Screws, Fasteners, etc

Chapter 8

A collection of threaded fasteners. (Courtesy of Clark Craft Fasteners)
CHAPTER 8

SCREWS, FASTENERS, CONNECTIONS

1. Thread geometry
2. Bolt strength
3. Mechanics of Screws
4. Preloaded joints... AXIAL LOAD
   - Preload forces
   - Total forces
   - Bolt & joint stiffness
   - Static failure
   - Fatigue failure
5. Bolted & riveted joints in shear
   - Failure modes
   - Eccentric loading

6. Misc. components
   - Keys, pins, retaining rings
   - Set screws
FIGURE 8-1

Terminology of screw threads.

Sharp vee threads shown for clarity; the crests and roots are actually flattened or rounded during the forming operation.
Some terminology

- pitch, \( p \)
- major (nominal) diameter, \( D, d \)
- Internal Thread (nset)
- External Thread (bolt)
- lead, \( l \) ... distance moved per turn
- Threads are normally right-handed
- American National (Unified) & Metric Threads
(c) triple
\[ L = 3p \]

(b) double
\[ L = 2p \]

(a) single
\[ L = p \]
Figure 8-2

Basic thread profile for metric M and MJ threads. $d_1$ = basic major diameter of internal (external) thread; $D_1$ = basic minor diameter of internal (external) thread; $D_2$ = basic pitch diameter.

$p = \text{pitch}$; $H = 0.5(3/16)$
### TABLE 8-2
Diameters and Area of Unified Screw Threads UNC and UNF

<table>
<thead>
<tr>
<th>SIZE DESIGNATION</th>
<th>NOMINAL MAJOR DIAMETER, in</th>
<th>COARSE SERIES—UNC</th>
<th>FINE SERIES—UNF</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>THREADS PER INCH $N$</td>
<td>TENSILE-STRESS AREA $A_t$, in$^2$</td>
<td>MINOR-DIAMETER AREA $A_m$, in$^2$</td>
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<tr>
<td>0</td>
<td>64</td>
<td>0.002 63</td>
<td>0.002 18</td>
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<tr>
<td>1</td>
<td>56</td>
<td>0.003 70</td>
<td>0.003 10</td>
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<tr>
<td>2</td>
<td>48</td>
<td>0.004 87</td>
<td>0.004 06</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>0.006 04</td>
<td>0.004 96</td>
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<tr>
<td>4</td>
<td>32</td>
<td>0.009 09</td>
<td>0.007 45</td>
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<tr>
<td>5</td>
<td>32</td>
<td>0.014 0</td>
<td>0.011 96</td>
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<tr>
<td>6</td>
<td>24</td>
<td>0.017 5</td>
<td>0.014 50</td>
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<tr>
<td>8</td>
<td>18</td>
<td>0.020 0</td>
<td>0.017 5</td>
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<tr>
<td>10</td>
<td>16</td>
<td>0.024 2</td>
<td>0.020 6</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>0.031 8</td>
<td>0.026 9</td>
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<tr>
<td>1/8</td>
<td>12</td>
<td>0.052 4</td>
<td>0.045 4</td>
</tr>
<tr>
<td>1/4</td>
<td>16</td>
<td>0.077 5</td>
<td>0.067 8</td>
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<td>5/32</td>
<td>14</td>
<td>0.106 3</td>
<td>0.093 3</td>
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<td>12</td>
<td>0.141 9</td>
<td>0.125 7</td>
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<td>0.182</td>
<td>0.162</td>
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<td>0.226</td>
<td>0.202</td>
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<tr>
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<td>0.334</td>
<td>0.302</td>
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<td>5</td>
<td>1.405</td>
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*This table was compiled from ANSI B1.1-1974. The minor diameter was found from the equation $d_m = d - 1.299 \times 0.038p$, and the pitch diameter from $d = d_p - 0.649 \times 519p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.*

ANSI Standards
<table>
<thead>
<tr>
<th>NOMINAL MAJOR DIAMETER $d$</th>
<th>COARSE-PITCH SERIES</th>
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<th>FINE-PITCH SERIES</th>
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<tr>
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<td>PITCH $p$</td>
<td>TENSILE-STRESS AREA $A_t$</td>
<td>MINOR DIAMETER AREA $A_r$</td>
<td>PITCH $p$</td>
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<td>5.03</td>
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<td>6.78</td>
<td>6.00</td>
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<td>817</td>
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<td>110</td>
<td>6</td>
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*The equations and data used to develop this table have been obtained from ANSI B1.1-1974 and B18.3.1-1978. The minor diameter was found from the equation $d_m = d - 1.226 \times 869p$, and the pitch diameter from $d_n = d - 0.649519p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.

