Fatigue Strength Under General Fluctuating Loads

- We have up to now looked at "alternating" or "fully-reversed" loading where stress fluctuates between $\pm \sigma_a \sim \pm \sigma_a$

- More generally, we can have $\sigma \sim \sigma_a$

- $\sigma_m$ can be +ve or -ve
Also

\[ \sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]  
alternating stress

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]  
mean stress

\[ \sigma_{\text{max}} + \sigma_{\text{min}} \]  
can be computed from max & min loading:

\[ F_{\text{max}} + F_{\text{min}} \]  
Force

\[ M_{\text{max}} + M_{\text{min}} \]  
Moment

\[ T_{\text{max}} + T_{\text{min}} \]  
Torque

\[ \text{max} \& \text{min} \]  
refer to loads at a particular location over time, e.g., rotating shaft, or other fluctuating forces, loads
Se + Sf apply directly to cases where $\Theta_m = 0$

When $\Theta_{\text{min}} = 0$, i.e., fluctuations between 0 and $\Theta_{\text{max}}$ we call it "repeated loading" and $\Theta_a = \Theta_{\text{max}}$ and $\Theta_m = \Theta_{\text{max}} / 2$

... e.g., loading of rotating gear teeth.
**Question:** How does the presence of a mean alternating stress affect fatigue strength/endurance?

**Ans:** if $\sigma_m > 0$ (tensile) 
Strength is reduced
if $\sigma_m \leq 0$ (compressive) 
Strength is unchanged.

**Graphically:**

[Diagram showing the Goodman line, with a line labeled "Safe," and axes labeled $\sigma_m$ and $\sigma_a$. The line intersects the axes at $\sigma_m$ and $\sigma_a$.]
Fig 7-14

Some data

Note: When $T_m = 0$, only have $T_a$
Then only need $S_a$ and $S_f$
Fatigue diagram showing various criteria of failure. For each criterion, specific points on or outside the respective line indicate failure.

Some point A on the Goodman line, for example, gives the strength $S_m$ as the limiting value of $\sigma_m$, which, paired with $\sigma_a$, is the limiting value of $\sigma_a$. For $S_m = 0$, or $\sigma_m = 0$, $S_a = 0$, or $\sigma_a = 0$. For $S_m > 0$, use Goodman line.
1. In this zone

\[ \frac{\sigma_a}{S_e} + \frac{\tau_m}{S_{ut}} = \frac{1}{n} \]

(also check that: \( S_{yt} > \frac{\sigma_{max}}{\tau_{max}} \)

for static failure \( n = \frac{S_{yt}}{\sigma_{max}} \)

2. In this zone

\[ n = \frac{S_e}{\sigma_a} \text{ or } \frac{S_f}{\sigma_a} \]
Example: Determine Factor of Safety

1) \( \sigma_a = 5 \text{ kpsi} \)
\( \sigma_m = 3 \text{ kpsi} \) \( \Rightarrow \) 1st quadrant
\( \sigma_e = 20 \text{ kpsi} \)
\( \sigma_{ut} = 100 \text{ kpsi} \)

Goodman Line
\( \frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ut}} = \frac{1}{n} \)
\( \frac{1}{n} = \frac{5}{20} + \frac{3}{100} = 0.28 \)
\( n = \frac{3.57}{0.28} \)

\( n = 3.57 \)

\( \frac{\sigma_s}{\sigma_{ut} + \sigma_m} \)
\( \sigma_s = \frac{\sigma_{ut}}{\sigma_{max}} \)
No mean stress

(ii) $\sigma_a = 5 \text{kpsi}$  \[ \text{STATIC} \]
$\sigma_m = 0 \quad \text{Rightarrow} \quad n_s = \frac{S_t}{\sigma_a + \sigma_m}$
$S_e = 20 \text{kpsi}$
$S_{ut} = 100 \text{kpsi}$

\[
n = \frac{S_e}{\sigma_a} = \frac{20}{5} = 4.0
\]

(iii) $\sigma_a = 5 \text{kpsi}$

$\sigma_m = -28 \text{kpsi}$
$S_e = 20 \text{kpsi}$
$S_{ut} = 100 \text{kpsi}$

\[
n = \frac{S_e}{\sigma_a} = \frac{20}{5} = 4.0
\]
Concept of Load-line

- Safety factor calc assumes load-line through origin
- Use this assumption unless told otherwise
Assignment #4
Due Mon Jan 17th

#1. 7-1 from text

#2. 7-5 (reversed bending, see sect 5-4 for tensile strength)

#3. 7-11 (infinite life and static failure (yielding))

