STRESS CONCENTRATION

- AT DISCONTINUITIES, e.g., HOLES, FILLETS, CHANGES IN CROSS SECTION

STRESSES MAY BE HIGHER THAN NOMINAL VALUE

- IMPORTANT FOR
  - BRITTLE MTLS IN STATIC LOADING
  - BRITTLE <u>AND</u> DUCTILE MTLS IN DYNAMIC OR OSCILLATORY LOADING. ... i.e. FATIGUE

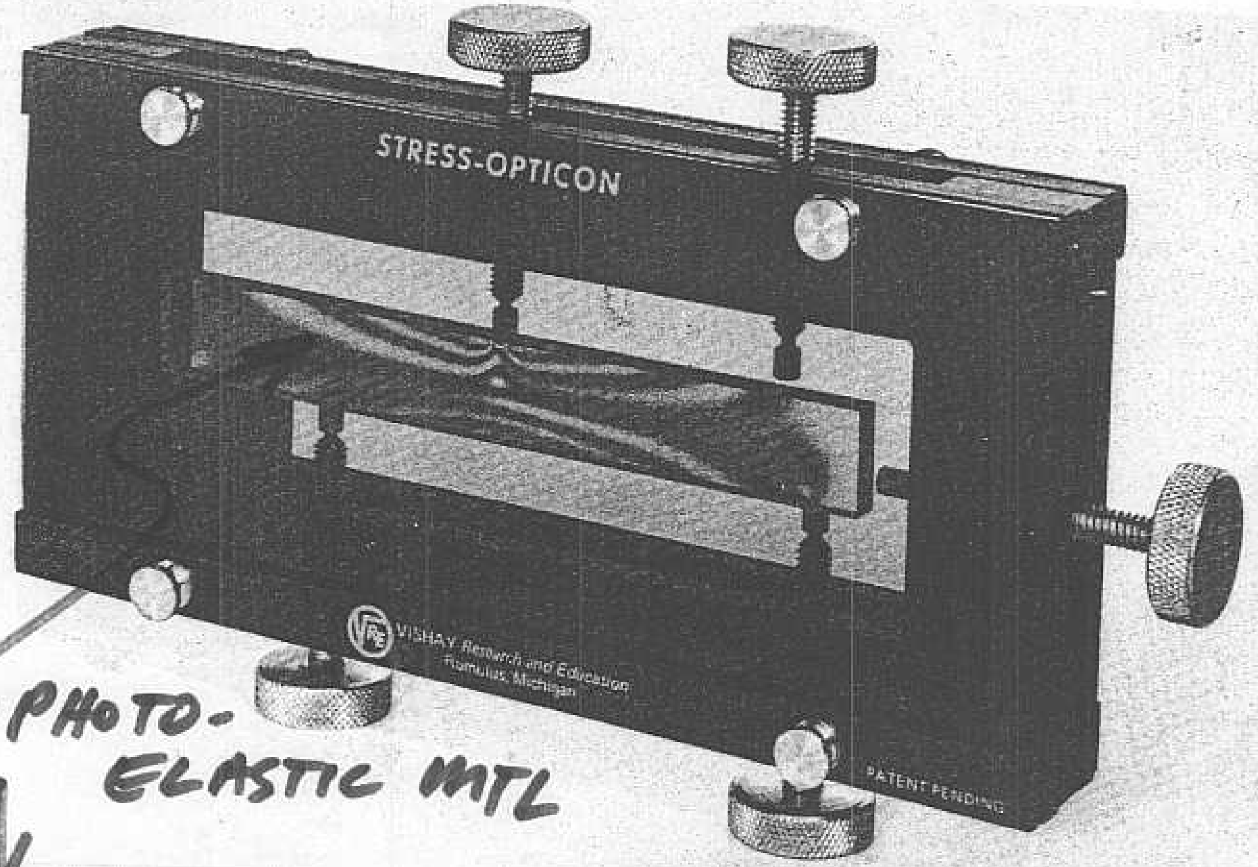
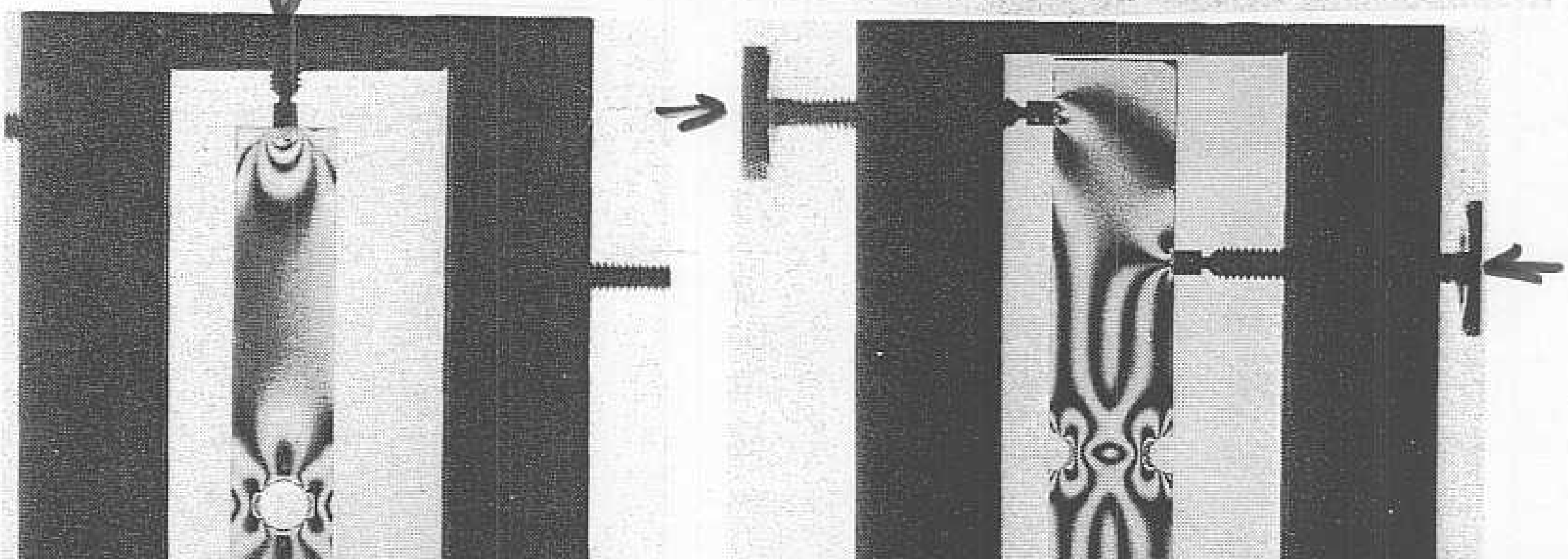


PHOTO-ELASTIC MTL



CLOSELY SPACED FRINGES = HIGHER STRESS

EXAMPLE

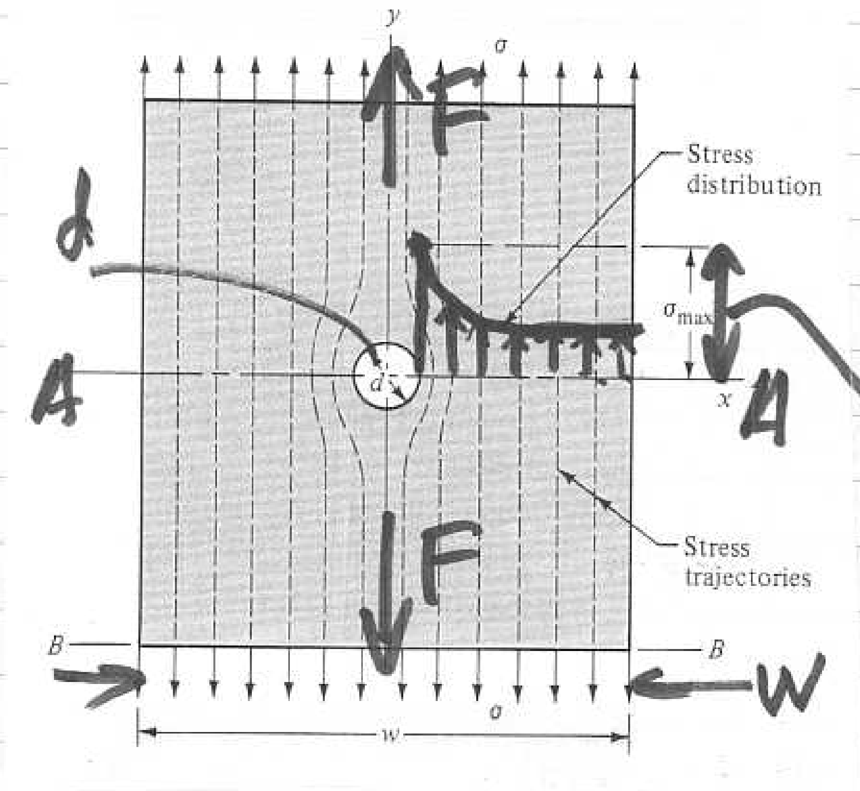


Fig 2-22

Tension

σ_{max}

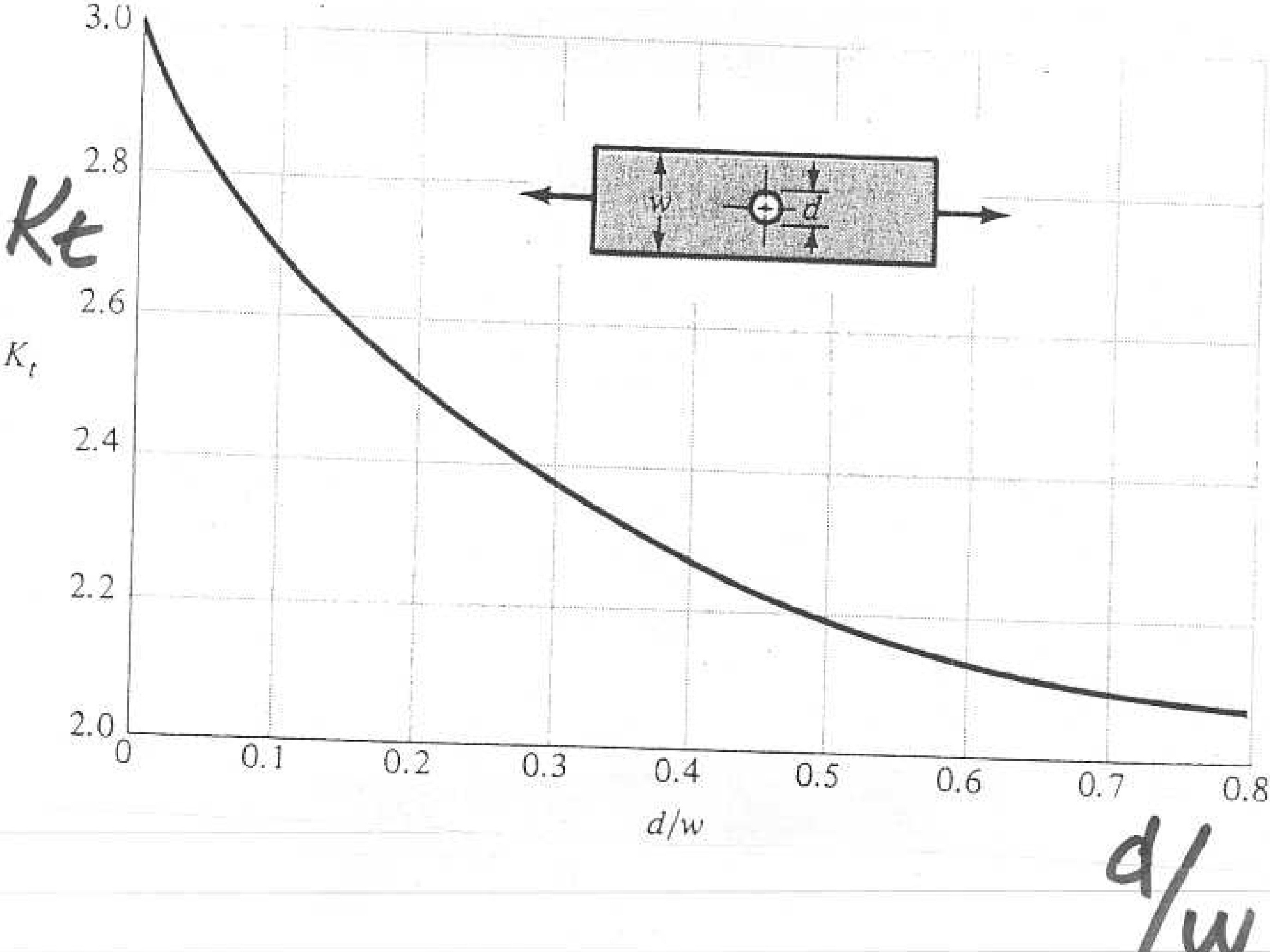
$$\sigma_{nom} = \frac{F}{A} = \sigma_0 \quad \text{AT A-A}$$