**Metric Threads**

E.g. M12 x 1.75

Φ pitch

**Tensile Area**

$A_t$
The spring constant, or stiffness constant, of an elastic member such as a bolt, as we learned in Chap. 3, is the ratio between the force applied to the member and the deflection produced by that force. We can use Eq. (3-4) and the results of Prob. 3-1 to find the stiffness constant of a fastener in any bolted connection.

The grip of a connection is the total thickness of the clamped material. In Fig. 8-12 the grip is the sum of the thicknesses of both members and both washers. In Fig. 8-13 the grip is the thickness of the top member plus that of the washer.

The stiffness of the portion of a bolt or screw within the clamped zone will generally consist of two parts, that of the unthreaded shank portion and that of the threaded portion. Thus the stiffness constant of the bolt is equivalent to the stiffnesses of two springs in series. Using the results of Prob. 3-1, we find

\[
\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1k_2}{k_1 + k_2}
\]

for two springs in series. From Eq. (3-4), the spring rates of the threaded and unthreaded portions of the bolt in the clamped zone are, respectively,

\[
k_T = \frac{A_T E}{l_T} \quad k_d = \frac{A_d E}{l_d}
\]
FIGURE 8-15
Typical stress-strain diagram for bolt materials showing proof strength \( S_p \), yield strength \( S_y \), and tensile strength \( S_{ut} \).

- Ultimate Strength
- Yield Strength
- Proof Strength...use in place of yield strength for bolts
<table>
<thead>
<tr>
<th>PROPERTY CLASS</th>
<th>SIZE RANGE, INCLUSIVE</th>
<th>MINIMUM PROOF STRENGTH, MPa</th>
<th>MINIMUM TENSILE STRENGTH, MPa</th>
<th>MINIMUM YIELD STRENGTH, MPa</th>
<th>MATERIAL</th>
<th>HEAD MARKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6</td>
<td>M5–M36</td>
<td>225</td>
<td>400</td>
<td>240</td>
<td>Low or medium carbon</td>
<td>4.6</td>
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<tr>
<td>4.8</td>
<td>M1.6–M16</td>
<td>310</td>
<td>420</td>
<td>340</td>
<td>Low or medium carbon</td>
<td>4.8</td>
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<tr>
<td>5.8</td>
<td>M5–M24</td>
<td>380</td>
<td>520</td>
<td>420</td>
<td>Low or medium carbon</td>
<td>5.8</td>
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<tr>
<td>8.8</td>
<td>M16–M36</td>
<td>600</td>
<td>830</td>
<td>660</td>
<td>Medium carbon, Q&amp;T</td>
<td>8.8</td>
</tr>
<tr>
<td>9.8</td>
<td>M1.6–M16</td>
<td>650</td>
<td>900</td>
<td>720</td>
<td>Medium carbon, Q&amp;T</td>
<td>9.8</td>
</tr>
<tr>
<td>10.9</td>
<td>M5–M36</td>
<td>830</td>
<td>1040</td>
<td>940</td>
<td>Low-carbon martensite, Q&amp;T</td>
<td>10.9</td>
</tr>
<tr>
<td>12.9</td>
<td>M1.6–M36</td>
<td>970</td>
<td>1220</td>
<td>1100</td>
<td>Alloy, Q&amp;T</td>
<td>12.9</td>
</tr>
</tbody>
</table>

*The thread length for bolts and cap screws is:

\[
L_T = \begin{cases} 
2d + 6 & \text{for } L \leq 125 \\
2d + 12 & \text{for } 125 < L \leq 200 \\
2d + 25 & \text{for } L > 200 
\end{cases}
\]

where \( L \) is the bolt length. The thread length for structural bolts is slightly shorter than given above.
### Table 8-4
SAE Specifications for Steel Bolts

