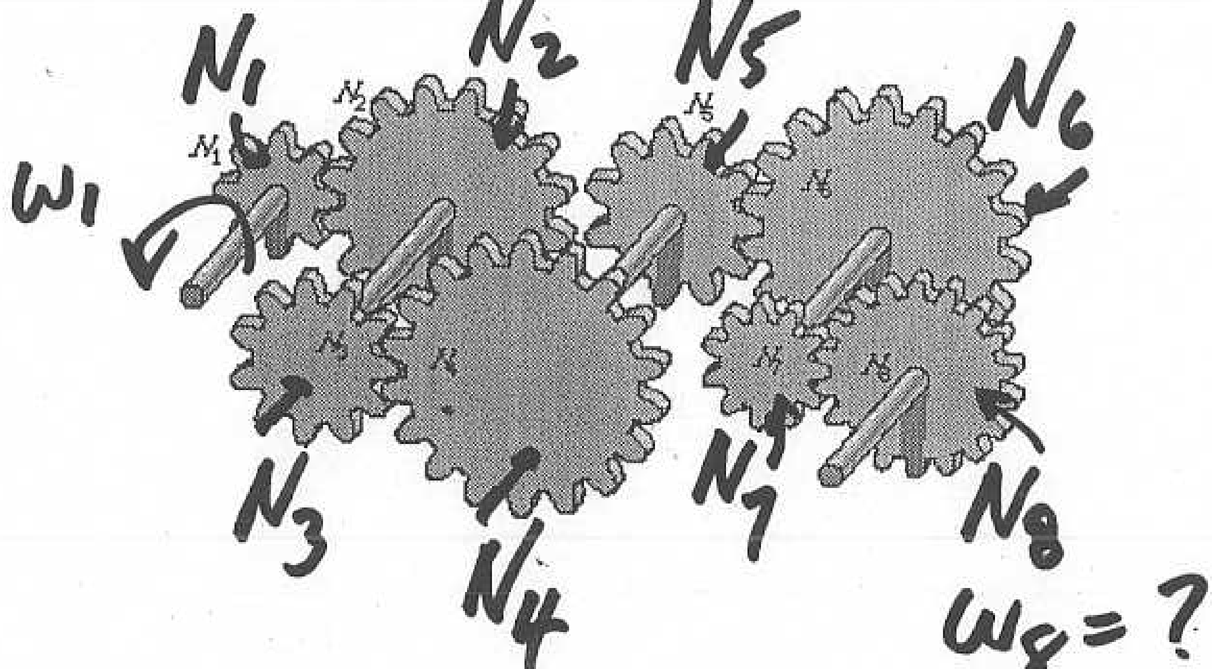
$$\frac{\omega_2}{\omega_1} = -\frac{N_1}{N_2} \qquad \frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3} \qquad etc.$$

$$\frac{\omega_5}{\omega_1} = \frac{\omega_5}{\omega_4}\cdot\frac{\omega_4}{\omega_3}\cdot\frac{\omega_3}{\omega_2}\cdot\frac{\omega_2}{\omega_1} = \left(-\frac{N_4}{N_5}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_2}{N_3}\right)\left(-\frac{N_1}{N_2}\right)$$

Hamrock, Jacobson and Schmid

$$\boxed{\frac{\omega_5}{\omega_1} = \frac{N_1}{N_5}}$$ ---> INTERMEDIATE N's don't matter

Compound Gear Train



$$\frac{\omega_2}{\omega_1} = -\frac{N_1}{N_2} \quad \frac{\omega_4}{\omega_3} = -\frac{N_3}{N_4} \quad \frac{\omega_6}{\omega_5} = -\frac{N_5}{N_6} \quad \frac{\omega_8}{\omega_7} = -\frac{N_7}{N_8}$$

AND $\omega_3 = \omega_2$ & $\omega_5 = \omega_4$ & $\omega_7 = \omega_6$

Therefore:

$$\omega_8 = \left(-\frac{N_7}{N_8}\right)\omega_7 = \left(-\frac{N_7}{N_8}\right)\omega_6$$