$$= \frac{F}{(w-d)t}$$

t ← thickness of plate

$$K_t = \frac{\sigma_{max}}{\sigma_0}$$

↳ Theoretical or Geometric Stress Concentration



Tables A-15

USING STRESS CONCENTRATION FACTORS

① CALCULATE nominal STRESS σ_0

② FIND K_t for <u>given geometry</u>

③ CALCULATE $\sigma_{max} = K_t \sigma_0$

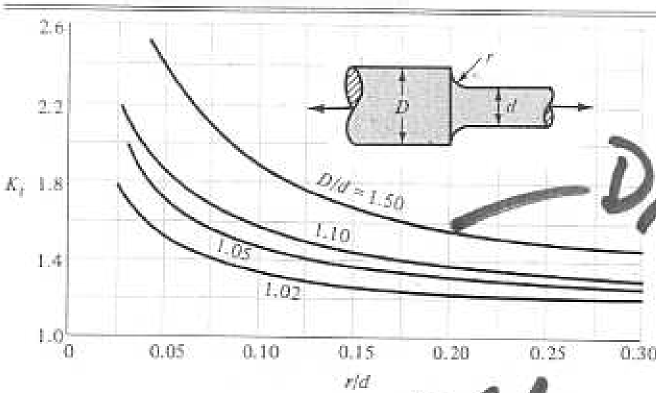
54

DEPENDS ON LOADING TYPE

TABLE A-15

Charts of Theoretical Stress-Concentration Factors K_t *(Continued)*

Kt



D/d=.5

FIGURE A-15-7

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

r/d

Kts

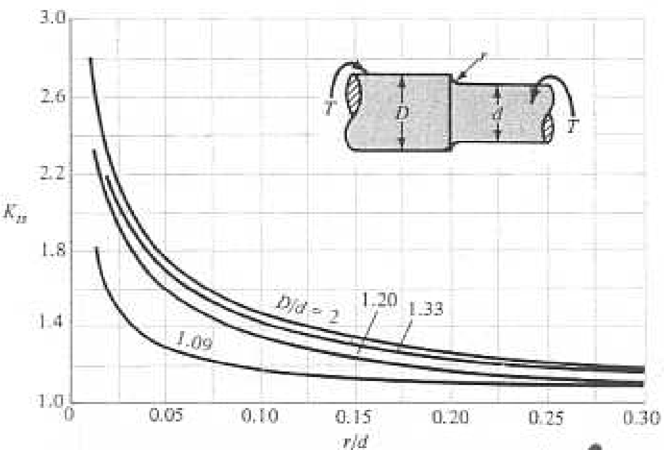


FIGURE A-15-8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

r/d

Kt

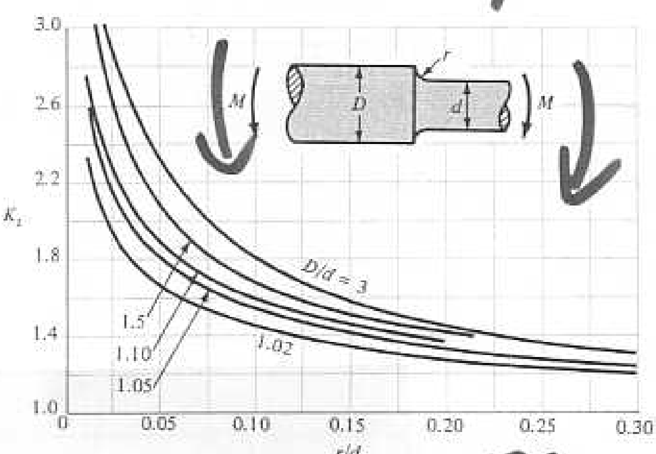


FIGURE A-15-9

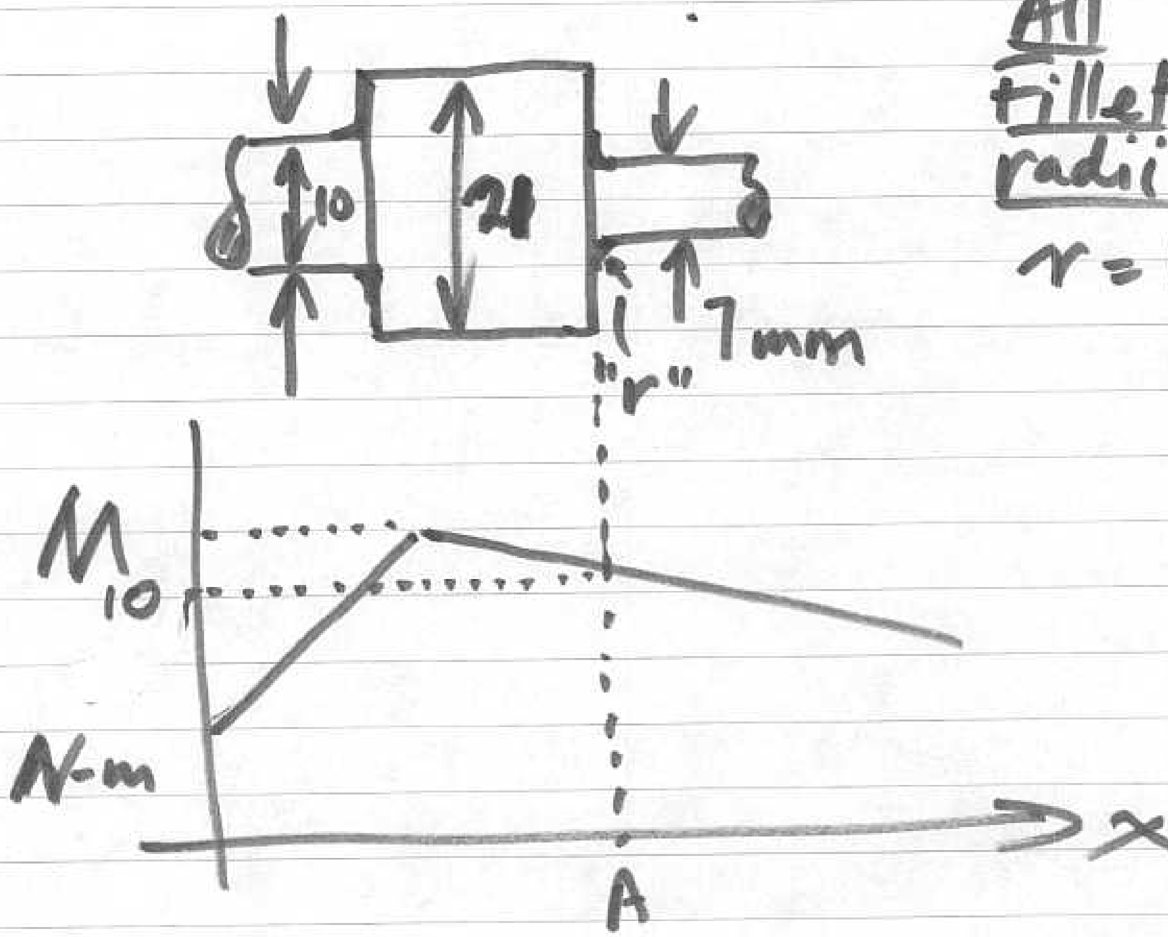
Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

r/d

EXAMPLE

All fillet radii

$r = 1\,mm$



- Determine the nominal bending stress at A
- What is K_t at A ?

$$\frac{r}{d} = \frac{1}{7} = \underline{0.14}$$

$$\frac{D}{d} = \frac{21}{7} = \underline{3}$$

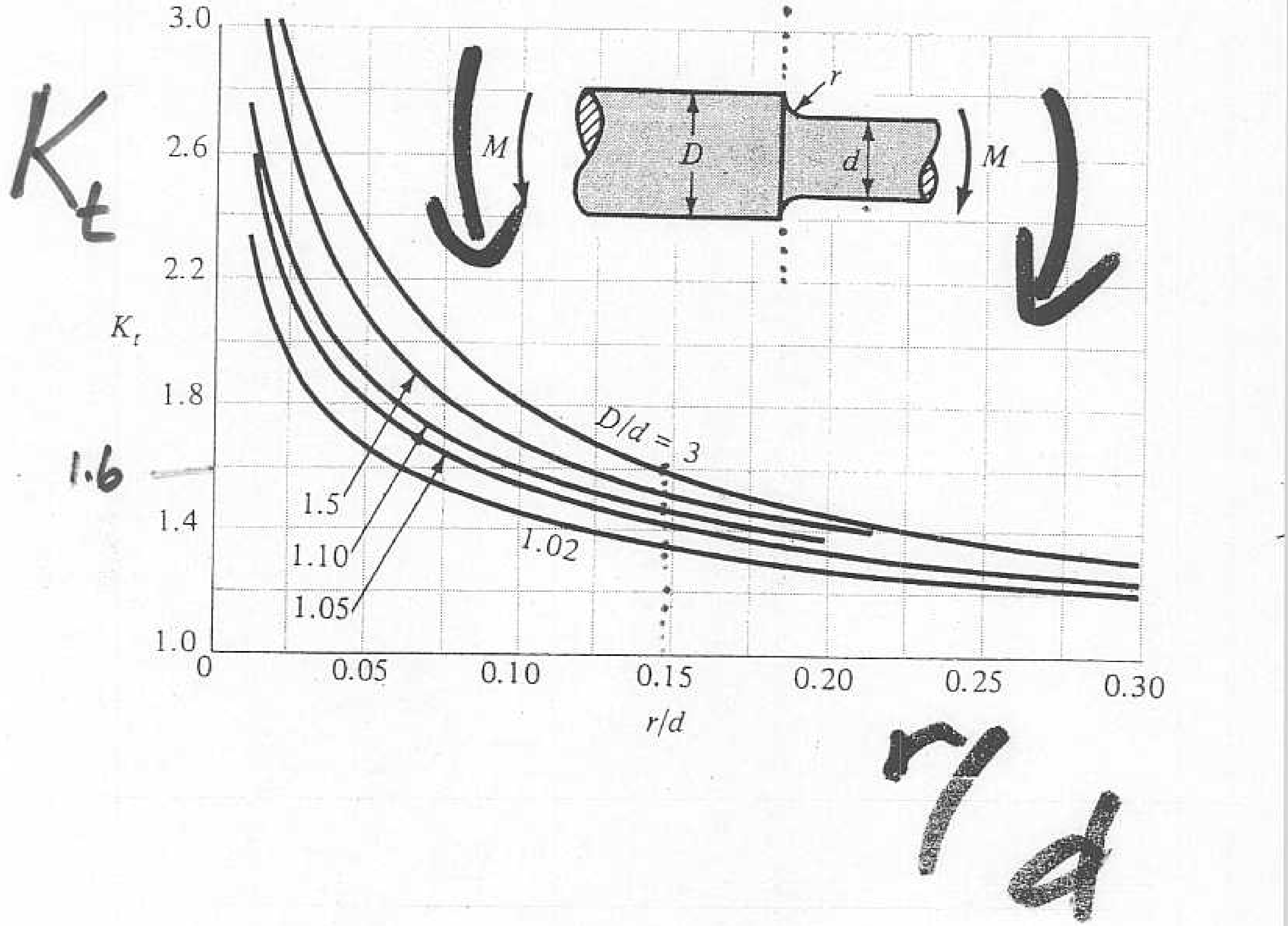


FIGURE A-15-9

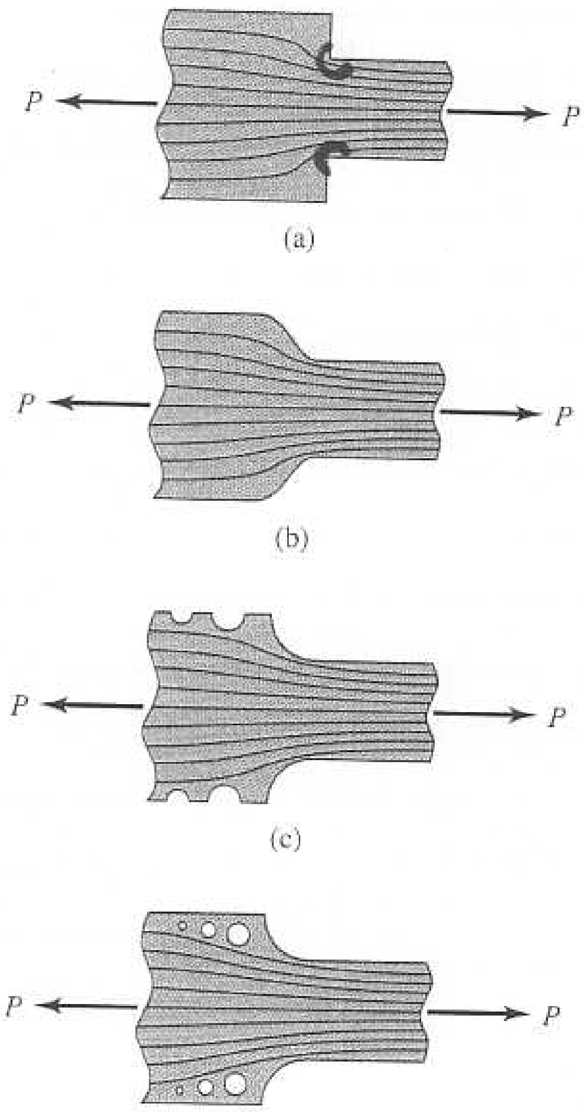
Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$