<table>
<thead>
<tr>
<th>SAE Grade No.</th>
<th>Size Range, Inclusive, in</th>
<th>Minimum Proof Strength, kpsi</th>
<th>Minimum Tensile Strength, kpsi</th>
<th>Minimum Yield Strength, kpsi</th>
<th>Material</th>
<th>Head Marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/4 - 1 1/4</td>
<td>33</td>
<td>60</td>
<td>36</td>
<td>Low or medium carbon</td>
<td><img src="image" alt="Head Marking" /></td>
</tr>
<tr>
<td>2</td>
<td>1/8 - 1 1/4</td>
<td>55</td>
<td>74</td>
<td>57</td>
<td>Low or medium carbon</td>
<td><img src="image" alt="Head Marking" /></td>
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<tr>
<td>4</td>
<td>1/2 - 1 1/4</td>
<td>65</td>
<td>115</td>
<td>100</td>
<td>Medium carbon, cold-drawn</td>
<td><img src="image" alt="Head Marking" /></td>
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<tr>
<td>5</td>
<td>1/4 - 1 1/2</td>
<td>85</td>
<td>120</td>
<td>92</td>
<td>Medium carbon, Q&amp;T</td>
<td><img src="image" alt="Head Marking" /></td>
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<tr>
<td>5.2</td>
<td>1/4 - 1</td>
<td>85</td>
<td>120</td>
<td>92</td>
<td>Low-carbon martensite, Q&amp;T</td>
<td><img src="image" alt="Head Marking" /></td>
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<tr>
<td>7</td>
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<td>105</td>
<td>133</td>
<td>115</td>
<td>Medium-carbon alloy, Q&amp;T</td>
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<td>8</td>
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<td>120</td>
<td>150</td>
<td>130</td>
<td>Medium-carbon alloy, Q&amp;T</td>
<td><img src="image" alt="Head Marking" /></td>
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<td>8.2</td>
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<td>120</td>
<td>150</td>
<td>130</td>
<td>Low-carbon martensite, Q&amp;T</td>
<td><img src="image" alt="Head Marking" /></td>
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</table>

*Don't use*
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<tr>
<th>ASTM DESIGNATION NO.</th>
<th>SIZE RANGE INCLUSIVE, in</th>
<th>MINIMUM PROOF STRENGTH, kpsi</th>
<th>MINIMUM TENSILE STRENGTH, kpsi</th>
<th>MINIMUM YIELD STRENGTH, kpsi</th>
<th>MATERIAL</th>
<th>HEAD MARKING</th>
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<td>120</td>
<td>92</td>
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<td>A325</td>
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<tr>
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<td>1/8-1 1/4</td>
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<td>105</td>
<td>81</td>
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<td>105</td>
<td>81</td>
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</table>
Other threads

Acme

Square
Mechanics of Threads... square thread

FIGURE 8-5
Portion of a power screw.

\[ \frac{F}{2} + \frac{F}{2} = F \]

Advance screw against load F.

Retract
Force diagrams: (a) lifting the load; (b) lowering the load.

"Raising"  "Lowering"

\[ \sum F = 0 \]
\[ \sum F = 0 \]

Mechanics of Screws

Torque to raise or lower
\[ T = \frac{Pd_m \times \text{diam.}}{2} \]
Raising Load

\[ H \Rightarrow \text{Horizontal} \]
\[ V \Rightarrow \text{Vertical} \]

\[
\Sigma F_H = P - N \sin \lambda - \mu N \cos \lambda = 0
\]
\[
\Sigma F_V = F + \mu N \sin \lambda - N \cos \lambda = 0
\]

Eliminate \( N \) and solve for \( P \):

\[
P = \frac{F \left( \sin \lambda + \mu \cos \lambda \right)}{\left( \cos \lambda - \mu \sin \lambda \right)}
\]

\[
= \frac{F \left( \frac{\sin \lambda}{\cos \lambda} + \mu \right)}{\left( 1 - \mu \frac{\sin \lambda}{\cos \lambda} \right)}
\]

\[
\frac{\sin \lambda}{\cos \lambda} = \tan \lambda = \frac{2}{\pi R \text{d}m} \rightarrow \text{Leaf circumference (mean)}
\[ P = F \left( \frac{2/\mu \Pi d_m + \mu}{1 - \mu e/\mu \Pi d_m} \right) \]

**Lower Load**

... same process leads to

\[ P = F \left( \mu - \frac{2}{\mu \Pi d_m} \right) \left( \frac{1}{1 + \mu e/\mu \Pi d_m} \right) \]