$$= \left(-\frac{N_7}{N_8}\right)\left(-\frac{N_5}{N_6}\right)\omega_5$$

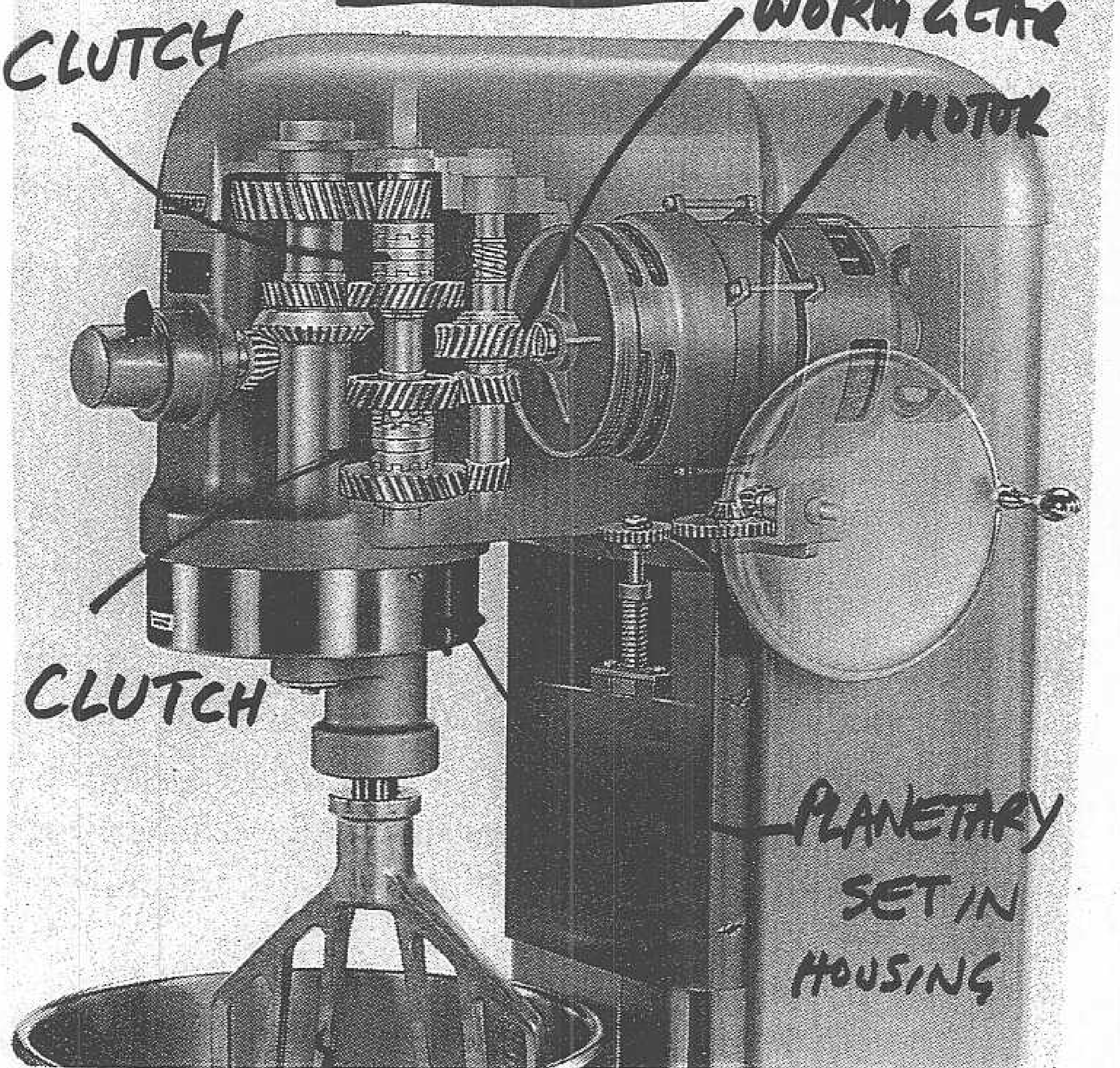
$$= \left(-\frac{N_7}{N_8}\right)\left(-\frac{N_5}{N_6}\right)\omega_4$$

... etc ... so that:

$$\omega_8 = \left(-\frac{N_7}{N_8}\right)\left(-\frac{N_5}{N_6}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_1}{N_2}\right)\omega_1$$

$$\boxed{\omega_8 = \left(\frac{N_7 N_5 N_3 N_1}{N_8 N_6 N_4 N_2}\right)\omega_1}$$

GEARTRAIN EX.



Gears inside industrial mixer.

[Courtesy of Hobart]

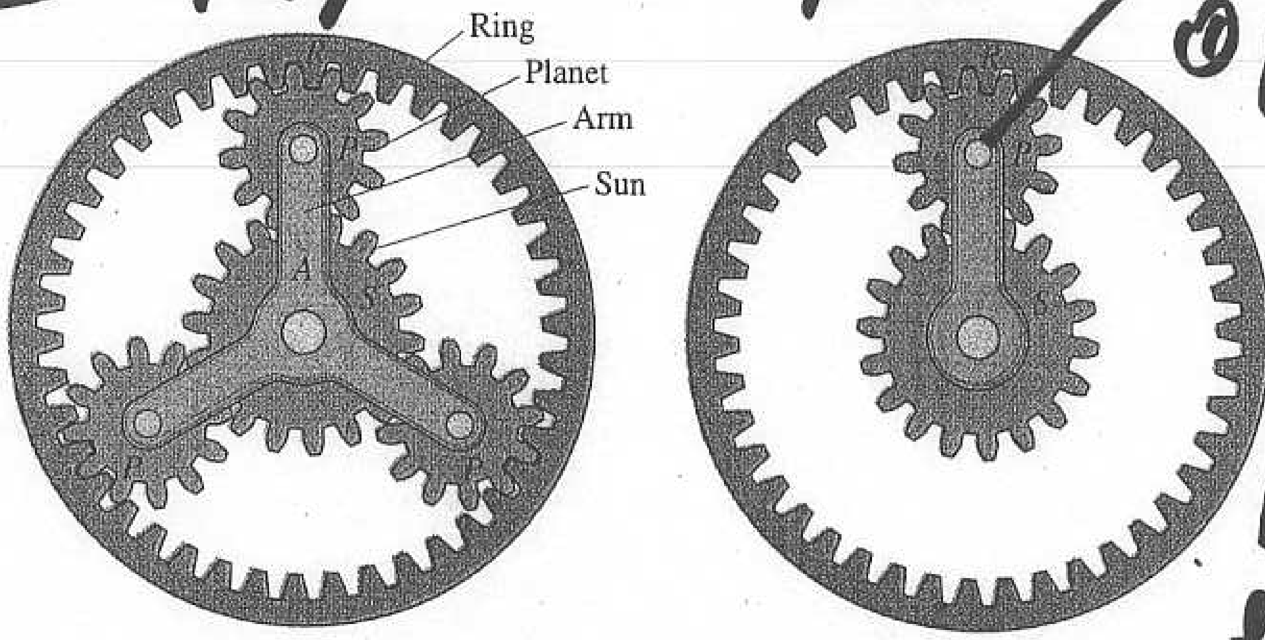
(1) BLADE ROTATES ON SHAFT
(2) BLADE ROTATES WITHIN BOWL?