#4. A 1.25 inch diameter hot-rolled steel rod has a 0.125 inch diameter hole drilled through it. Its (ultimate) tensile strength is 60 kpsi. The rod is subjected to a reversed (alternating) torque of 2000 in-lbs. Estimate the factor of safety for infinite life and against yielding. What would be the safety factor if a life of 20,000 cycles were needed? ...$S_y = 45 \text{ kpsi}$
Torsional Fatigue (Shear) ... Alternating + Mean Stress

Quasi-Goodman Diagram

\[ \frac{\sigma_a}{\sigma_y} + \frac{\tau_m}{\tau_y} = \frac{1}{n} \]

For Quasi-Goodman Line 1st quadrant

\[ \frac{\tau_m}{\tau_y} = \frac{1}{n} \]

For Yielding

\[ \sigma_y = 0.577\sigma_y \] for Yielding
WHAT IF LOADING IS COMBINED?
I ALTERNATING ONLY.
E.g. $O_xa$, $O_ya$, $O_{xya}$

- Calculate Von-Mises $(\sigma'_a)$
  (using principal stresses)
- Obtain fully correct $S_e$ or $S_f$
  for bending
- Conservative approach.
- Define safety factor, $n$

$$n = \frac{S_e}{\sigma'_a} \geq \frac{S_f}{\sigma'_a}$$

infinite life \rightarrow finite life
II  ALTERNATING + MEAN

e.g. \( \sigma_x, \sigma_y, T_{xy} \Rightarrow \sigma_a' \)
and \( \sigma_{xm}, \sigma_{ym}, T_{xym} \Rightarrow \sigma_m' \)

\[ \begin{align*}
\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_t} &= \frac{1}{n} \\
\frac{\sigma_u}{S_e} &= \frac{1}{n}
\end{align*} \]
ADDITIONAL NOTE FOR COMBINED LOADING PROBS

\[ T_{x_a}, T_{y_a}, T_{xy_a} \rightarrow T_{x_m}, T_{y_m}, T_{xy_m} \rightarrow T_{x_m}, T_{y_m}, T_{xy_m} = T_{m} \]

\[ \Delta \]

Apply stress concentration factors \( K_f \) and \( K_s \) (shear) to increase alternating (not mean) stresses. Do not use \( k_e = \frac{1}{K_f} \) to reduce strength in these cases.

\[ \Delta \]

Do not use \( K_f \) for mean stresses (except for brittle materials).
SECTION 7-16 DEALS
WITH CUMULATIVE FATIGUE
WHEN DIFFERENT LEVELS
OF FLUCTUATING LOADS
ARE PRESENT AT
DIFFERENT TIMES
"Palmgren-Miner Rule"
... WE WILL NOT CONSIDER

SECTION 7-17 FRACTURE
MECHANICS APPROACH
... WE WILL NOT CONSIDER
FATIGUE

7-18 SURFACE STRENGTH

- Failure by pitting, surface fatigue in gears & rolling bearings & other contacts
- Repeated loading assumed

SURFACE FATIGUE STRENGTH

\[ \sigma_c = (0.4 \cdot H_B - 10) \text{ kpsi} \]

or \[ \sigma_c = (2.76 \cdot H_B - 70) \text{ MPa} \]

STEEL

Compare with 5-20 for \( S_u \)

\[ S_4 = 0.45 \cdot H_B \text{ kpsi} \]

\[ S_u = 3.10 \cdot H_B \text{ MPa} \]
NOTE THAT, GENERALLY

\[ S_c > 0.55u \]

... this is due to the hydrostatic compression during contact loading

... also compare from NOT \( \sigma_a \)

... \( S_c \) applies for EXACTLY \( 10^8 = N \) cycles of loading

... NO ENDURANCE LIMIT

... if \( S_c \) @ some other value of \( N \) is needed, call it \( S_{cn} \)
For contact fatigue it's been found that:

\[ S_{cn} a = \text{Constant} \quad \text{Palmer} \quad 1940's \]

\[ 3 \leq a \leq 3.3 \]

\[ S_{c1} N_1 = S_{c2} N_2 \]

\[ a \]

\[ S_{c2} = \left( \frac{N_1}{N_2} \right)^{1/a} S_{c1} \]