**Torque req'd:**

\[ \frac{\text{Raise}}{T = \frac{F \Pi d_m (\mu - \frac{2}{\mu \Pi d_m})}{2}} \quad \frac{\text{Lower}}{T = \frac{F \Pi d_m (\mu - \frac{2}{\mu \Pi d_m})}{2}} \]
If \( \mu > \frac{L}{\pi d_m} \)

or \( \mu \pi d_m > L \)

... Lowering torque is the

... Screw/thread is "SELF-LOCKING"

**NON-SQUARE THREADS**

\[
\frac{F d_m (L + \pi \mu d_m \sec \alpha)}{2 (\pi d_m - \mu L \sec \alpha)}
\]

Diagram:
- \( \alpha \) - Thread angle
- \( 2\alpha \) - Thread angle
- \( 2L \) - Additional notation
- \( L \) - RAISING TORQUE
- \( \alpha \) - cos \alpha
- \( F \) - Force
- \( \mu \) - Coefficient of friction
- \( d_m \) - Diameter
- \( \sec \alpha \) - Secant of angle
There is also friction at the collar/washer.

\[ T_c = \mu_c F d_c \frac{1}{2} \]

Typically, \( d_c = \frac{5d}{4} \)

Total torque is:

\[ T = F_i d \left[ \tan \lambda + \mu_c \frac{\text{seed}}{1 - \mu_c \tan \lambda \text{seed}} + \frac{5}{8} \right] \]

\[ \text{... E-19} \]

\( F_i \) is initial load on a screw or bolt.
Normally can set K so that
\[ T = K F_i d \]

→ dis
Nonjial
Major dia.

\[ \therefore \text{see table 8-10} \]

<table>
<thead>
<tr>
<th>BOLT CONDITION</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonplated, black finish</td>
<td>0.30</td>
</tr>
<tr>
<td>Zinc-plated</td>
<td>0.20</td>
</tr>
<tr>
<td>Lubricated</td>
<td>0.18</td>
</tr>
<tr>
<td>Cadmium-plated</td>
<td>0.16</td>
</tr>
<tr>
<td>With Bowman Anti-Seize</td>
<td>0.12</td>
</tr>
<tr>
<td>With Bowma-Grip nuts</td>
<td>0.09</td>
</tr>
</tbody>
</table>

- Std 60° Thread angle
- Various surface treatments
- WHAT SHOULD F_i be??
UNDERSTANDING LOAD & PRELOAD

1. Tighten bolt to preload $F_i$
2. Bolt in tension
3. Mat'l in grip in compression

Bolt pressing against mat'l

No external load... yet
MATL & BOLT STIFFNESS

Material or Grip (Compression)

External load, \( P \)

\( \text{MTL} \)

\( \text{KM} \)

\( k_6 \) (LIKE ROD)

\( k_6 + \text{KM} \) will be given (in AAT 31)
When external load, \( P \), is applied ... Tension:
\[
\frac{P}{E}
\]

1. Bolt gets more tension
2. Material sees less compression.

\[
P = P_b + P_m
\]
\( F_i \rightarrow \text{initial preload} \)
\( P \rightarrow \text{external load} \)

\[ F_b = F_i + P_b \]

\[ F_m = P_m - F_i \]

\[ P_m = k_m \delta \quad P_b = k_b \delta \]

\[ \therefore \quad \frac{P_m}{k_m} = \frac{P_b}{k_b} \]

\[ P_b = \frac{k_b P_m}{k_m} \]
But \( P = P_m + P_b \)

\[ P_m = P - P_b \]

\[ P_b = \frac{k_b P}{k_m} \]

\[ P_m = \frac{k_b (P - P_b)}{k_m} \]

and \( P_b = \frac{k_b P}{k_b + k_m} \)

Also \( P_m = \frac{k_m P}{k_b + k_m} \)

\( \text{Total load on bolt:} \)

\[ F_b = F_i + P_b = \frac{k_b P}{k_b + k_m} + F_i \]

\[ F_m = \frac{k_m P}{k_b + k_m} - F_i \]

\( \text{for contact } F_m < 0 \)
EXAMPLE

A ½" UNC site grade 7 bolt grips two plates of steel. The grip is 2". The recommended preload is 13,000 lbs.