$\sigma_0 = \frac{Mc}{I}$.

$$\sigma_0 = \frac{32(10)(0.007/2)}{\frac{\pi(0.007)^4}{64}} = \underline{296\ MPa}$$

$\sigma_{max} = 296\,k_t = 473.6$ MPa (MAY YIELD)

Reducing Kt :

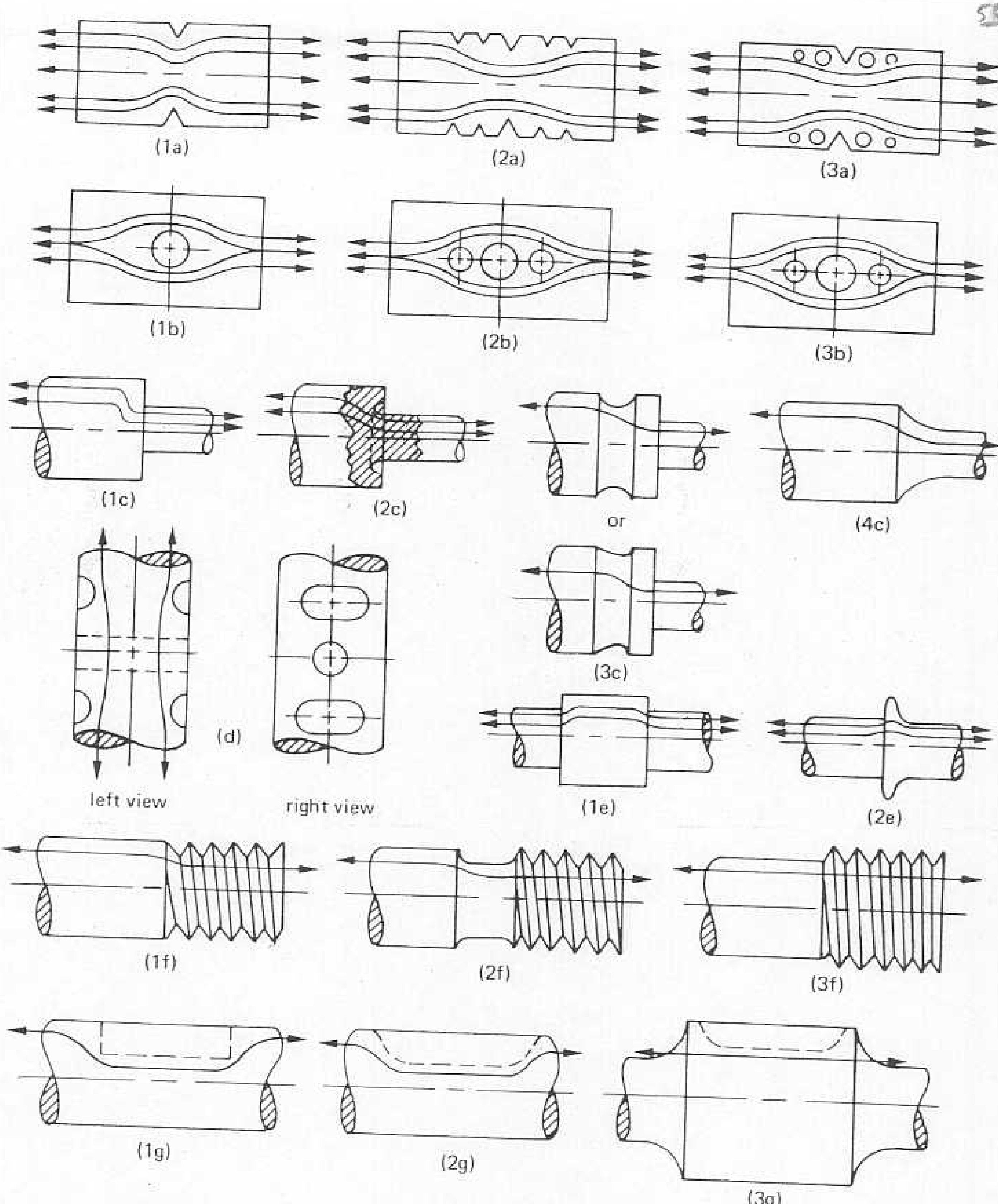


(a)

(b)

(c)

(d)



WORSE ⟵⟶ BETTER

Reducing K_t, con'd

SUMMARY OF LECT 1-7

CH1

- Design Process
- Strength, Stress, allowable stress, factor of safety

CH2

- Simple stress calcs
  - tension/compression
  - shear
  - torsion
  - bending – tension/comp, shear
- Locating maximum stress for given geometry & loading
- Principal stresses: ① calculate from $\sigma_x, \sigma_y, \tau_{xy}$ ② plot on Mohr's circle ③ 2-D & 3-D Mohr's circles
- Thick walled cylinders & interference fits
- Contact Stresses
- Stress Concentration Factors

CH 1 omit 1-10

CH 2 omit 2-17, 2-20

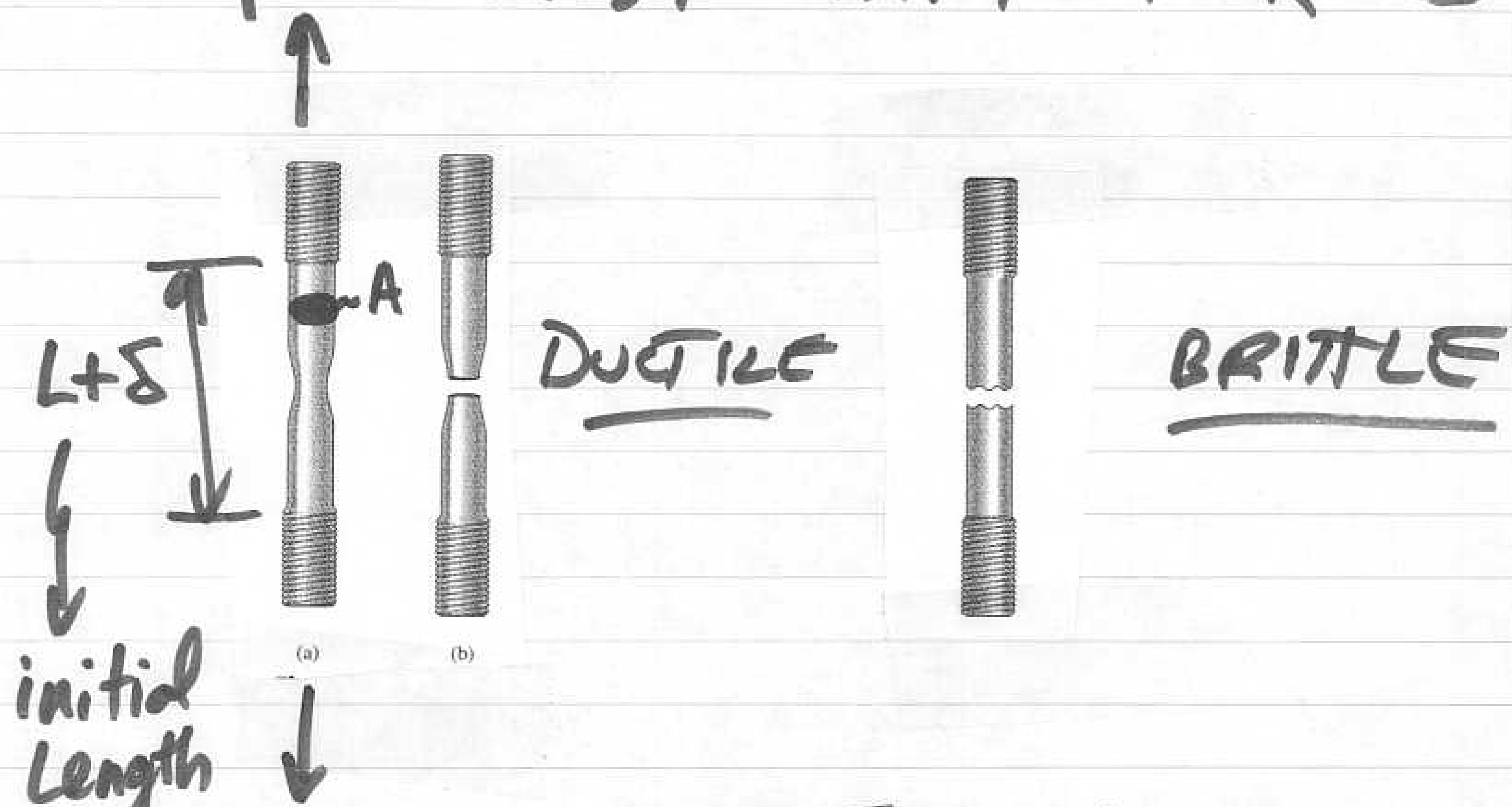
CH 3 omit

CH 4 omit

CH5

MATERIAL PROPERTIES

... **STRENGTH** IS MOST IMPT FOR US

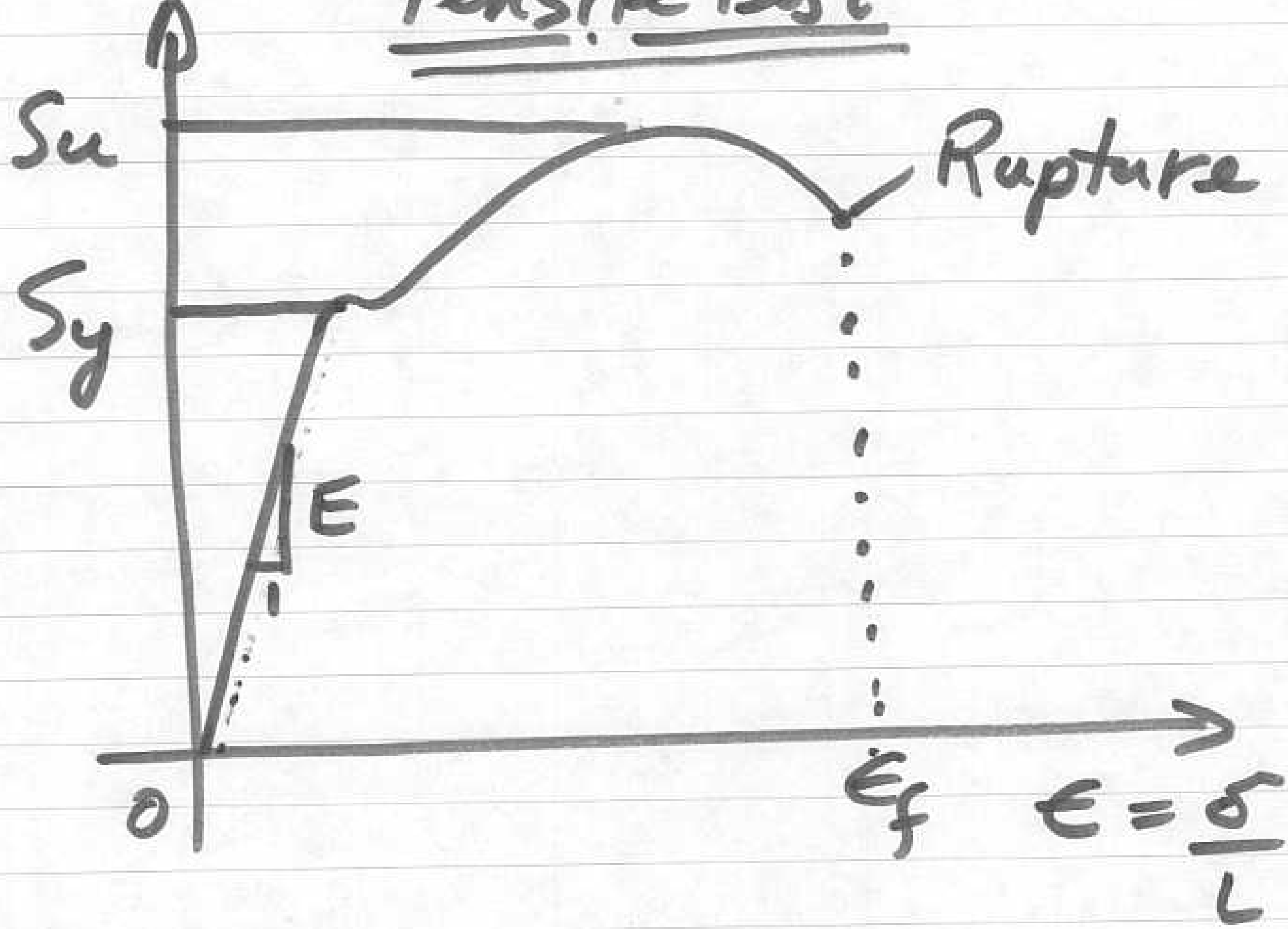


TENSILE TEST SAMPLES

$$\epsilon = \frac{\delta}{L} \qquad \%\,\text{Elongation} = \frac{\delta}{L} \times 100\%$$