NOTE: $N_{ring} = N_{sun} + 2N_{planet}$

Planet axis

① Planet rotates about axis

② And it sees one additional rotation will lead rotation of arm

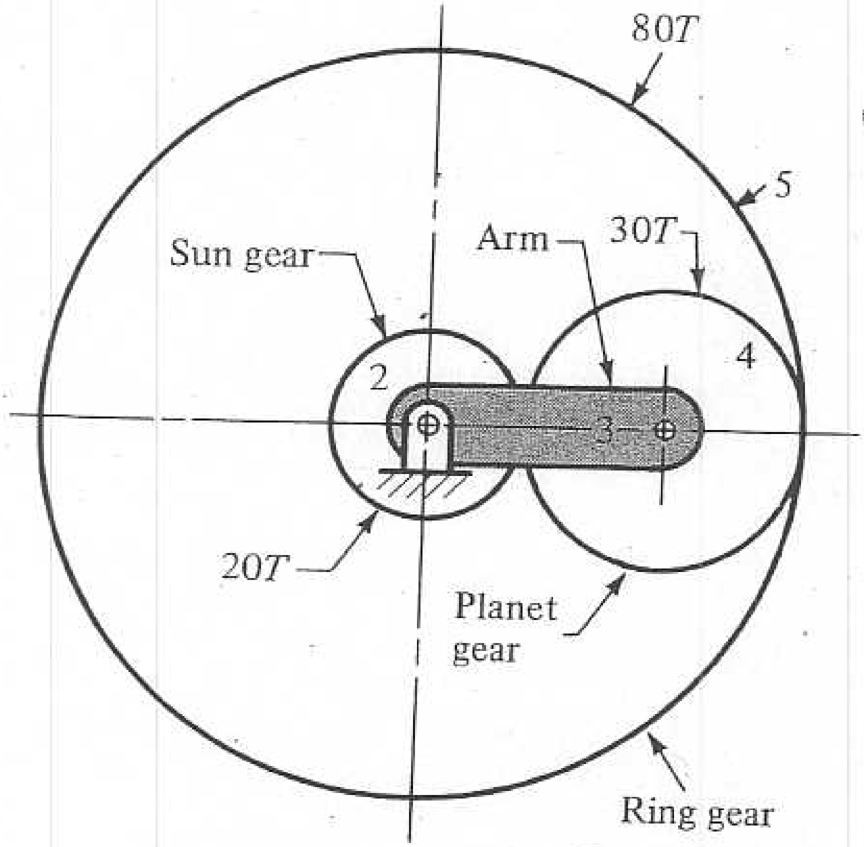


(a) (b)

Illustration of planetary gear.
(a) With three planets (typical); (b) with one planet (for analysis only).

PLANETARY OR EPICYCLIC GEARS

TWO DEGREES OF FREEDOM



Example: ① Fixed ring (One Input)
② Sun rotates at 600 radians/sec (2nd input)
③ Find ω_{arm} and ω_{planet}

Governing eqs to be used

$$-\frac{N_{sun}}{N_{ring}} = \frac{\omega_{ring} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} \quad \cdots (1)$$

also

$$-\frac{N_{sun}}{N_{planet}} = \frac{\omega_{planet} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} \quad \cdots (2)$$

and

$$\frac{N_{planet}}{N_{ring}} = \frac{\omega_{ring} - \omega_{arm}}{\omega_{planet} - \omega_{arm}} \quad \cdots (3)$$

Eqs applicable for values of ω_{arm}

Gear ratios with arm fixed .... i.e $\underline{\omega_{arm} = 0}$

Start with ω_{arm} & eq (1)

$$-\frac{20}{80} = \frac{0 - \omega_{arm}}{600 - \omega_{arm}}$$

$$\omega_{arm} = \frac{150}{1.25} = \underline{\underline{120 \text{ radians/s}}}$$

Now find ω_{planet} from eq (2)

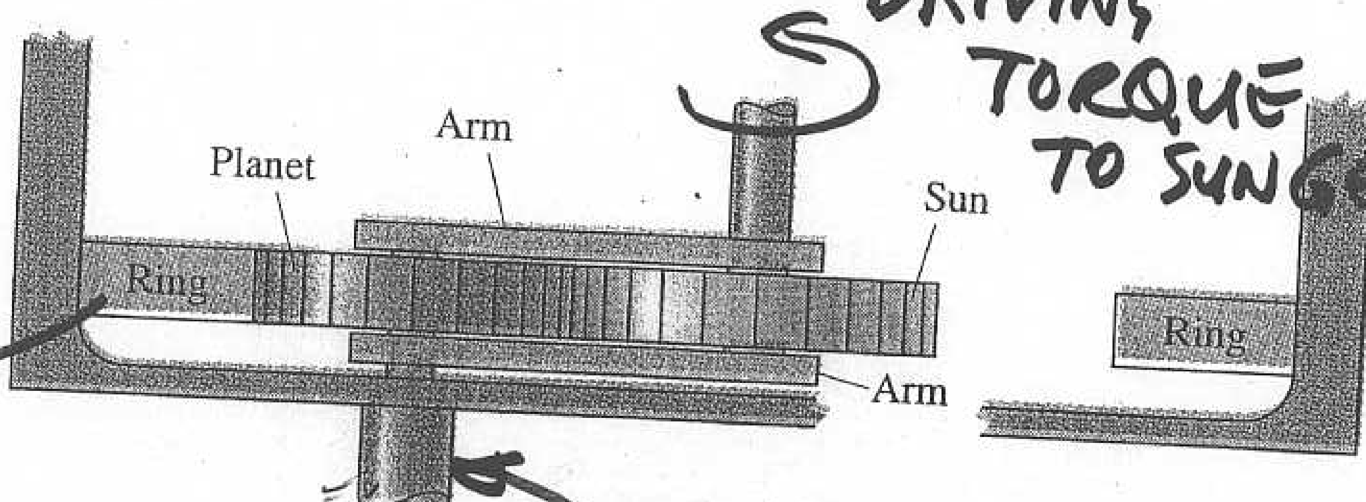
$$-\frac{20}{30} = \frac{\omega_{planet} - 120}{600 - 120}$$