Let \( N_1 = 10^8 \), \( S_{c1} = S_c \)

\[ N_2 = N \quad S_{c2} = S_{cn} \]

\[ S_{cn} = \left( \frac{N_1}{N_2} \right)^{ya} S_c \]
E.g. if $a = 3$
and $N = 10^5$

$$\frac{Sc_N}{Sc} = \left(\frac{10^8}{10^5}\right)^{\frac{1}{3}} 5c$$

$$Sc_N = 10^5 Sc$$

At any # of cycles
Factor of safety is:

$$n = \frac{Sc_n}{\sigma_{p_{max}}}$$
MAE 311 Miniproject 1

Due March 19, 2003

Worth five per cent of course grade

To be submitted:

A written report with a title page plus 2-3 pages of written description, plus any illustrations or figures. This is to be an individual report, written in your own words. Cite all sources of information. Relate to MAE 311 course material as appropriate.
Choose one of three topics:

1. A recent newsworthy mechanical failure. e.g., the Alaska Airlines crash, recently traced to a maintenance problem. Discuss nature of failure, technical aspects, failure mechanisms or design issues, as appropriate. **Dow Jones Interactive**

2. A commercial mechanical product or component that has become available within the past ten years. Identify and describe the product or component. What’s technically interesting about
it? What seems to have prompted the introduction of the product, new materials, demand for the product, cost/technical advantage, innovation, some combination? **MACHINE DESIGN, DESIGN NEWS**

3. Document how and why a longstanding product or component has evolved over the past 20 to 60 years. How is the product better? What advances in technology, regulation or competitive pressures have contributed to the evolution?
Table 13.1 Characteristics of Hair Dryers and Their Casings

<table>
<thead>
<tr>
<th>Model and Date</th>
<th>Power (W)</th>
<th>Weight (kg)</th>
<th>Parts</th>
<th>Fasteners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schott, 1940</td>
<td>300</td>
<td>1.0</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Ormond, 1950</td>
<td>500</td>
<td>0.85</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Morphy-Richards, 1960</td>
<td>400</td>
<td>0.82</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Pifco, 1965</td>
<td>300</td>
<td>0.80</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Braun, 1986</td>
<td>1200</td>
<td>0.27</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Fig 13.2 Hair dryers: (a) a metal hair dryer of about 1950; (b) a bakelite dryer, almost identical in form to (a); (c) a plastic dryer of 1960, still influenced by "metal" thinking, but with attractive moulding; (d) a dryer of 1965 — it has fewer fasteners than (c), but is undistinguished in design; (e) a hair dryer of 1986, exploiting fully and effectively the properties of polymers, and with a racy, youthful look. Their characteristics are given in Table 13.1.
Dodge Tomahawk

With the Dodge Tomahawk, DaimlerChrysler's designers have wrought an industrially styled platform for the Dodge Viper's aluminum 8.3-L OHV V10. The Tomahawk, which was named after both the hatchet and cruise missile, rolls on four wheels, but is effectively a motorcycle. The front and rear wheels roll in closely spaced pairs, but are spread far enough apart that the Tomahawk can stand on its own, without a kickstand.

Each of the front wheels is carried by a separate automotive-style unequal-length control-arm suspension system, and the wheels steer individually, rather than as a pair about an axis between them. The control arms are fabricated from polished billet aluminum. Steering effort is acceptably light when stationary, and would lighten further in motion. The design team carefully studied previous single-sided control-arm-design bikes, such as the Elf (petroleum)-backed Grand Prix racing bike of the late 1980s to aid the development, according to designer Mark Walters.

Each of the rear wheels is carried on its own motorcycle-style single-sided swing arm, and each is driven by its own chain drive from the Tomahawk's two-speed transmission. Each of the four 20-in wheels carries a rim-mounted brake rotor made of stainless steel in the front and cast iron in the rear. Front wheels carry a pair of four-piston aluminum calipers each, for 16 pistons total, and the rears carry a single four-piston caliper each, for eight pistons total. The wheels wear custom-made Dunlop symmetrical motorcycle tires. The Tomahawk has no frame, so all suspension parts attach to mounting plates on the engine.

Cooling the huge engine could be a challenge, and the Tomahawk uses a pair of hidden aluminum radiators mounted above the engine's intake manifold and fed by a turbine-style cooling fan. The riding characteristics of a four-wheeled motorcycle are also of interest, but DaimlerChrysler claims the Tomahawk can lean to a 45° angle.

The engine's unmuffled exhaust passes between the rear wheels, and the unequal-length headers give the engine a crackle that is not present in the Viper. The lightened flywheel also lends the Tomahawk the urgency of a race machine, even when stationary.

The Tomahawk was built by the vintage racing specialist shop RM Motorsports, whose technicians have tested the bike by riding it about 45 mi (72 km). They report that it handles much like a drag racing bike, so it isn't meant for carving corners, but it can be ridden safely on public roads.