DUCTILE MATERIALS

Tensile Test

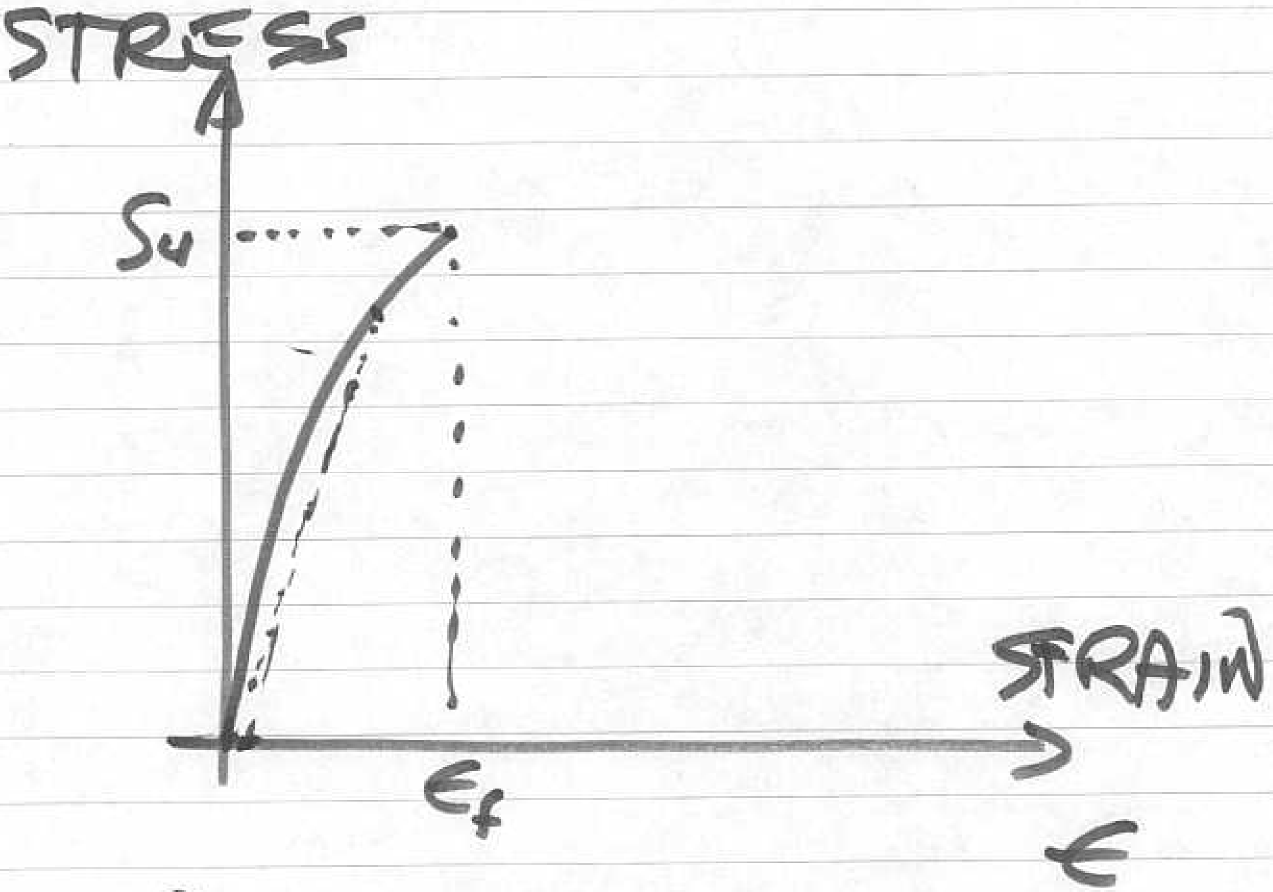


$S_u \Rightarrow$ Ultimate Strength
Tensile Compressive

$S_y \Rightarrow$ Yield Strength

- Ductile if $\epsilon_f > 5\%$. May be as high as 20-30%
- MOST METALS are DUCTILE

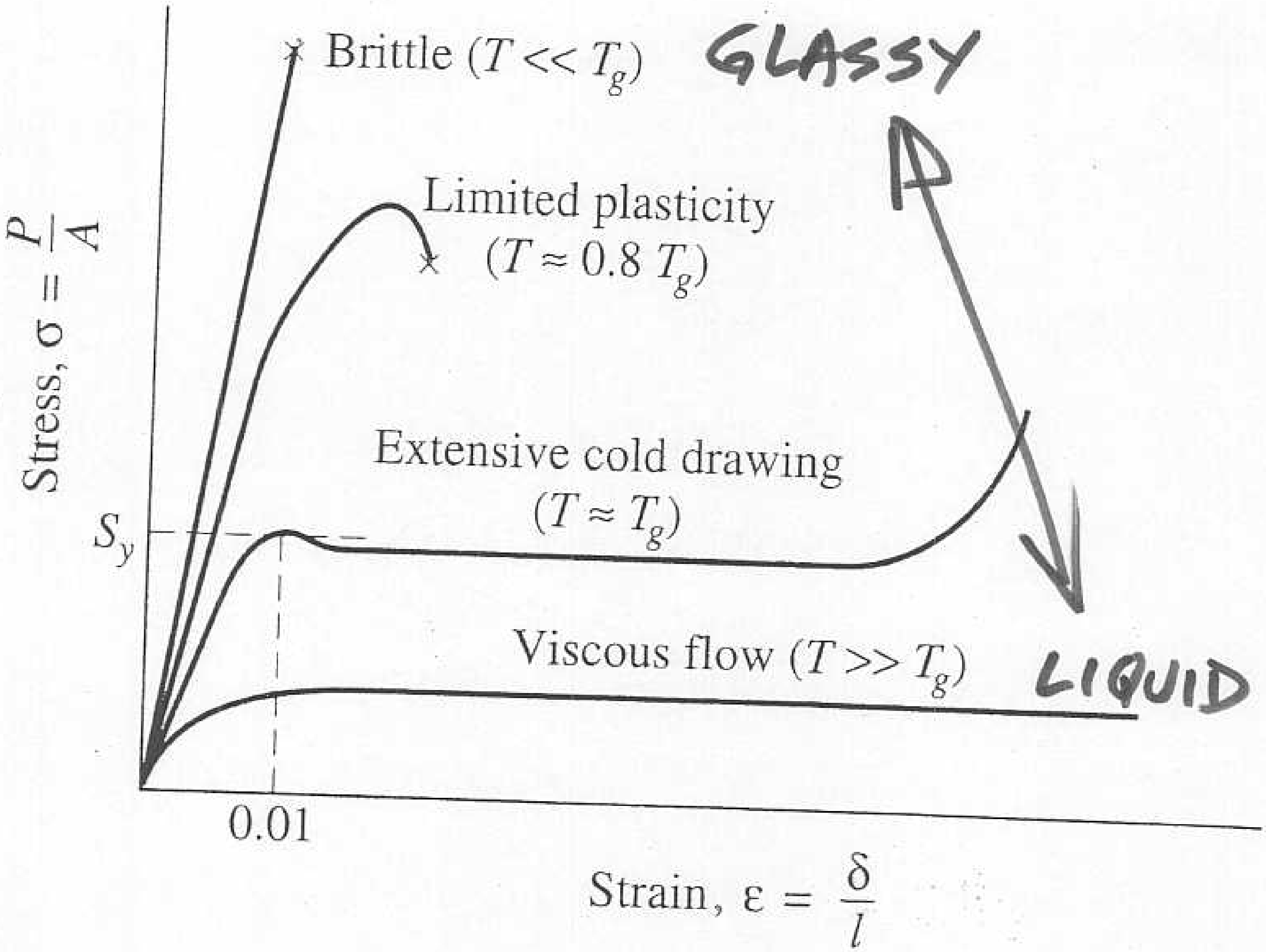
BRITTLE MATERIALS



- $\% \epsilon_f \ll 5\%$
- Ceramics, cast metals, concrete, some hardened metals, etc.
- Often $S_{uc} \gg S_{ut}$

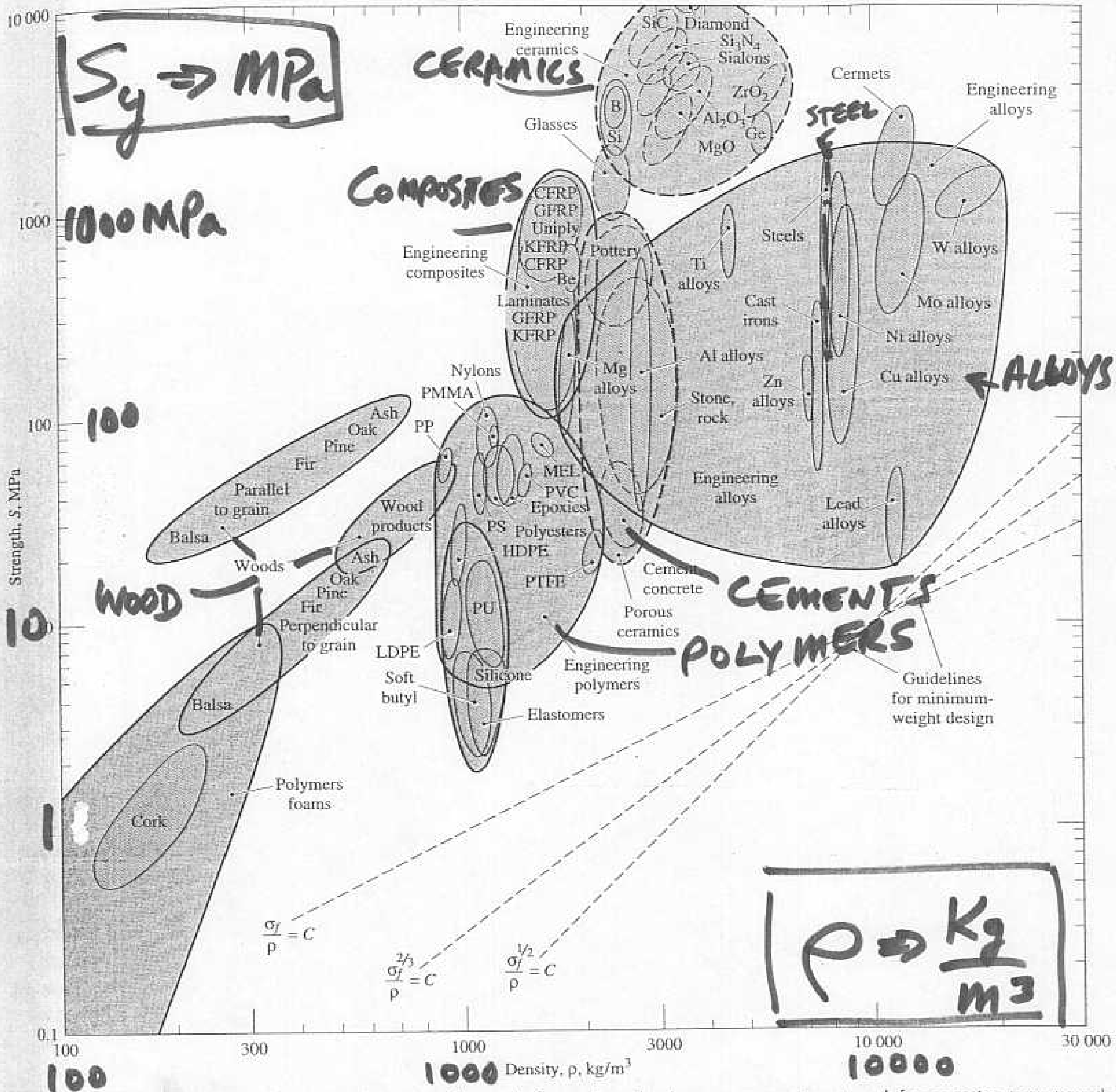
ultimate compressive strength

ultimate tensile strength



Stress-strain diagram for polymer below, at, and above its glass transition temperature T_g.