1. What will be the load on the bolt and the grip (material) if an external load of 10,000 lbs is applied?

\[
F_b = F_i + \frac{k_b P}{k_b + k_m}
\]

From Table 8-7 \( k_b = 2.57 \text{ Mlbs/in} \)
\( k_m = 12.69 \text{ Mlbs/in} \)

\[
F_b = F_i + \frac{2.57P}{15.26} = 13000 + 0.168(10000)
\]
\[
F_b = 14,680 \text{ lbs} \quad \text{i.e. } F_b \text{ ↑ by 1680 lbs}
\]
\[ F_m = \frac{km \cdot P}{km + kb} - F_i \]
\[ = \frac{12.69 \cdot 10000}{15.26} - 13,000 \]
\[ F_m = 8320 - 13,000 = -4680 \text{ lbs} \]

\[ F_m \text{ b by 8320 lbs} \]

2. At what value of external load will the joint separate?

This will occur when \( F_m = 0 \)

\[ + \frac{km \cdot P}{km + kb} - F_i = 0 \]

\[ P = \left( kb + km \right) \frac{F_i}{km} \frac{1}{0.832} \]

\[ @ \text{Separation} \ P = 15,625 \text{ lbs} \]
**STATIC Failure**

\[ F_b = \frac{k_b P}{k_b + k_m} + F_i = C P + F_i \]

**Stress on Bolt:**

\[ \sigma_b = \frac{F_b}{A_t} = \frac{C P + F_i}{A_t} \]

"Tensile Area"

Allowing for a factor of Safety on "P" \( \Rightarrow n P \)

\[ \sigma_b = \frac{C n P}{A_t} + \frac{F_i}{A_t} \]

\[ \text{[i.e. no F.S. on F_i]} \]
at "failure" \( \sigma_b = \frac{5p}{p} \)

Solve for "\( n \)"

\[
n = \frac{5pA_t - F_i}{CP}
\]

... Also .... The "Proof Load"

\[ F_p = 5pA_t \]

... The recommended preload on bolted connections is

\[ F_i = 0.75F_p = 0.75 \times 5pA_t \]

..... "Reusable"

OR \[ F_i = 0.90F_p = 0.90 \times 5pA_t \]

..... "Permanent"
CONT'D EXAMPLE

... \( \frac{3}{4} \) " UNC Grade 7 bolt

... \( F_i = 13,000 \) lbs

... What is the factor of safety for bolt failure at \( P_1 = 10,000 \) lbs and at \( P_2 = 15,625 \) lbs

\[ n = \frac{\sum P A \varepsilon - F_i}{C P} \]

\[ n_i = \frac{107 \text{ ksi} \cdot (0.142^2) - 13,000}{(0.168)(10,000)} \]

\[ n_i = 1.31 \] @ 10,000 lbs
\[ N_2 = \frac{107k \cdot (0.142 \text{ in}^2)}{0.168 \cdot (15,625)} \]

\[ N_2 = 0.83 \]

Joint (bolts) would fail due to "yielding" before separation.
Fatigue Failure

- Stress concentrations at roots of the threads and under head of bolt (fillet)

2.1 < Kf < 3.8 Table 8-11

... machined finish.
For our purposes just use Table 8-12
- Fully corrected Endurance Limits, $Se$

<table>
<thead>
<tr>
<th>GRADE OR CLASS</th>
<th>SIZE RANGE</th>
<th>ENDURANCE LIMIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAE 5</td>
<td>$\frac{1}{2}$–1 in</td>
<td>18.6 kpsi</td>
</tr>
<tr>
<td></td>
<td>$1\frac{1}{8}$–$1\frac{1}{2}$ in</td>
<td>16.3 kpsi</td>
</tr>
<tr>
<td>SAE 7</td>
<td>$\frac{1}{4}$–$1\frac{1}{2}$ in</td>
<td>20.6 kpsi</td>
</tr>
<tr>
<td>SAE 8</td>
<td>$\frac{1}{4}$–$1\frac{1}{2}$ in</td>
<td>23.2 kpsi</td>
</tr>
<tr>
<td>ISO 8.8</td>
<td>M16–M36</td>
<td>129 MPa</td>
</tr>
<tr>
<td>ISO 9.8</td>
<td>M1.6–M16</td>
<td>140 MPa</td>
</tr>
<tr>
<td>ISO 10.9</td>
<td>M5–M36</td>
<td>162 MPa</td>
</tr>
<tr>
<td>ISO 12.9</td>
<td>M1.6–M36</td>
<td>190 MPa</td>
</tr>
</tbody>
</table>

TABLE 8-12
Fully Corrected Endurance Limits for Bolts and Screws with Rolled Threads
Use Goodman diagram for Fatigue