$$\omega_{planet} = -\frac{20}{30}(480) + 120$$

$$\omega_{planet} = \underline{\underline{-240 \text{ radians/s}}}$$

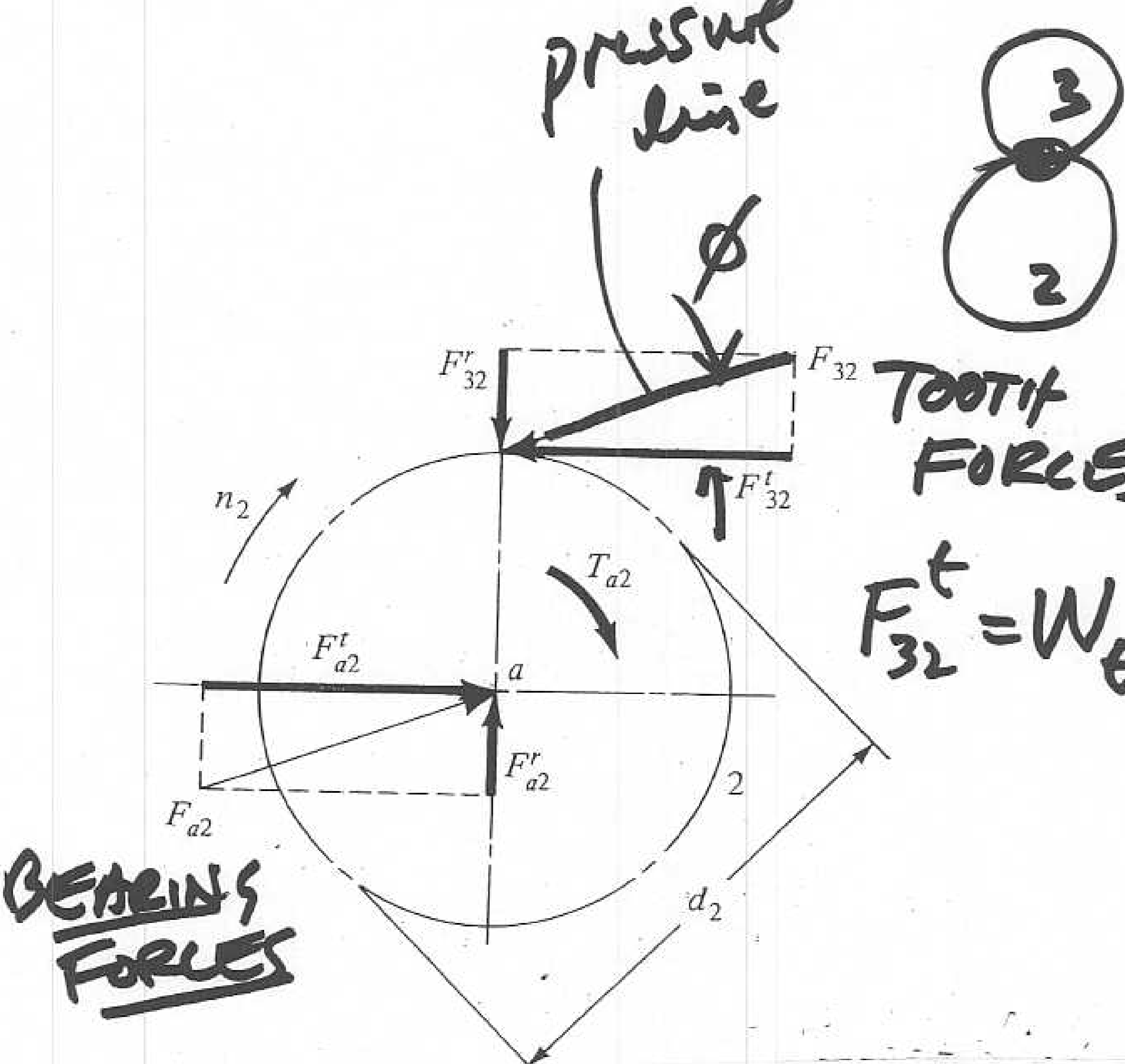
INDUSTRIAL MIXER (Revisited)

DRIVING TORQUE TO SUN GEAR



RING GEAR IS FIXED

BLADE ATTACHES HERE... spins around planet axis & moves around bowl



$F^t_{32} = W_t$

$W_t = \dfrac{HV}{33000}$

V ft/min $= \dfrac{\pi d n}{12}$ (d in in, n in rpm)