POLYMER BEHAVIOR AT VARIOUS TEMPS



Strength plotted against density (yield strength for metals and polymers, compressive strength for ceramics, tear strength for elastomers, and tensile strength for composites). The guidelines of constant S/ρ, $S^{2/3}/\rho$, and $S^{1/2}/\rho$ allow selection of materials for minimum-weight, yield-limited design. [M.F. Ashby, *Materials Selection in Mechanical Design*, Pergamon Press, 1992.]

STRENGTH vs DENSITY

FOR VARIOUS (MOST) MATERIALS

TABLE A-21

OBTAIN STRENGTH FROM TABLES
A-20 to A-24

Mechanical Properties of Some Heat-Treated Steels
[These Are Typical Properties for Materials Normalized and Annealed. The Properties for Quenched and Tempered (Q&T) Steels Are from a Single Heat. Because of the Many Variables, the Properties Listed Might Be Considered Attainable but Should Not Be Treated as Average or as Minimum. In All Cases, Data Were Obtained from Specimens of Diameter 0.505 in, Machined from 1-in Rounds, and of Gauge Length 2 in. Unless Noted, All Specimens Were Oil-Quenched]

S_{ut} & S_y

1	2	3	4	5	6	7	8
AISI NO.	TREATMENT	TEMPERATURE, °C (°F)	TENSILE STRENGTH, MPa (kpsi)	YIELD STRENGTH, MPa (kpsi)	ELONGATION, %	REDUCTION IN AREA, %	BRINELL HARDNESS
1030	Q&T*	205 (400)	848 (123)	648 (94)	17	47	495
	Q&T*	315 (600)	800 (116)	621 (90)	19	53	401
	Q&T*	425 (800)	731 (106)	579 (84)	23	60	302
	Q&T*	540 (1000)	669 (97)	517 (75)	28	65	255
	Q&T*	650 (1200)	586 (85)	441 (64)	32	70	207
	Normalized	925 (1700)	521 (75)	345 (50)	32	61	149
	Annealed	870 (1600)	430 (62)	317 (46)	35	64	137
1040	Q&T	205 (400)	779 (113)	593 (86)	19	48	262
	Q&T	425 (800)	758 (110)	552 (80)	21	54	241
	Q&T	650 (1200)	634 (92)	434 (63)	29	65	192
	Normalized	900 (1650)	590 (86)	374 (54)	28	55	170
	Annealed	790 (1450)	519 (75)	353 (51)	30	57	149
1050	Q&T*	205 (400)	1120 (163)	807 (117)	9	27	514
	Q&T*	425 (800)	1090 (158)	793 (115)	13	36	444
	Q&T*	650 (1200)	717 (104)	538 (78)	28	65	235
	Normalized	900 (1650)	748 (108)	427 (62)	20	39	217
	[illegible]	[illegible]					

6p

- STRENGTH OF MATERIALS USED IN TABLES A-20 to A-24 and in TEXT

- HARDNESS & STRENGTH ARE RELATED ... see . 5-4

  HARDNESS ↑ ⟺ STRENGTH

- STRENGTH DEPENDS ON MATERIAL PREPARATION AND HEAT TREATMENT
  e.g. cold working   tempering
- STRENGTH DECREASES WITH INCREASING TEMP FOR MOST MATERIALS

- SURFACE PROPERTIES MAY DIFFER FROM BULK PROPERTIES

  ON PURPOSE   OR NOT

e.g. HEAT TREAT,

Announcements

① First Test:
Monday Feb 24 IN CLASS

② Assignment #2 Extension
until MONDAY FEB 3

③ Class notes thru Mon
JAN 27 up on website.

FAILURE OF MECHANICAL COMPONENTS

CH 6

STATIC/STEADY LOADING

- Ductile Mtls 1
- Brittle Mtls 2

CH 7

VARIABLE, OSCILLATING FLUCTUATING LOADS

FATIGUE FAILURE

- FINITE LIFE
- INFINITE LIFE
- VARIABLE LOADS ONLY

3

- VARIABLE + STEADY LOADS 4

NOTE: Figs 6-1 → 6-11 Are examples of fatigue

FAILURE UNDER STEADY (STATIC) LOADING

LOAD APPLIED $\longrightarrow$ COMPONENT YIELDS/FRACTURES

Failure Theories

... explain/postulate why components fail under complex loading... we will deal mostly with plane stress

i.e, $\sigma_x, \sigma_y, \tau_{xy}$ ... may be non-zero

$\sigma_z, \tau_{xz}, \tau_{yz} \Rightarrow$ are ZERO

... result in σ_1, σ_2 principal stresses & $\sigma_3 = 0$

Failure theories

STATIC LOADING OF DUCTILE MATERIALS

1. Maximum normal stress theory (don't use)

(MSST)

2. Maximum Shear stress Theory (Tresca Theory)
... Good somewhat conservative

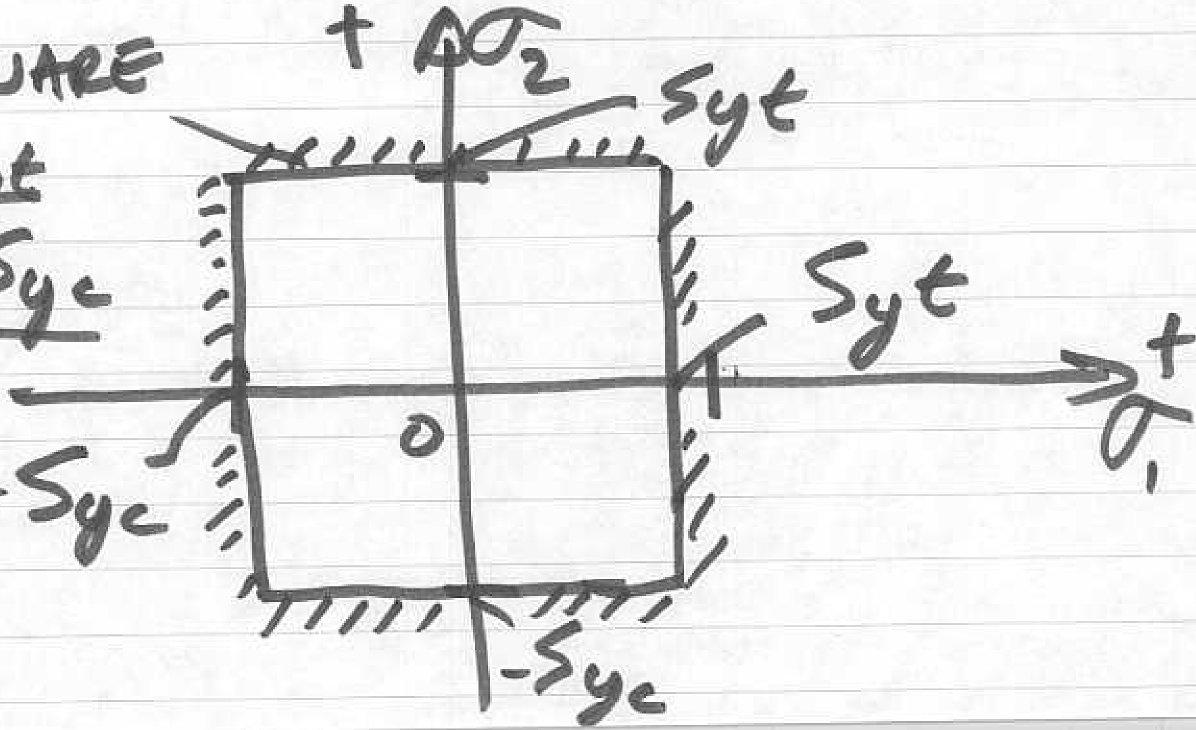
3 Distortion Energy Theory
.... Generally best .... a little better Than Max shear.

MAX NORMAL STRESS THEORY

"IF MAX PRINCIPAL STRESS EXCEEDS YIELD STRENGTH (IN A TENSILE OR COMPRESSIVE TEST) COMPONENT WILL FAIL, i.e. YIELD"

.... CAN PLOT "SAFE REGION" AS FUNCTION OF σ_1 & σ_2 THE TWO NON-ZERO PRINCIPAL STRESSES

SQUARE when $S_{yt} = S_{yc}$



If it's right!

ACCORDING TO THEORY

- ANY COMBINATION OF σ_1, σ_2 WITHIN BOX WILL HAVE FACTOR OF SAFETY > 1

- ALL POINTS ON EDGES OF BOX HAVE FACTOR OF SAFETY $n = 1$

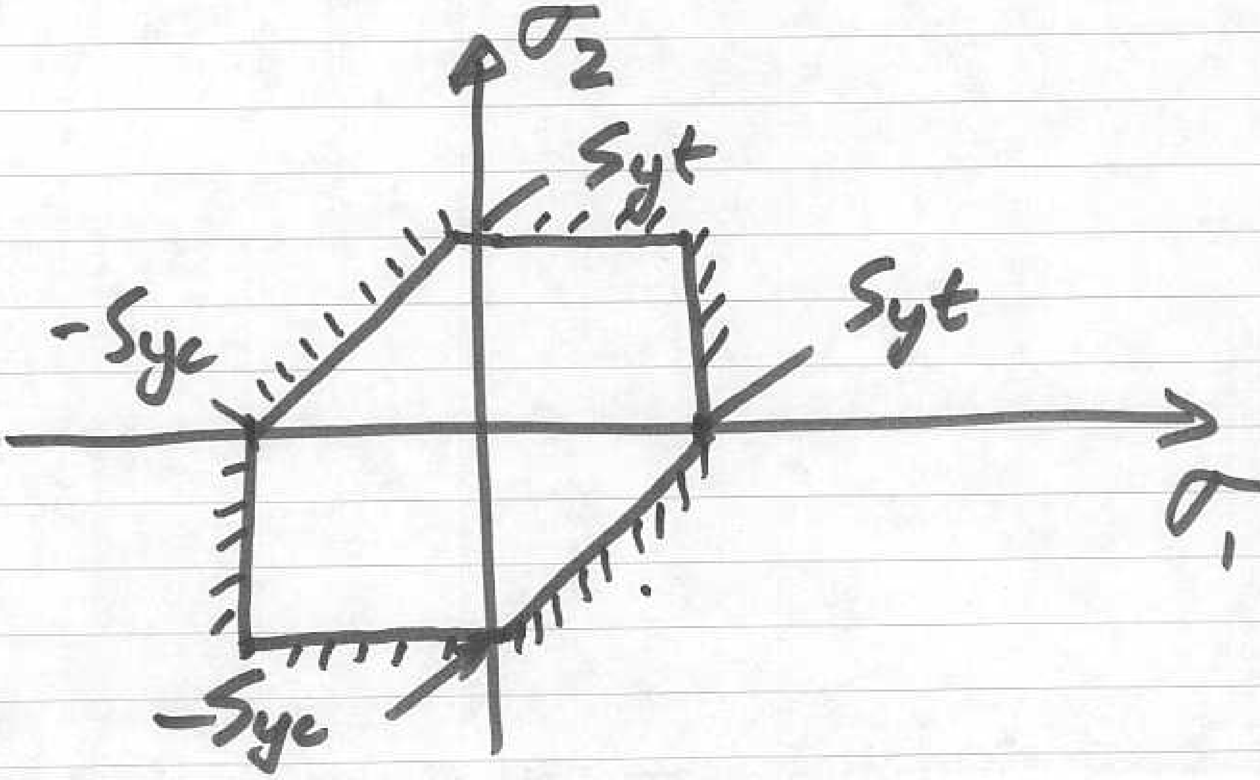
$\therefore$ n will be The smallest of

$$\underbrace{\frac{S_{yt}}{\sigma_1}, \frac{S_{yt}}{\sigma_2}}_{\substack{\sigma_1 \text{ or } \sigma_2 \text{ IN} \\ \text{TENSION (+VE)}}}, \underbrace{\frac{-S_{yc}}{\sigma_1}, \frac{S_{yc}}{\sigma_2}}_{\substack{\sigma_1 \text{ or } \sigma_2 \text{ IN} \\ \text{COMPRESS (-VE)}}}$$