DaimlerChrysler has made no official decision on the Tomahawk, but it is openly contemplating production of 100 bikes, to be built by RM, which would sell for $150,000-200,000. Computer projections put top speed at a theoretical 300 mph (483 km/h).

Dan Carney
VARIATION ON EXAMPLE 7-8

42mm

\[ d = 34mm \]

\[ \frac{D - d}{2} \]

\[ \frac{42 \times 4mm + 151}{1018 \text{ COLD DRAWN STEEL TUBE}} \]

6mm hole = a

\[ T = \text{const} = 120 \text{ N-m} \]

\[ M = \text{Reversed bending} = 150 \text{ N-m} \]
Find Factor of Safety for Infinite Life:

From A-20

\[ \sigma_{ut} = 440 \text{ MPa} \]

\[ \sigma_{yt} = 370 \text{ MPa} \]

\[ \sigma_e' = 220 \text{ MPa} \]

\[ \tau_{a, \text{ trym}} \]

For M.T. Stress

Determine \( k_a, k_b, k_c, k_d, k_e = 1 \)

\[ \sigma_e = k_a k_b \sigma_e' \]

\[ k_a = \frac{a \sigma_{ut}}{b} = \frac{1.58(440)}{7.62} = 0.942 \]

\[ k_b = \left( \frac{7.62}{7.62} \right)^{-0.1133} = \left( \frac{42}{7.62} \right)^{-0.1133} \]

\[ k_b = 0.824 \]

\[ \sigma_e = (0.942 \times 0.824)(220) = 170 \text{ MPa} \]
Still need $K_f = 1 + \theta (K_t - 1)$

From Fig A-16 $K_t = 2.37$

Also $Z_{net} = 3.31 \times 10^3 \text{ mm}^3$

$J_{net} = 15.5 \times 10^4 \text{ mm}^4$

$\theta = \frac{M}{T D}$

$\theta = 0.78$ from $5-16$ Fig

$k_f = 1 + 0.78(2.37 - 1) = 2.07$

$k_f = \frac{0}{0_{nom}}$

$k_f = k_f \frac{M}{Z_{net}}$

$= \frac{2.07(150)}{3.31 \text{ cm}^3} = 93.9 \text{ MPa}$

$\tau_{y_m} = \frac{T(D)}{J_{net}^2} = 16.3 \text{ MPa} = \tau_m$
TABLE A-16

Approximate Stress-Concentration Factors $K_r$ for Bending of a Round Bar or Tube with a Transverse Round Hole.

The Nominal Bending Stress is $\sigma_0 = M/Z_{net}$, where $Z_{net}$ is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

Values of $A$ are listed in the table. Use $d = 0$ for a solid bar.

<table>
<thead>
<tr>
<th>$a/D$</th>
<th>$d/D$</th>
<th>$A$</th>
<th>$K_r$</th>
<th>$A$</th>
<th>$K_r$</th>
<th>$A$</th>
<th>$K_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>0.9</td>
<td>0.92</td>
<td>2.63</td>
<td>0.91</td>
<td>2.55</td>
<td>0.88</td>
<td>2.42</td>
</tr>
<tr>
<td>0.075</td>
<td></td>
<td>0.89</td>
<td>2.55</td>
<td>0.88</td>
<td>2.43</td>
<td>0.86</td>
<td>2.35</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td>0.85</td>
<td>2.49</td>
<td>0.85</td>
<td>2.36</td>
<td>0.83</td>
<td>2.27</td>
</tr>
<tr>
<td>0.125</td>
<td></td>
<td>0.82</td>
<td>2.41</td>
<td>0.82</td>
<td>2.32</td>
<td>0.80</td>
<td>2.20</td>
</tr>
<tr>
<td>0.15</td>
<td></td>
<td>0.79</td>
<td>2.39</td>
<td>0.79</td>
<td>2.29</td>
<td>0.76</td>
<td>2.15</td>
</tr>
<tr>
<td>0.175</td>
<td></td>
<td>0.76</td>
<td>2.38</td>
<td>0.75</td>
<td>2.26</td>
<td>0.72</td>
<td>2.10</td>
</tr>
<tr>
<td>0.20</td>
<td></td>
<td>0.73</td>
<td>2.39</td>
<td>0.72</td>
<td>2.23</td>
<td>0.72</td>
<td>2.07</td>
</tr>
<tr>
<td>0.225</td>
<td></td>
<td>0.69</td>
<td>2.40</td>
<td>0.68</td>
<td>2.21</td>
<td>0.65</td>
<td>2.04</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td>0.67</td>
<td>2.42</td>
<td>0.64</td>
<td>2.18</td>
<td>0.61</td>
<td>2.00</td>
</tr>
<tr>
<td>0.275</td>
<td></td>
<td>0.66</td>
<td>2.48</td>
<td>0.61</td>
<td>2.16</td>
<td>0.58</td>
<td>1.97</td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td>0.64</td>
<td>2.52</td>
<td>0.58</td>
<td>2.14</td>
<td>0.54</td>
<td>1.94</td>
</tr>
</tbody>
</table>