H horsepower

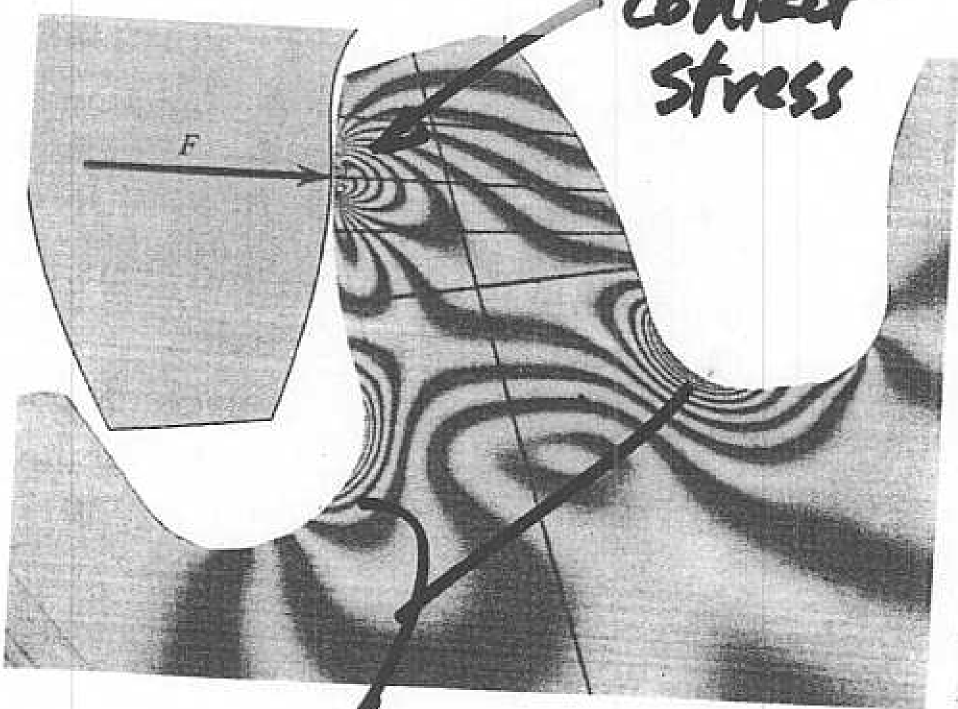
American Units

$W_t = \dfrac{H(60)(10^3)}{\pi d n}$

W_t in kN, H in kW, d in mm, n in rpm.

SI Units

LOADING OF TEETH



contact stress

bending with stress concentration

SIZE/STRENGTH OF GEAR TEETH

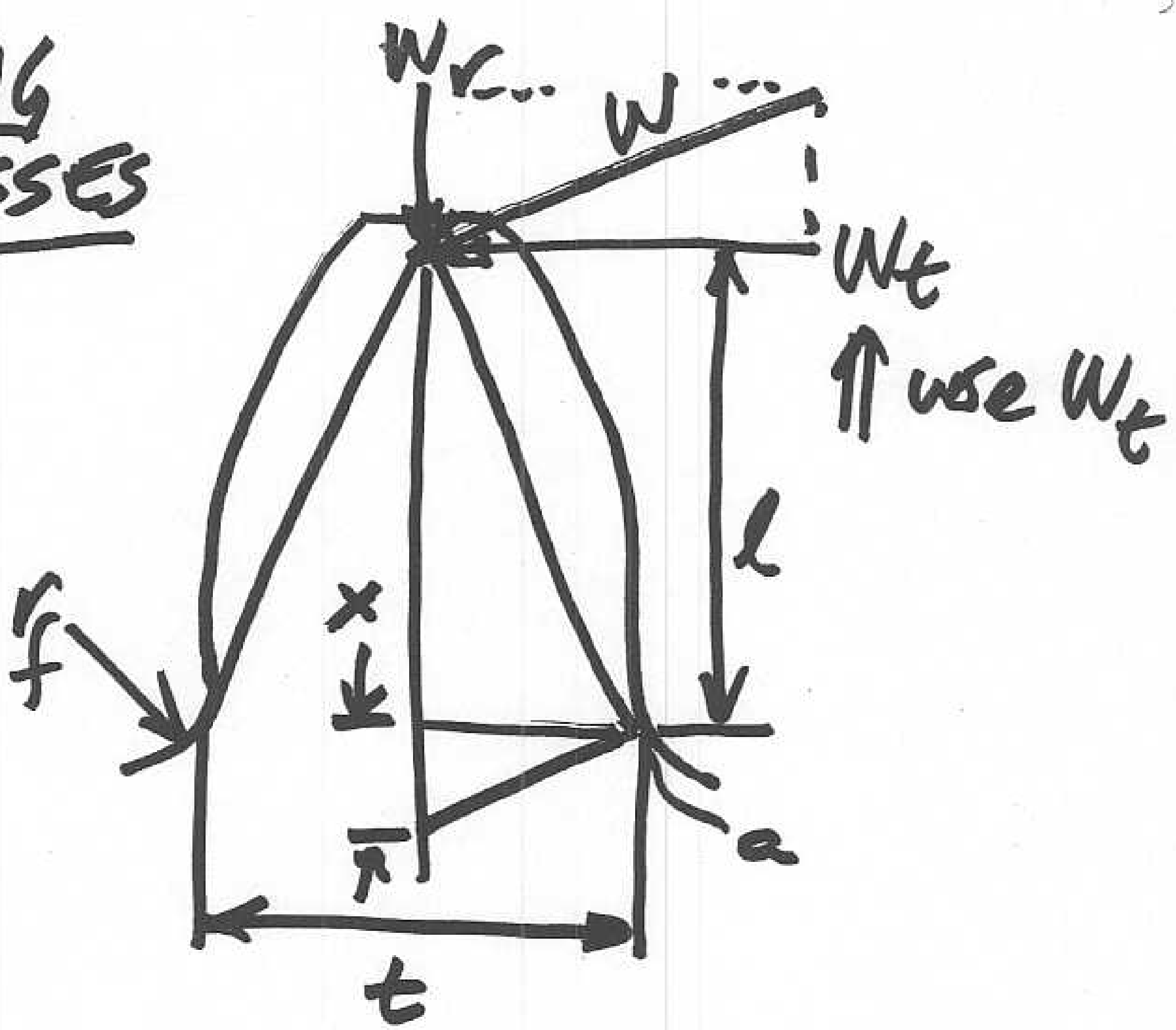
Chapter 14

- We will cover methods in 14-1 & 14-2
- We will NOT use AGMA stress formulas which cover the same mechanics as 14-1 & 14-2 but include more correction factors

---

- In practice use AGMA formulas

BENDING STRESSES



$$\frac{0.3}{P} = r_f$$

For 20° full depth (inches)

Like a beam subjected to W_t

Stress @ a

$$\sigma = \frac{Mc}{I} = \frac{W_t l (t/2)}{\frac{F t^3}{12}}$$

$$= \frac{6 W_t l}{F t^2}$$

$\boxed{F \text{ is face width}}$

Lewis Form Factor, Y (dimension-less qty)

$$Y = \frac{2xP}{3}$$ ← Diametral Pitch

Y depends on tooth shape
e.g. pressure angle &
whether full-depth
or stub teeth.

