from eq. 2-8

\[ \sigma_1, \sigma_2 = \frac{60 - 0 + \sqrt{(60 - 0)^2 + 40^2}}{2} \]

\[ \sigma_1, \sigma_2 = 80, -20 \text{ MPa} \]

\[ \gamma_{\text{max}} = 180 \left( \frac{20}{2} \right)^{1/2} = 50 \text{ MPa} \]

\[
\tan 2\phi_p = \frac{2(40)}{60 - 0} = \frac{4}{3}
\]

\[ 2\phi_p = 53^\circ \quad \phi_p = 26.5^\circ \]
Also look at Examples 2-4, 2-5, 2-6 in text.

- No need to formally use vectors but you may if you are used to it
THICK-WALLED CYLINDERS

inner and outer pressures $p_i \neq p_o$

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

NOTE: if $r_i = 0$ (solid cylinder)

$$\sigma_{r_o} = -p_o \quad \text{and} \quad \tau_{r_o} = 0$$
Special Case, \( p_o = 0 \)

As usual, positive values indicate tension and negative values, compression.

The special case of \( p_o = 0 \) gives

\[
\sigma_t = \frac{r^2 p_i}{r_o^2 - r_i^2} \left( \frac{r_o^2}{r_i^2} \right)
\]

\[
\sigma_r = \frac{r^2 p_i}{r_o^2 - r_i^2} \left( \frac{r_i^2}{r_o^2} \right)
\]

The equations of set (2-51) are plotted in Fig. 2-24 to show the distribution of stresses over the wall thickness.

**Tangential**

**Radial**

*FIGURE 2-24*

Distribution of stresses in a thick-walled cylinder subjected to internal pressure.

(a) Tangential stress distribution

(b) Radial stress distribution

tension compression
\[ \delta = |\delta_i| - |\delta_o| \]

**PRESS FIT GENERAL SITUATION:**

SHAFT > nominal size by \( \delta_i \)

HUB < nominal size by \( \delta_o \)

INTERFERENCE \( \delta = |\delta_i| + |\delta_o| \)

**NOTE** \( r_i, r_o \) have different meaning from single thicke-walled cylinder eq's.
\[ \delta = |\delta_i| - |\delta_o| \]

**PRESS FIT**

**GENERAL SITUATION:**

SHAFT > nominal size by \( \delta_i \);
HUB < nominal size by \( \delta_o \).

INTERFERENCE \( \delta = |\delta_i| + |\delta_o| \)

**NOTE** \( r_i \) & \( r_o \) have different meaning from single thick-walled cylinder eq's.
STRESSES AT INTERFACE

TANGENTIAL
- HUB IN TENSION
- SHAFT IN COMPRESSION

RADIAL
- BOTH IN COMPRESSION

INNER (SHAFT) \( + \) \(-10\) \( \downarrow \)

OUTER (HUB) \( + \) \(-\) \( \downarrow \)

RADIAL
Applying thick-walled cylinder equation for shaft and hub

Shaft @ R

\[ \sigma_{It} = -p \frac{R^2 + r_i^2}{R^2 - r_i^2} \quad \cdots 2.57 \]

Pressure @ interface

Hub

\[ \sigma_{Ot} = -p \frac{r_o^2 + R^2}{r_o^2 - R^2} \quad \cdots 2.58 \]

Need to find p @ interface

i.e. \[ \sigma_r = -p \]

Then we have \( \sigma_f \) & \( \sigma_r \) values
The tangential (circumferential) strain $\varepsilon_{ot}$ can be related to both the interference:

$$\varepsilon_{ot} = \frac{\delta_0}{R}$$

and the stress

$$\varepsilon_{ot} = \frac{\sigma_{ot}}{E_0} - \frac{\gamma_{0r}}{E_0}$$

So that (using 2-57 and 2-58) we have

$$\delta_0 = \frac{pR}{E_0} \left( \frac{R_0^2 + R^2}{n_0^2 - R^2} + \gamma_0 \right)$$

Similarly

$$\delta_i = \frac{pR}{E_i} \left( \frac{R^2 + r_i^2}{R^2 - r_i^2} - \gamma_0 \right)$$
recall that $\delta = \delta_i + \delta_0$, we can solve previous 2 eqs. for $p$. If one unit $E, \gamma$

\[ p = \frac{E \delta}{R} \left( \frac{R_0^2 - R^2}{2R(R_0^2 - r_i^2)} \right) \]

Note $R_0 > R$ and $R > r_i$.

\[ \therefore \text{p is positive number} \]

but The radial stress at interface $\sigma_r = -p$ (i.e. compressive)
Steel shaft & hub

E.g. \( R_i = 0 \), \( R = 1" \) \( r_c = 2" \)

\( \delta = 0.002" \) \( E = 30 \times 10^6 \text{ psi} \)

From eq. 2-60

\[
p = \frac{30 \times 10^6 \times 2 \times 10^{-3} \left(\frac{(2^2-1^2)}{2(1)(2^2)}\right)}{1}
\]

\[
= 30 \times 10^6 \times 2 \times 10^{-3} \times \frac{3}{8}
\]

\[
p = 22.5 \times 10^3 \text{ psi}
\]

1 inch length has area, \( A \)

\[
\pi d(1) = 2\pi R(1) = 12.56 \text{ in}^2
\]

\( N = 282,000 \text{ lb} \)

\( \mu N = 0.3(282,000) = 84,600 \text{ lb} \)
SHRINK FIT

The hub is sized smaller than the shaft for proper interference.