Load line with changing $P$

Operating Condition

$S_e$

$S_a$

$S_m$

"Alternating $\sigma_i = \frac{E_i}{A_t}$"

External load fluctuations between 0 and $P$ **
For Fatigue

\[ n = \frac{S_a}{\sigma_a} \]

\[ \sigma_a = \frac{C P}{2} \left( \frac{1}{A_t} \right) = \frac{C P}{2A_t} \]

\[ \sigma_m = \frac{E_t}{A} + \frac{C P}{2} \cdot \left( \frac{1}{A_t} \right) \]

From geometry of Goodman diagram:

\[ S_a = \frac{S_{ut} - F_i/A_t}{1 + S_{ut}/S_e} \]

For Finite Life use \( S_f \) and \( N \) cycles
Returning to example let \( P \) fluctuate from 0 to 10,000 lbs. Let \( S_e = 20.6 \) kpsi and Table 8-12 
\[ S_{ut} = 133 \text{ kpsi} \]
\[ (\frac{C}{0.142}) \]
\[ \sigma_a = \frac{C \times 10,000}{2} \]
\[ = 35.2 \text{ kpsi} = 5.91 \text{ kpsi} \]
\[ S_a = 133k - \frac{13k}{0.142} \]
\[ 1 + \frac{133}{20.6} \]
\[ = 41.45 / 7.46 = 5.56 \text{ kpsi} \]
\[ n = \frac{5a}{\Delta a} = 0.94 \]
Fails??

i.e. Finite Life

Replace \( \sigma_a \) by \( \sigma_f \) in eq. for \( \sigma_a \)

Exercise: Find \( N \) if a safety factor of 1.3 is needed.
Direct Loading of Bolted and Riveted Joints

- Failure of bolts/rivets
  - bending* (b) 8-21
  - shear* (c) 8-21

- Failure of materials/plates or beams joined
  - tension of members* (d) 8-21
  - bearing stresses/crushing* (e) 8-21
  - shear & tensile tear-out (f) & (g) 8-21

*CALCULATE n
rivet/bolt 1½ diameters from edge


- **Grip**
  \[ t \]

- **Bolt or Rivet**
  \[ F = \frac{Mc}{I} \]

- **Bending**
  \[ \sigma = \frac{F}{t} \]

- **Shear**
  \[ \tau = \frac{F}{A} \]

- **Tension**
  \[ \sigma = \frac{FA}{A} \]
  \[ A = \frac{t}{2} (w - 2d) \]

- **Compression**
  \[ F_{Ed} = \sigma \]

- **Bearing**
  \[ \text{Projected Stress Area} \]

- **Shear Tearout**
  \[ \text{Stay away from edge} \]

- **Tensile Tearout**
Secondary Stress/Force P

n \rightarrow \text{# of bolts}

F_i \rightarrow \text{Primary Force on Joint Contd.}

F_i = V_i \rightarrow \text{Beam Analysis}

\text{Load Analysis}

\text{Eccentric Load Max.}
\[
\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4 + A_5 x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum A_i x_i}{\sum A_i}
\]

\[
\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4 + A_5 y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum A_i y_i}{\sum A_i}
\]

- Centroid is sometimes obvious from inspection or can be found from eq 8-42.
- If all areas equal:
  \[
  \bar{x} = \frac{\sum x_i}{n} \quad \text{and} \quad \bar{y} = \frac{\sum y_i}{n}
  \]

Array of bolts \( A_1, \ldots, A_n \)

Figure 8-22
Secondary Shear, \( F'' \)

\[ \text{bolt loads} \quad \{ F_a'', F_b'', F_c'' \ldots \text{etc} \} \]

\[ \text{distance of bolt from centroid} \quad \{ r_a, r_b, r_c \ldots \text{etc} \} \]

\[ M = F_a'' r_a + F_b'' r_b + \ldots \]

Moment at joint

\[ \text{ASSUME (not a bad assumption)} \]

\[ \ldots \text{The more distant (from centroid) bolts see bigger loads, i.e.} \]

\[ \frac{F_a''}{r_a} = \frac{F_b''}{r_b} = \frac{F_c''}{r_c} \ldots = \text{const} \]
Combining eqs for \( M \) & load sharing, we have

\[ F''_n = \frac{M_{r_n}}{r_A^2 + r_B^2 + r_C^2 + \ldots} \]

\( n \)th bolt

Combine \( F' \) and \( F'' \) vectorially at each bolt or rivet.

i.e. \( \frac{V}{n} + \frac{F''_n}{n} \)

direction of \( V \) of \( \frac{1}{n} \)
EXAMPLE 8-4

Shown in Fig. 8-24 is a 15- by 200-mm rectangular steel bar cantilevered to a 250-mm steel channel using four bolts. On the basis of the external load of 16 kN, find:

(a) The resultant load on each bolt
(b) The maximum bolt shear stress
(c) Bearing stress (my)
(d) Bonding in bar

\[ F = 16kN \]

\[ F = 16kN \]

Dimensions in millimeters.