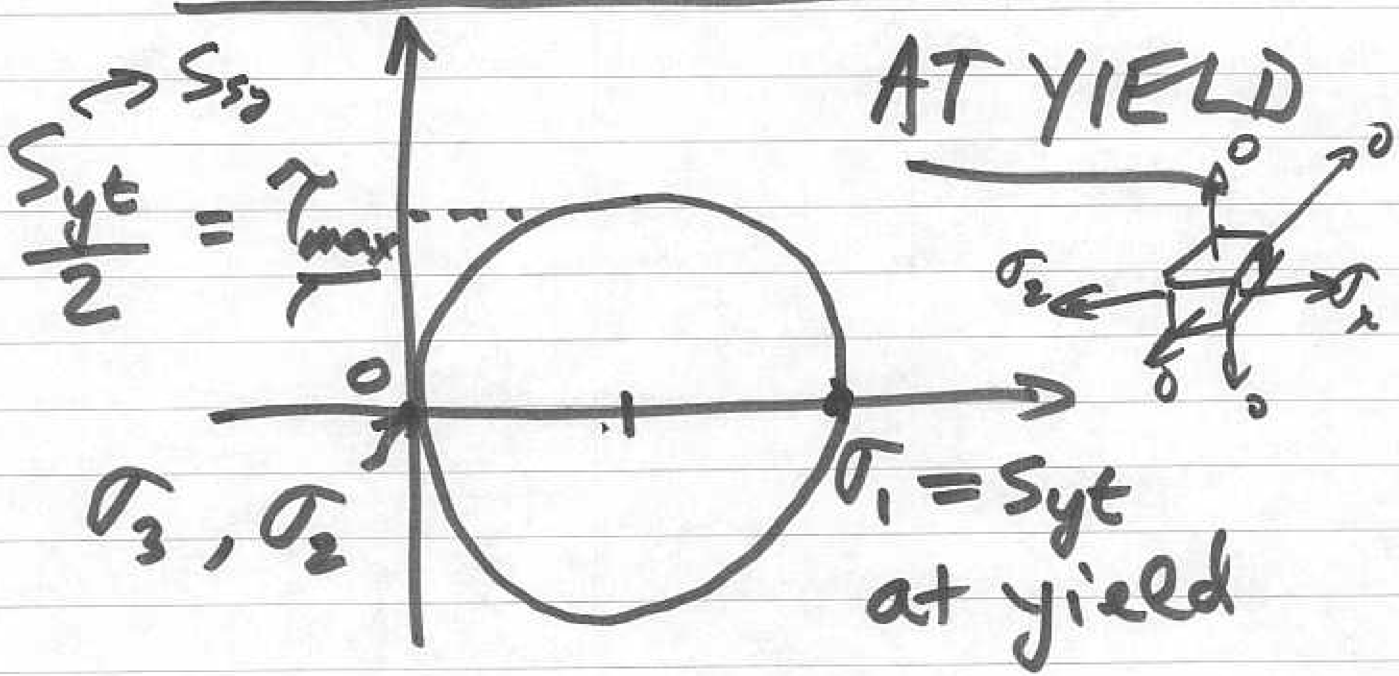
MAXIMUM SHEAR STRESS THEORY

"YIELDING BEGINS WHEN THE MAX SHEAR STRESS IN AN ELEMENT BECOMES EQUAL TO THE MAX SHEAR STRESS IN A TENSILE/COMPRESSIVE TEST AT YIELD"

.. OFTEN USED IN STRUCTURAL CODES



τ_{max} in tensile test is:

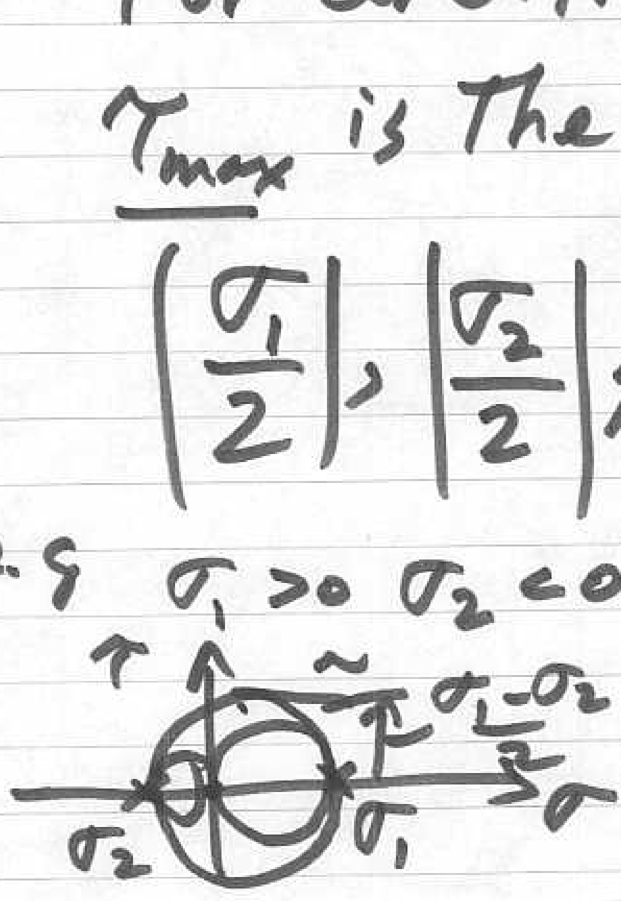


For arbitrary σ_1, σ_2

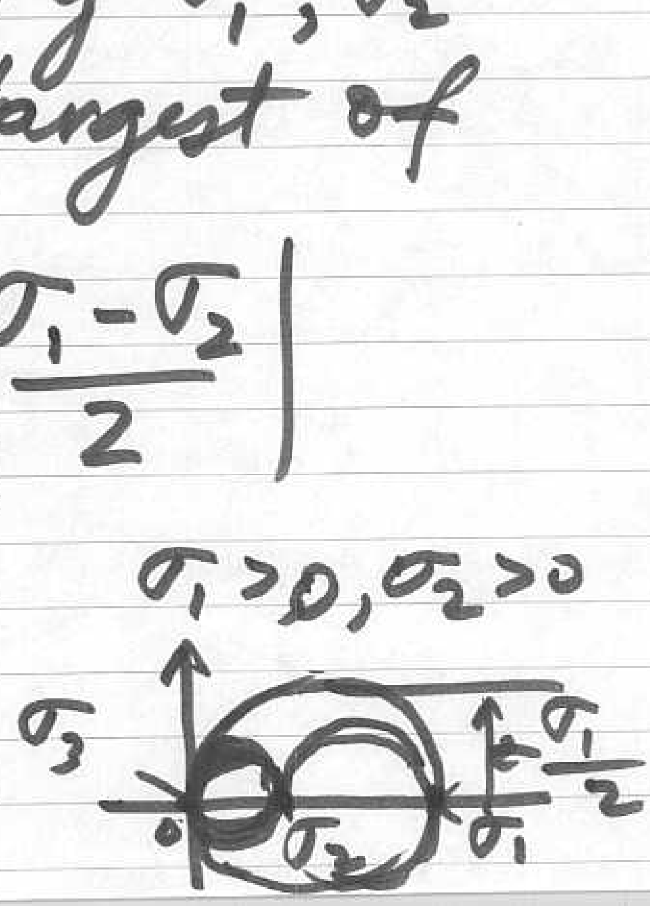
τ_{max} is the largest of

$$\left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|, \left|\frac{\sigma_1 - \sigma_2}{2}\right|$$

e.g. $\sigma_1 > 0 \quad \sigma_2 < 0$



$\sigma_1 > 0, \sigma_2 > 0$



... IF WE ORDER THE PRINCIPAL STRESSES SUCH $\sigma_a \geq \sigma_b \geq \sigma_c$

... THEN FACTOR OF SAFETY, n BECOMES:

$$\boxed{n = \frac{S_y}{\sigma_a - \sigma_c}}$$

... ALSO USE THE TERM "SHEAR STRENGTH" WHICH IN THIS THEORY SAYS $S_{sy} = 0.5\, S_y$

DISTORTION ENERGY (AKA VON-MISES) THEORY

... "FAILURE OCCURS WHEN SHEAR DEFORMATION EQUALS SHEAR DEFORMATION (DISTORTION) IN A TENSILE TEST AT YIELD PT"

Corollary:

... PURE (HYDROSTATIC) COMPRESSION $\sigma_1 = \sigma_2 = \sigma_3$ WILL NOT CAUSE FAILURE

... THIS THEORY EMPLOYS THE VON-MISES STRESS σ'

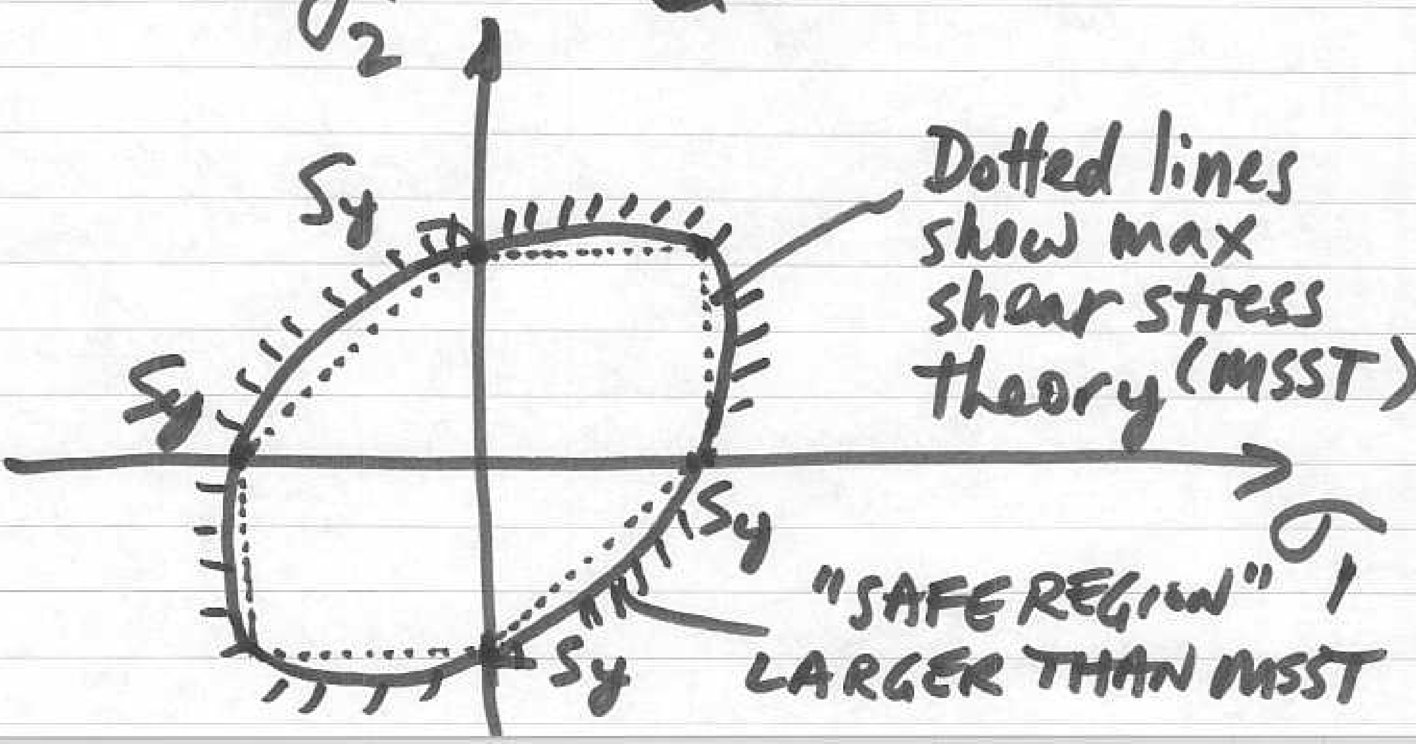
$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}\right]^{1/2}$$