Hub is heated to increase its size.

\[ \epsilon_T = \alpha \Delta T \]

\[ \frac{S}{R} = \frac{\Delta R}{R} = \alpha \Delta T \]

It's slipped on and allowed to cool.

(\( \alpha_{\text{steel}} = 10^{-5}/\text{oC} \) (200°C for previous example))
CONTACT STRESSES

- When convex-convex contacts or convex-concave contacts form, localized contact stresses occur.

- Can cause surface failure in ball bearings, roller bearings, gears, ball joints etc.

- General form of contact patch is ellipse.

- We will look at circular and line contacts.
"Point Contacts"

FIGURE 11.1
Nomenclature of a ball bearing.
(Courtesy of New Departure—Hyatt Division, General Motors Corporation.)

Rolling-Contact bearings

(a) Deep groove
(b) Filling notch
(c) Angular contact
(d) Shielded
(e) Sealed
(f) External self-aligning
(g) Double row
(h) Self-aligning
(i) Thrust
(j) Self-aligning thrust
"LINE CONTACTS"

**Figure 11-3**

Types of roller bearings:
(a) straight roller; (b) spherical roller thrust; (c) tapered roller thrust; (d) needle; (e) tapered roller; (f) steep-angle tapered roller. (Courtesy of The Timken Company.)
**Contact Stresses**

Two spheres held in contact by force $F$. Contact stress has an elliptical distribution at face of contact of width $2a$.

---

**Spheres**

Avg contact pressure

$$P_a = \frac{F}{A} = \frac{F}{\pi a^2}$$

$$P_{max} = \frac{3}{2} P_a$$

---

From Hertz Theory

$$a = \sqrt{\frac{3F}{8} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)}$$

$d$ positive for convex surface

$d$ negative for concave surface
Magnitude of the stress components below the surface as a function of the maximum pressure for contacting spheres. Note that the normal stresses are all compressive stresses and are slightly below the surface while the shear components are approximated by 0.5 of the maximum value at the surface.

FIGURE 2.33

depth below surface

depth

Ratio of stress to \( p_{\text{max}} \)

\[
\frac{\text{STRESS}}{p_{\text{max}}} \]

\[
\alpha \quad \beta \\
\gamma \quad \delta
\]
Example

Two steel spheres \( d_1 = d_2 = d \)
\( E_1 = E_2 = E = 207 \text{ GPa} \)
\( \nu_1 = \nu_2 = \nu = 0.3 \)
\( F = 100 \text{ N} \)

Find: Contact radius "a"

\[ \frac{p_{\text{max}}}{\text{location of max shear}} = \frac{a}{d_1 + \frac{d_2}{2}} \]

\[ a = 3 \sqrt{\frac{3F (1-\nu^2) E_1 + (1-\nu_2^2) E_2}{8 \left( \frac{1}{d_1} + \frac{1}{d_2} \right)}} \]

\[ = \sqrt{\frac{3F (8) (1-\nu^2)}{8 E} \cdot \frac{d}{x} \cdot \left( \frac{3}{2} \right)} \]
\[ a = \sqrt[3]{\frac{3F}{8\frac{(1-\mu^2)}{E}}} \]

\[ = 3 \sqrt[3]{\frac{300 (0.91)(0.01)}{8 \times 207 \times 10^9 \text{ Pa}}} \]

\[ a = 0.118 \times 10^{-3} \text{ m} = 0.118 \text{ mm} \]

\[ p_{\text{max}} = \frac{3F}{2\pi a^2} \quad @ \quad 0.4a = 0.05 \text{ mm} \]

\[ = 3.429 \text{ GPa} \]

\[ = 480,000 \text{ psi} \]

VERY HIGH ....

Expect plastic deformation
(a) Two cylinders held in contact by force $F$ uniformly distributed along cylinder length $l$. (b) Contact stress has an elliptical distribution at face of contact of width $2b$.

\[
b = \sqrt{\frac{2F}{\pi l} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) / \left( \frac{1}{d_2} + \frac{1}{d_2} \right)}
\]

\[
p_{max} = \frac{2F}{\pi b l}
\]
Example: Two Steel Cylinders

d_1 = d_2 = 10 mm = 0.01 m

Y_1 = Y_2 = 0.03 = Y

E_1 = E_2 = E = 207 GPa

F = 100 N  \quad l = 10 mm = 0.01 m

Find: b, \ p_{\text{max}}, \ \text{location of } \ p_{\text{max}}

b = \sqrt{\frac{2F (1-Y^2) \cdot d}{\pi \ell \cdot E}}

= \sqrt{\frac{2(100)(0.01)(0.91)(0.01)}{\pi (0.01) 207 \times 10^9}}

b = 0.0167 mm = 1.67 \times 10^{-5} m