Centroid by observation by symmetry.

M at centroid = 425 F \( \text{N-m} \)

\[ V = \frac{16kN}{2} \]

\[ V = \frac{16kN}{2} \]
Primary Shear

\[ V = 16 \text{ kN} \quad F' = \frac{V}{n} = 4 \text{ kN} \]

Secondary Shear

\[ F'' = \frac{M \rho}{(R_1^2 + R_2^2 + R_3^2 + R_4^2)} = \frac{Mr^2}{4r^2} \]

Since all \( r \)'s are equal

\[ F'' = \frac{M}{4r} \quad M = (16 \times 0.425) \quad M = 6800 \text{ N-m} \]

\[ F'' = \frac{6800 \text{ N-m}}{4(96) \text{ mm}} = \frac{17.7 \text{ kN}}{} \]
\[ \mathbf{F}_{A}'' \perp \mathbf{r}_{A}, \text{ etc} \]
\[ \mathbf{F}' \text{ acts vertically} \]
\[ \mathbf{M} \& \mathbf{V} \text{ are reaction moment and shear} \]
\[ \mathbf{F}_{A} = \mathbf{F}_{B} > \mathbf{F}_{C} = \mathbf{F}_{D} \]
From geometry:

\[ F_A = F_A^{'} + F_A^{''} \]

\[ F_A = 21 \text{ kN} = F_B \]

\[ E = F_D = 13.8 \text{ kN} \]

\[ \tau_A = \frac{F_A}{A} = \frac{21 \text{ kN}}{200 \text{ mm}^2} = 104 \text{ MPa} \]

Use major diameter.

If bolt is grade 5.8, \( S_p = 380 \text{ MPa} \)

\[ \frac{S_p}{S_s} = \frac{S_{ssp}}{S_p} = \frac{190}{104} = 1.82 \]

Factor of safety against shear.
c) Max bearing stress (@ A or B)

\[ \sigma_b = \frac{F_A}{A_b} \]

\[ A_b = t d \]

\[ \Rightarrow \text{Projected Area of Bolt diam.} \]

\[ \text{Thickness of thinner member (hole or bolt)} \]

\[ A_b = (10 \text{ mm})(16 \text{ mm}) = 160 \text{ mm}^2 \]

\[ \sigma_b = \frac{21 \text{ kN}}{160 \text{ mm}^2} = -131 \text{ MPa} \]

\[ \Rightarrow \eta_b = \frac{S_f}{\sigma_b} \]

\[ \text{S Factor} \]
d) Critical bending of bar

- occurs across A-B due to reduction in area

Assumption.

\[ M_{AB} = 16 \, kN \times (350 \, mm) \]
\[ M_{AB} = 5600 \, N \cdot m \]

\[
\sigma_{bend} = \frac{M_{bend}}{I}
\]

\[
I = I_{bend} - 2\left( I_{holes} + d^2 A \right)
\]

\[ I = 8.26 \times 10^6 \, mm^4 \]

\[
\sigma_{bend} = \frac{(5800)(100 \, mm)}{8.26 \times 10^6 \, mm^4}
\]

\[ \sigma_{bend} = 67.8 \, MPa \]
- SAFETY FACTORS
  FOR BEARING STRESS &
  BENDING OF BAR/BEAM
  DEPEND ON MTL AND
  ASSUMED FAILURE THEORY

- BUT MTL'S ARE OFTEN
  WEAKER THAN BOLTS SO
  THIS COULD RESULT IN A
  LOWER SAFETY FACTOR
  THAN FOR THE BOLTS
  OR RIVETS.
Assignment #6

Due Mon, March 24

8-11  Use \( \frac{k_b}{k_b + k_m} = C = 0.213 \)

8-21  Use \( C = 0.213 \)

8-24

8-30

8-37  (Assume bracket pivots about lower edge)

8-39