FOR PLANE STRESS, $\sigma_3 = 0$

and $\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$

FAILURE (YIELD) OCCURS WHEN $\sigma' \geq S_y$

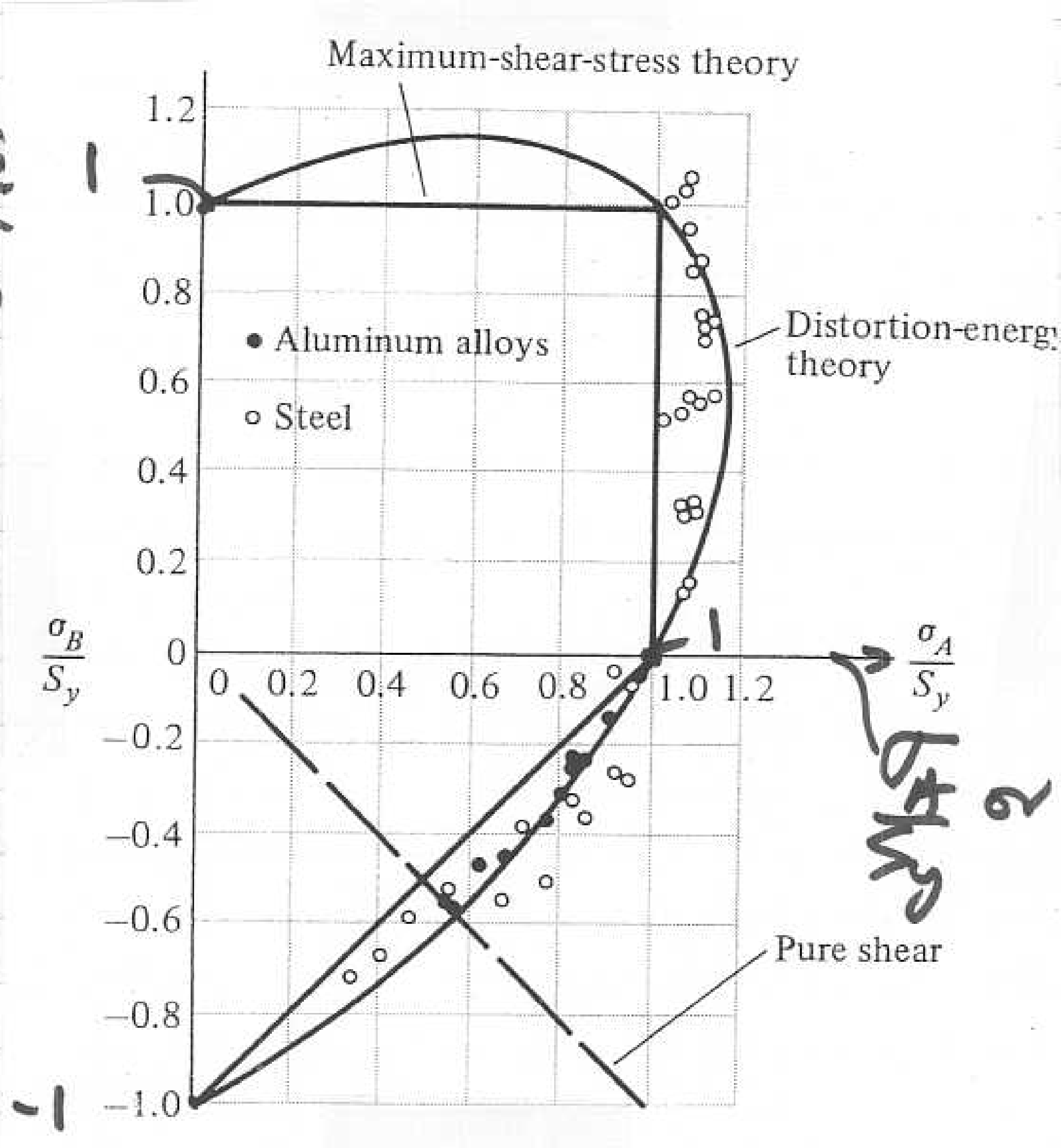
AND THE FACTOR OF SAFETY IS $n = \dfrac{S_y}{\sigma'} = \dfrac{S_y}{\sigma'}$



SOME FAILURE DATA

Fig 6-25

$\frac{\sigma_3}{S_y}$ or $\frac{\sigma_2}{S_y}$



$\frac{\sigma_A}{S_y}$ or $\frac{\sigma_1}{S_y}$

DUCTILE MTL

FOR PURE SHEAR OR TORSION

$$S_{sy} = 0.5\,S_y$$ as per MSST

$$S_{sy} = 0.577\,S_y$$ as per Dist. Energy

$$n = \frac{S_{sy}}{\tau_{max}}$$

Static Failure of Ductile Materials

- USE ① Max Shear Stress or ② Distortion Energy Theory .... assume $S_{yt} = S_{yc}$ ..... ignore other failure Theories.
- Generally Distortion energy is preferred
- No stress concentration ... DUCTILE

Example: Determine Factory of Safety for:

$\sigma_y = -8\,kpsi$ $\sigma_x = 20\,kpsi$

$\tau_{xy} = 12\,kpsi$ $S_y = 50\,kpsi.$

* **Find Principal Stresses**

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

and $\sigma_1 = 24.4\,kpsi$ $\sigma_2 = -12.4\,kpsi$

① **Max Shear stress Theory, n_s**

$$n_s = \frac{S_y}{\sigma_a - \sigma_c} = \frac{50}{24.4-(-12.4)} = \underline{\underline{1.36}}$$

Max Princ. stress Min Princ. Stress

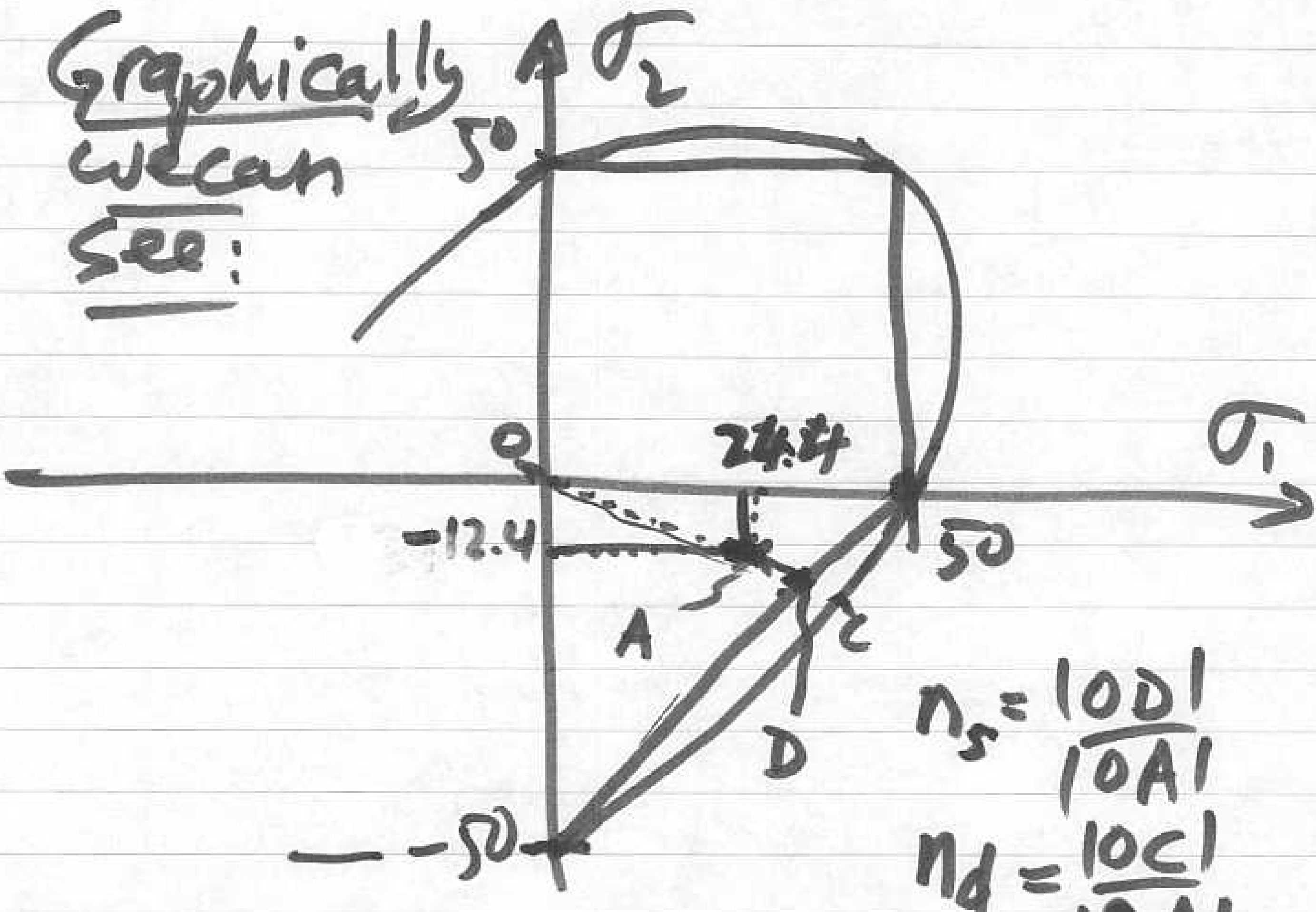
② Distortion Energy, n_d

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$\sigma' = \underline{32.5 \text{ kpsi}}$$

$$n_d = \frac{S_y}{\sigma'} = \frac{50}{32.5} = \underline{\underline{1.54}}$$

Graphically we can see:



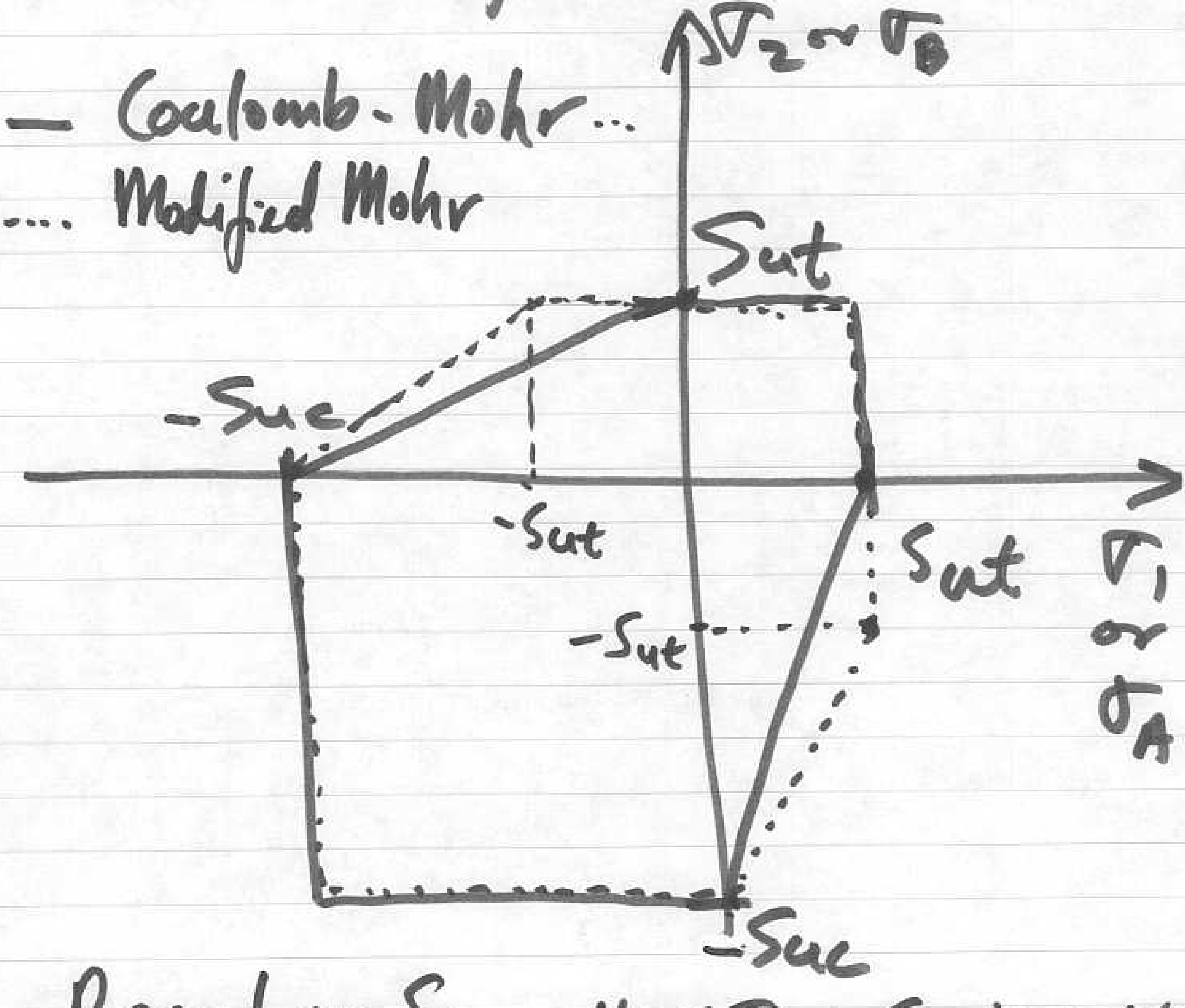
$$n_s = \frac{|OD|}{|OA|}$$

$$n_d = \frac{|OC|}{|OA|}$$

For Brittle Materials

① Coulomb - Mohr

② Modified Mohr



Based on S_u ... NOTE That S_{uc} is a +ve No.