$K_e = 2.37$ from interpolation

$a/D = \sqrt[4]{2} = 0.143, \quad d/D = \sqrt{2} = 0.81$
To find principal stresses $\sigma'$

$\sigma'_2 = \sigma'_a = \sigma_a = 93.8 \text{ MPa} \quad \text{Pure bending}$

$\tau_m = \sigma'_m = 16.3 \text{ MPa} \quad \text{Pure shear}$

$\tau_m = -\sigma'_m = -16.3 \text{ MPa} \quad \text{(see next page)}$

Factor of safety ... FATIGUE

$$\frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

$$= \frac{93.8}{170} + \frac{28.3}{440} = 0.62$$

$$n = 1.62$$
RECALL: PURE SHEAR

\[ \sigma_m' = \sqrt{\frac{\sigma_1^2 - \sigma_2 \sigma_2 + \sigma_2^2}{2}} \]

\[ = \sqrt{\frac{16.3^2 + 16.3^2 + 16.3^2}{2}} \]

\[ = \sqrt{797} \]

\[ = 28.3 \text{ MPa} \]
CHECK FOR YIELDING

\[ \sigma' = \sqrt{\sigma_1^2 - \sigma_0^2 + \sigma_2^2} \]

for only \( \sigma^1, \theta_x \), \( \theta_y \)

\[ \sigma_{\text{max}} + \sigma_0 = \sigma_{\text{max}} \quad \tau_{\text{y},\text{max}} = \tau_x + \tau_y \]

THEN:

\[ \sigma_{\text{max}}' = \sqrt{\frac{\sigma_{\text{max}}^2}{3} + 3 \tau_{\text{y},\text{max}}^2} \]

\[ \frac{45.3}{48.9} = \frac{\sigma_{\text{max}}'}{\sigma_{\text{max}}} \]

\[ = 53.4 \text{ MPa} \]

\[ n = \frac{54.7}{53.4} = 370 \]

\[ \frac{\sigma_{\text{max}}'}{\sigma_{\text{max}}} \]

\[ n = 6.93 \]

\[ \frac{\sigma_{\text{max}}'}{\sigma_{\text{max}}} \]

No stress concentration

Much larger \( n \) than for fatigue
COMBINED ALTERNATING MEAN LOADS... EXAMPLE

AS SHAFT ROTATES
MOMENT IN VERTICAL & HORIZONTAL PLANES CAUSE alternating stress, \( \sigma_a \)

STEADY TORQUE BETWEEN C & D CAUSES \( \tau_m \)
**Moment**

*X-Z Plane*

\[ M_y (N \cdot m) \]

\[ M_y = 118.75 \text{ N} \cdot \text{m} \]

*Vert.*

**Moment**

*X-Y Plane*

\[ M_z (N \cdot m) \]

\[ M_z = 37.5 \text{ N} \cdot \text{m} \]

\[ M_z = 75 \text{ N} \cdot \text{m} \]

*Horiz.*

**Torque**

\[ T_x (N \cdot m) \]

\[ T_x = 7.5 \text{ N} \cdot \text{m} \]
LOCATION OF MINIMUM
FACTOR OF SAFETY

... since diameter is constant
look for location of max
loading

... max bending moment @ C

\[ M_{\text{max}} = \sqrt{M_y^2 + M_z^2} \]
\[ = \sqrt{118.75^2 + 32.5^2} \]
\[ = 124.5 \text{ N-m} \]

... also have torque T
acting @ C

... \( \therefore \) min 'n' is @ C.

... may decrease if diam const.
POSSIBLE QUESTIONS ...

MTL KNOWN

÷ SHAFT DIAMETER FOR GIVEN LIFE AND SAFETY FACTOR

÷ GIVEN DIAMETER, WHAT IS SAFETY FACTOR FOR INFINITE OR FINITE LIFE

÷ WHAT SORT OF SURFACE FINISH SHOULD BE SPECIFIED?

÷ \( \sigma_t \)

* IF NOT SPECIFIED, ASSUME SAFETY FACTOR \( n = 1 \)