Developed by Wilfred Lewis
in 1892

IN TERMS OF Y

$$\sigma = \frac{W_t P}{FY}$$

Table 14-2

N NUMBER OF TEETH	Y	N NUMBER OF TEETH	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Y for 20° Full-depth

(Note That $N < 12$ is not used)

$0.245 < Y < 0.485$

Nominal max stress ("static")

... include velocity factor K_v ... accts for impacts & other dynamic effects

$$K_v = \frac{1200}{1200 + V}$$

V ft/min

American Units

$$K_v = \frac{6.1}{6.1 + V}$$

V m/s

SI

Bending stress calc becomes

American

$$\sigma = \frac{W_t P}{K_v F Y}$$

SI

$$\sigma = \frac{W_t}{K_v F m Y}$$

Use Goodman "Approach"...

Stress fluctuates between 0 and σ

$$\therefore \sigma_a = \sigma_m = \frac{\sigma}{2}$$

- but we don't use σ_a & σ_m directly
- Instead use k_e in two parts

$$k_e = k_e'\left(\frac{1}{K_f}\right)$$

accounts for using σ instead σ_a & σ_m

$= 1.33$

obtained in usual way

- Obtain strength S_e via $S_e = k_a k_b k_c k_d k_e S_e'$

$k_a \Rightarrow$ surface finish

$k_b \Rightarrow$ size obtain equiv. diameter

$k_c = 1$ loading (bending)

$k_d = 1$ Temperature

$$k_e = \frac{1.33}{K_f}$$

AND:

$$\frac{S_e}{\sigma} = n_\infty$$

FOR BENDING

INFINITE LIFE IN FATIGUE

Surface Durability

- Based on Hertzian stresses
- Compute σ_c (MAX comp).

Compression

$$\sigma_c = -\left[\frac{1}{\pi\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_p^2}{E_g}\right)}\right]^{1/2} \left[\frac{W_t}{C_v F \cos\phi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2}$$

"Equivalent Modulus" C_P in AGMA Standards

∴ use radii @ Pitch line

∴ $C_v = K_v$ (from before)

$$n_c = \frac{S_c}{|\sigma_c|}$$

S_c ← Comp fatigue strength

$|\sigma_c|$ ← Comp. stress. from eq.

n_c ← Factor of safety for contact fatigue

∴ Above is for 10^8 cylces

∴ S_c is reduced for large #, increased for smaller

$$S_c = 0.4H_B - 10 \text{ kpsi}$$

H_B → Brinnell Hardness

OR

$$= 2.76\,H_B - 70 \text{ MPa}$$

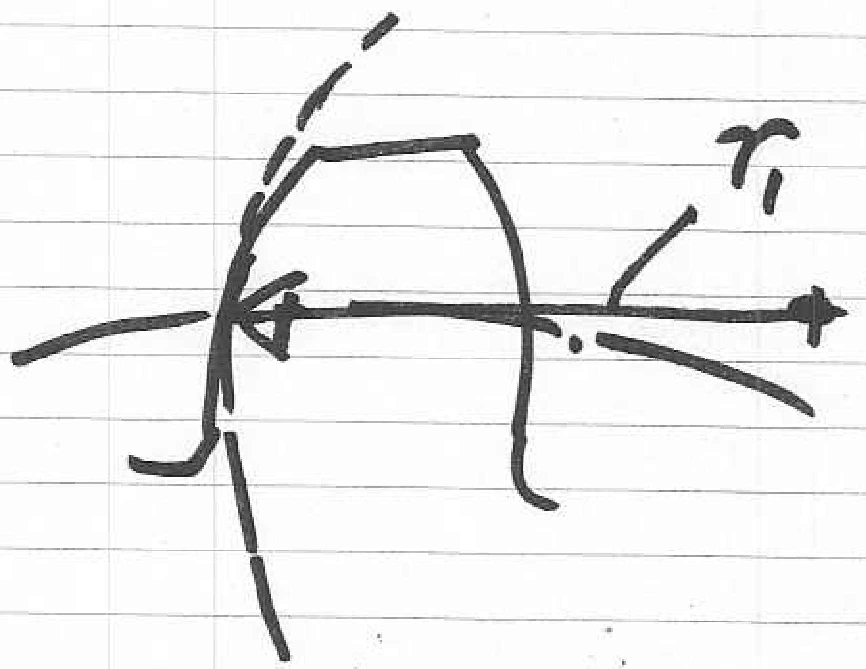
For Steels, Eq. 7-60

NOTE

For Steels: $S_u = 0.45\,H_B$ kpsi

$S_u = 3.10\,H_B$ MPa   5-20(eq)

Radius of curvature of gear teeth @ pitch line



$$r_1 = \frac{d_p \sin\phi}{2} \quad \text{Pinion}$$

$$r_2 = \frac{d_G \sin\phi}{2} \quad \text{Gear.}$$

(Slight)

<u>Example:</u> Variation

on Ex 14-1, 14-2, 14-3

following methodology

develop on preceding 9 pages

of notes. (TO BE USED FOR

DESIGN PURPOSES IN THIS

COURSE)

GEAR: Spur Gear

$P = 8$ teeth/in

$F = 1.5$ in.