FAILURE THEORY	FIRST QUADRANT $\sigma_A \geq 0,\ \sigma_B \geq 0$	FOURTH QUADRANT $\sigma_A \geq 0,\ \sigma_B < 0$	
Coulomb-Mohr (C-M)	$\sigma_A = \frac{S_{ut}}{n}$	$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$	C-M
Modified-Mohr (M-M)	$\sigma_A = \frac{S_{ut}}{n}$	$\sigma_A = \frac{S_{ut}}{n}$	$\sigma_B \geq -S_{ut}$
		$\sigma_A - \frac{S_{ut}\sigma_B}{S_{uc} - S_{ut}} = \frac{S_{uc}S_{ut}}{n(S_{uc} - S_{ut})}$	$\sigma_B < -S_{ut}$

M-M.

Formulae for Factor of Safety:

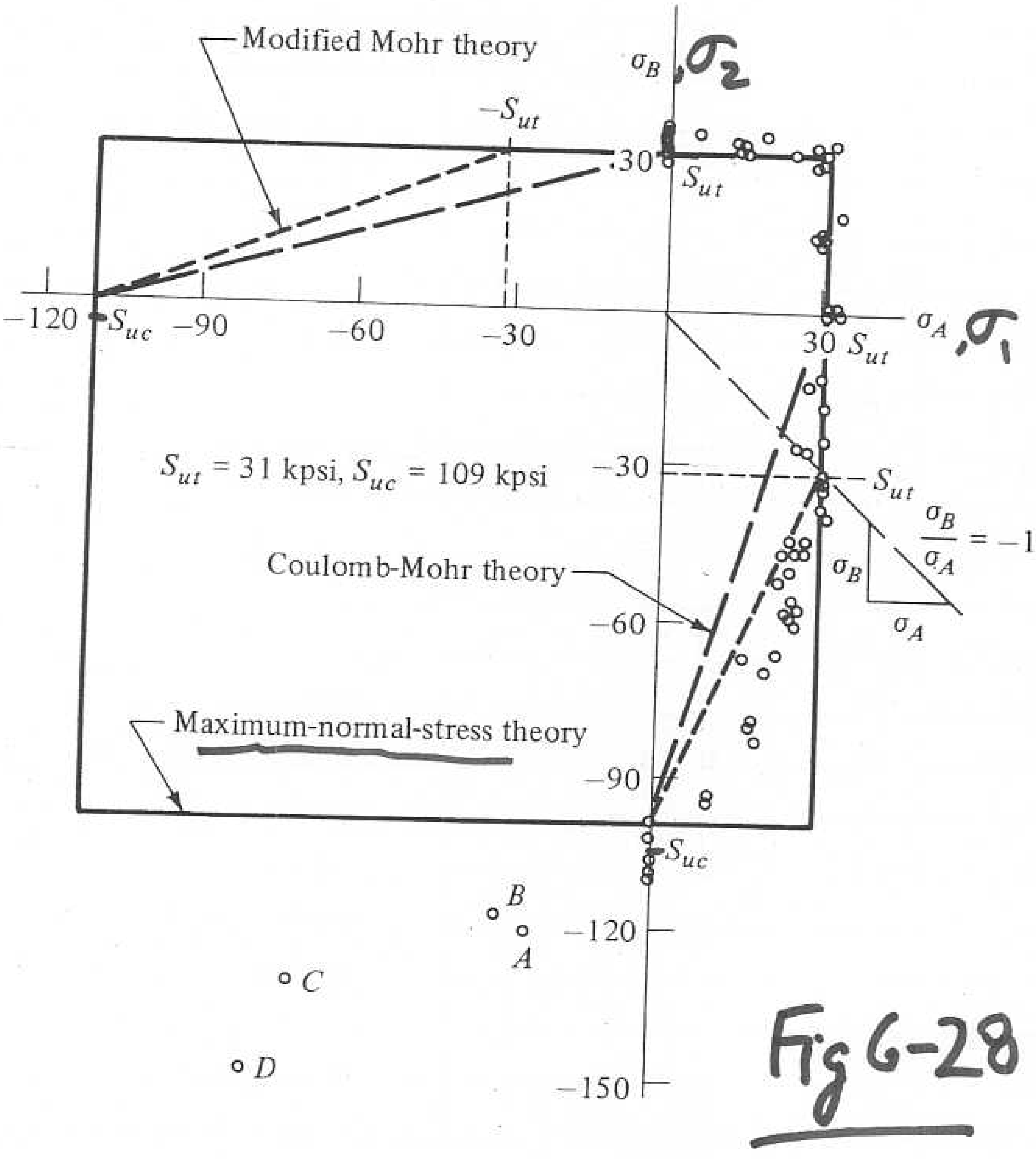
insert S_{ut}, S_{uc} ⇐ +ve Number

$\sigma_A \Leftrightarrow \sigma_1$

$\sigma_B \Leftrightarrow \sigma_2$

Solve for n. $\boxed{\text{factor of safety}}$

80



Modified Mohr theory
σ_B, σ_2
$-S_{ut}$
30
S_{ut}
-120
S_{uc}
-90
-60
-30
σ_A, σ_1
30 S_{ut}
S_{ut} = 31 kpsi, S_{uc} = 109 kpsi
-30
S_{ut}
$\frac{\sigma_B}{\sigma_A} = -1$
Coulomb-Mohr theory
σ_B
σ_A
-60
Maximum-normal-stress theory
-90
S_{uc}
B
-120
A
C
D
-150

Fig 6-28

Example (A)

Given: Brittle Mtl

$S_{ut} = 31$ kpsi

$S_{uc} = 109$ kpsi

For principal stresses:

$\sigma_1 = 15$ kpsi and $\sigma_2 = -20$ kpsi

Find:

Factor of safety, n → C-M, → M-M

NOTE Stresses in 4Th quadrant:

Stresses in 4th QUAD

According to C-M

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_2}{S_{uc}} = \frac{1}{n}$$

$$\frac{15}{31} - \frac{(-20)}{109} = \frac{1}{n}$$

$$\frac{1}{n} = .483 + .183 = .666$$

$\boxed{n = 1.5}$ → by Coulomb-Mohr Theory

According to M-M

Note $\sigma_2 \geq -S_{ut}$

i.e., $-20 \geq -31$

$$\therefore n = \frac{S_{ut}}{\sigma_1} = \frac{31}{15} = 2.07$$

$\boxed{n = 2.07}$ → by Modified-Mohr

Example (B)

÷ as A but with $\sigma_1 = 15$ kpsi

and $\sigma_2 = -35$ kpsi

... STILL IN 4TH QUAD ...

According to Coulomb-Mohr

$$\frac{1}{n} = \frac{\sigma_1}{S_{ut}} - \frac{\sigma_2}{S_{uc}}$$

$$= \frac{15}{31} - \frac{(-35)}{109}$$

$$\boxed{n = 1.24} \Rightarrow C\text{-}M$$

According to Modified-Mohr

$\sigma_2 = -35$ kpsi < 31 kpsi $= -S_{ut}$

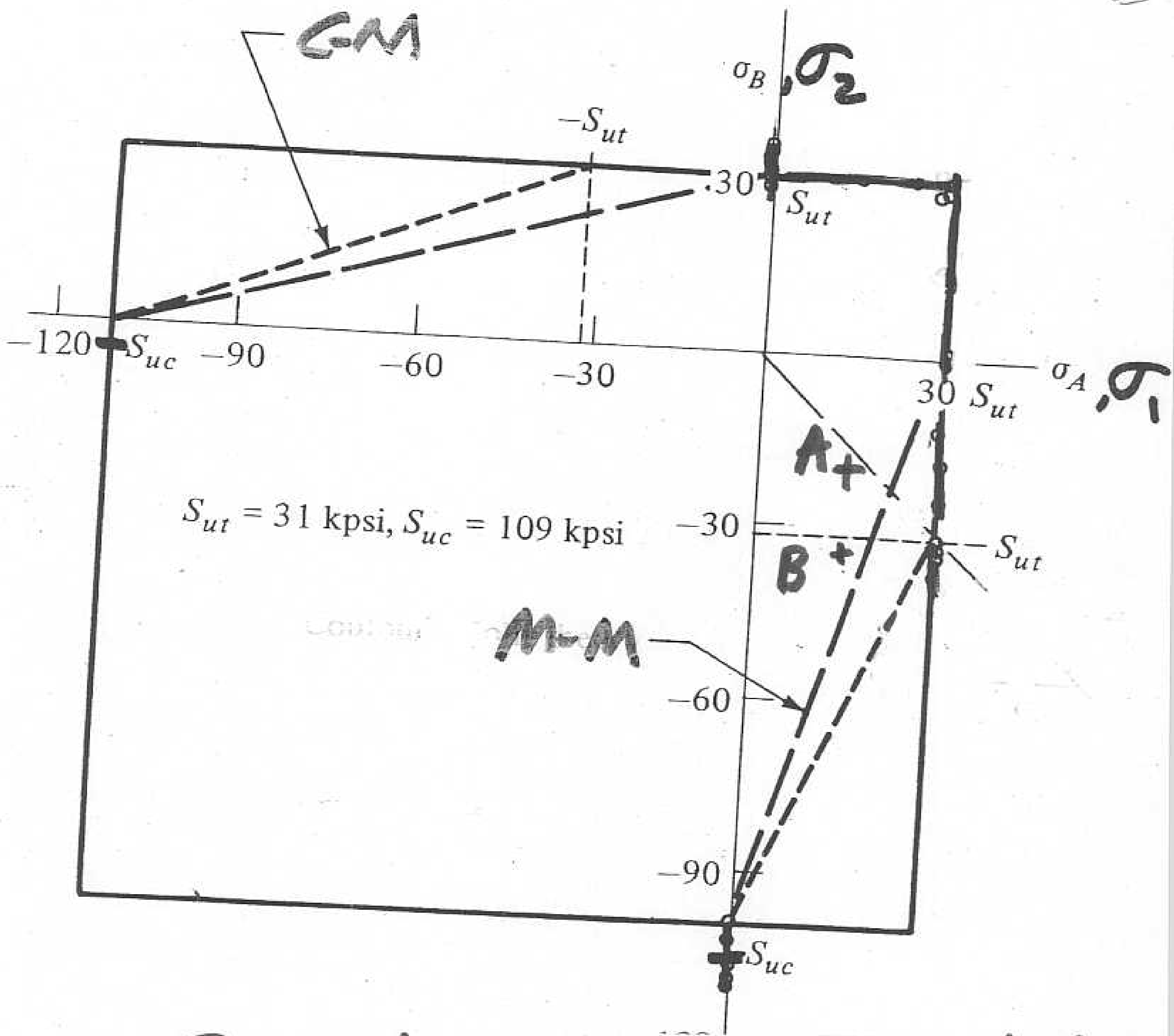
$$\therefore \quad \sigma_1 - \frac{S_{ut}\sigma_2}{S_{uc}-S_{ut}} = \frac{S_{uc}S_{ut}}{n(S_{uc}-S_{ut})}$$

$$15 - \frac{(31)(-35)}{(109-31)} = \frac{(109)(31)}{n(109-31)}$$

$$15 + 13.9 = \frac{43.3}{n}$$

$$n = \frac{43.3}{28.9}$$

$$\boxed{n = 1.50}$$ by Modified-Mohr



Examples A & B illustrated on σ_1 vs. σ_2 plot