<u>Table A-20</u> $N = 16$ teeth

$S_{ut} = 55$ kpsi $\phi = 20°$

$S_{yt} = 30$ kpsi AISI 1020 Steel (as rolled)

$n = 1200$ rev/min.

I

Determine HP capacity based on "Static failure"

(No stress concentration or modifying factors)

... use factor $n_s = 3$

... bending only

i.e. $n_s = \frac{S_y}{\sigma} = 3$

$$W_t = \frac{K_V F Y \sigma}{P}$$

$$K_V = \frac{1200}{1200 + V} = \frac{1200}{1200 + 628}$$

$$\boxed{K_V = 0.656}$$

$$V = \frac{\pi d n}{12} = \frac{\pi (N/P)(1200)}{12} = 628 \text{ ft/min}$$

2"

$$\sigma = \frac{S_y}{n_s} = \frac{30}{3} = \underline{10\,kpsi}$$

From Table 14-2 $Y = 0.296$
(for $P = 8$ & $N = 16$)

$$\therefore W_t = \frac{(0.656)(1.5)(0.296)(10^4)}{8}$$

$$\boxed{W_t = 364\ lbs}$$

HP is $H = \dfrac{W_t V}{33{,}000}$

$$\boxed{H = 6.93\ hp}$$

II Determine hp capacity for infinite life (BENDING)

$$\sigma = \frac{S_e}{n_\infty} = \frac{S_e}{3}$$

Need to determine S_e

$$S_e = k_a k_b k_c k_d k_e S_e'$$

$$S_e' = 0.504\, S_{ut}$$

$$= (0.504)(55) = \underline{27.7\ \text{kpsi}}$$

$k_a = a S_{ut}^{\,b}$ $a = 14.4$, $b = -0.718$ > Table 7-4

$$\boxed{k_a = 0.811}$$

SEE BELOW

$$k_b = \left(\frac{d_e}{0.3}\right)^{-0.1133} = \left(\frac{0.495}{0.3}\right)^{-.1133} = \underline{\underline{.94}}$$

$\div$ To find $d_e = 0.808(hb)^{1/2}$

(Equiv. diam $= 0.808(Ft)^{1/2}$

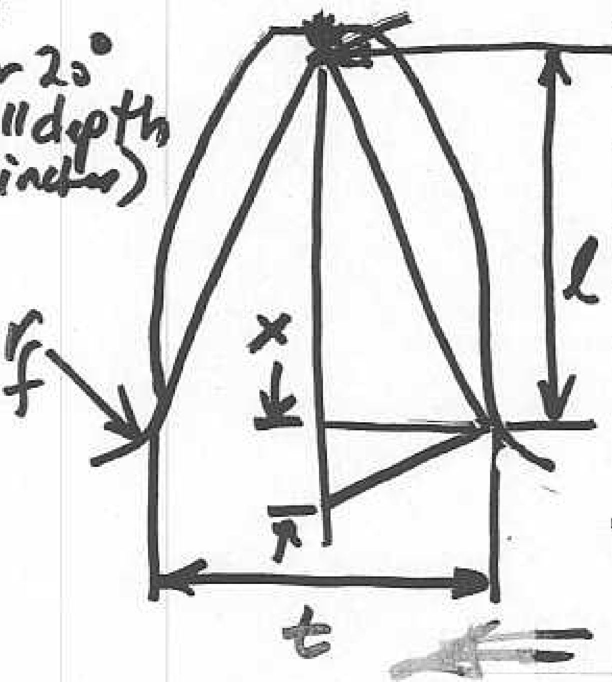
eq. 7-19)

$\div$ need t; (tooth thickness)

$t = (4\ell x)^{1/2}$

$\frac{0.3}{P} = r_f$

For 20° full depth (inches)



$$\ell = \frac{1}{P} + \frac{1.25}{P} = \underline{0.281\text{ in}}$$

$$x = \frac{3Y}{2P} = \frac{3(.296)}{2(8)}$$

$$x = \underline{0.0555}\text{ in}$$

$$t = (4\ell x)^{1/2} = \underline{0.25\text{ in}}$$

$$d_e = .808\left[(1.5)(.25)\right]^{1/2}$$

$$d_e = 0.495\text{ in}$$

$k_c = 1$ bending

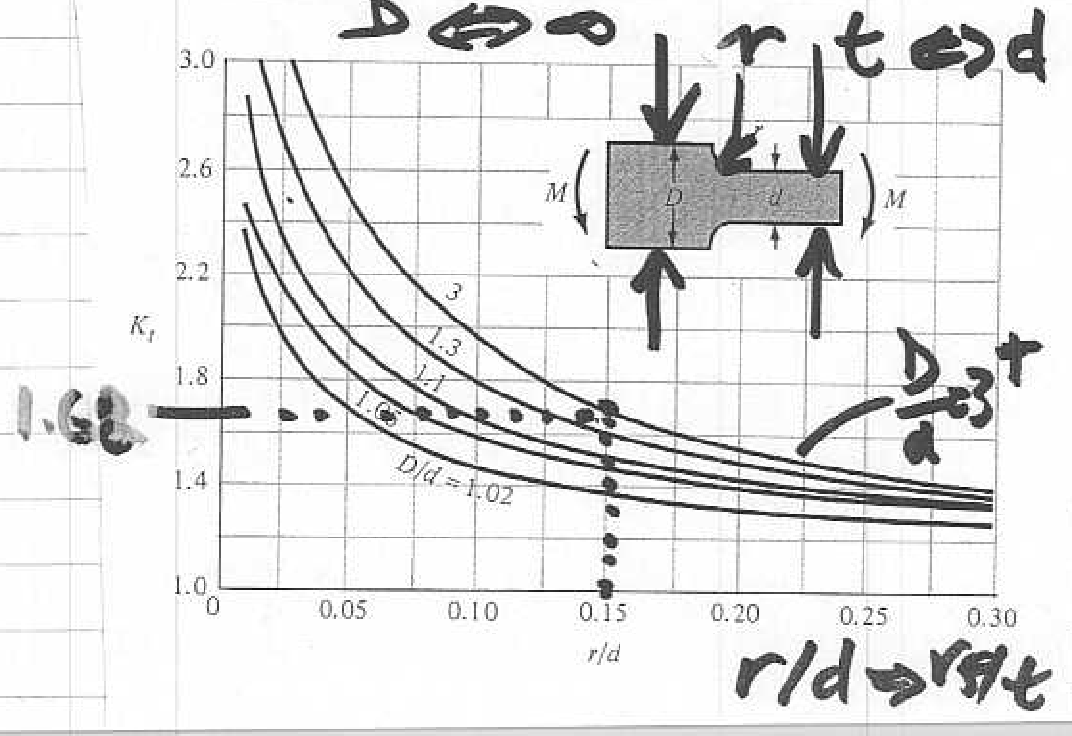
$k_d \doteq 1$ No temperature effect

$k_e = 1.33\left(\frac{1}{K_f}\right)$

$K_f = 1 + q(K_t - 1)$

$q = 0.78$ (from Fig 5-16)

K_t from Fig A-15-6



$r_f = \frac{0.3}{P}$

$= 0.0375$ in

$\frac{r_f}{t} = \frac{.0375}{.25}$

$\left(\frac{r}{d}\right) = 0.15$

$$\therefore K_f = 1 + (0.78)(1.68 - 1)$$
$$= \underline{1.53}$$

$$\therefore k_e = 1.33\left(\frac{1}{1.53}\right) = \underline{\underline{0.87}}$$

Now

$$S_e = (0.811)(0.945)(1)(1)(0.87)(27.7)$$

$$\boxed{S_e = 18.4 \text{ kpsi}}$$

$$\sigma = \frac{S_e}{n} = \frac{18.4}{3} = \underline{\underline{6.1 \text{ kpsi}}}$$

$$W_t = \frac{K_v F Y}{P}(6180) = \underline{224 \text{ lb}}$$

$$H = \left(\frac{224}{364}\right)(6.93) = \underline{\underline{4.26 \text{ hp}}}$$

Recall for static failure was 10 kpsi ≈ 60% of static

– 16 tooth pinion meshes with 50 tooth

III Determine factor of gear safety against surface fatigue (10^8 cycles)

$n = \frac{S_c}{\sigma_c}$ for $W_t = 224$ lb & $H = 4.26$ hp

(both AISI 1020 steel)

$$C_p = \left[\frac{1}{\pi\left(\frac{(1-\nu_p^2)}{E_p} + \frac{(1-\nu_G^2)}{E_G}\right)}\right]^{1/2}$$

$E_p = 30$ Mpsi $= E_G$ $\quad \nu_p = \nu_G = 0.3$

$C_p = \underline{2085}$

$$r_1 = \frac{2 \sin 20^\circ}{2} = \underline{0.342''} ; \text{ also } r_2 = \underline{1.069 \text{ in}}$$

$$\sigma_c = -2085\left[\frac{W_t}{C_v F \cos\phi}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right]^{1/2}$$

$$= -2085\left[\frac{224\left(\frac{1}{.342}+\frac{1}{1.069}\right)}{\underbrace{.656}_{\text{Same as } K_v}(1.5)\cos 20^\circ}\right]^{1/2}$$

$$\boxed{\sigma_c = -63{,}756 \text{ psi}}$$ Max Contact Stress

$S_c = 0.4\,H_B - 10$ kpsi

note that $H_B = \frac{S_{ut}}{0.45} = \frac{55}{0.45} = \underline{\underline{122}}$

$\underline{S_c = 38{,}800 \text{ kpsi}} = \underline{(0.4(122) - 10)}$ kpsi

$$n = \frac{S_c}{|\sigma_c|} = \frac{38\,800}{63756} = \underline{\underline{0.61}}$$

But 222 lb $= W_t$ includes some factor of safety (bending fatigue ... $n_\infty = 3$

If we recalculate σ_c with

$$\frac{224}{3} = 74.7\text{ lb} = W_t$$

$$\sigma_c' = 36,809$$

$$n = \frac{38,800}{36,809} = \underline{1.05}$$

SURFACE FAILURE BEFORE BENDING FAILURE

STILL MUCH LESS THAN "3" AS IN BENDING

(Fatigue)

AGMA BENDING STRENGTH, St (like Se)

AGMA CLASS	COMMERCIAL DESIGNATION	HEAT TREATMENT	MINIMUM HARDNESS AT SURFACE	CORE	S_t psi	S_t MPa
A-1 through A-5 STEELS	—	Through-hardened and tempered	180 BHN	—	25–33 000	(170–23
			240 BHN	—	31–41 000	(210–28
			300 BHN	—	36–47 000	(250–32
			360 BHN	—	40–52 000	(280–36
			400 BHN	—	42–56 000	(290–39

SURFACE FATIGUE STRENGTH Sc

high fatigue Sc

AGMA CLASS	COMMERCIAL DESIGNATION	HEAT TREATMENT	MINIMUM HARDNESS AT SURFACE	psi	MPa
A-1 through A-5 STEELS	—	Through-hardened and tempered	180 BHN and less	85–95 000	(590–660)
			240 BHN	105–115 000	(720–790)
			300 BHN	120–135 000	(830–930)
			360 BHN	145–160 000	(1000–1100)
			400 BHN	155–170 000	(1100–1200)

SOME STANDARD GEAR STEELS

Note high hardnesses HB

high surface strength

For GEARS WE USE MUCH STRONGER STEELS