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A New Algorithm to Optimize Barker Code Sidelobe Suppression Filters

Binary coding waveform sidelobe reduction after matched filtering is an active research topic both in radar system and, in some cases, spread spectrum communication applications. The authors suggest a new algorithm and present a rather general method, by which optimized sidelobe suppression filters for Barker codes can be obtained with the peak output sidelobe 2.62 dB lower than the results found in the literature (for 13 bit Barker code). This optimization algorithm is promising also for other binary coding waveforms, such as truncated pseudonoise (PN) sequences and concatenated codes.

I. INTRODUCTION

Because of their unique autocorrelation properties, some binary codes, such as Barker codes, are often found in modern pulse radars. They yield a pulse compression waveform by which the radar can achieve a relatively high range resolution with low average power. Barker coding waveforms are sometimes used also in spread spectrum communication, especially in asynchronous direct sequence systems [2] because of the simplicity to implement their correlation.

The following characteristic features for a pulse suppression filter are important: the output peak sidelobe level (PSL), the output mean square sidelobe level (MSSL), the loss in signal-to-noise ratio as compared with the matched filter (LSNR), and the complexity of hardware structure. In the application of high resolution synthetic aperture radars the mean square sidelobe level should be as low as −25 to −30 dB and the moving target indicator (MTI) radars require a sidelobe level performance at least as good as the synthetic aperture radars, in order to assure certain low reflectivity or "blind zone". The pulse doppler (PD) radars require the maximum sidelobe to be lower than −30 dB to prevent a sidelobe of a strong echo from masking the main lobe of a weak echo.

Unfortunately, after matched filtering the peak sidelobes for 11 bit and 13 bit Barker codes are −20.83 dB and −22.28 dB, respectively, far from the required level of −30 dB or lower. In order to further suppress the sidelobes, there are generally two methods which have currently often been used: one is utilizing an additional weighting network after the matched filter. One possibility is the R-G sidelobe reduction filters which were first introduced by A. W. Rihaczek and R. M. Golden in [3], and synthesized in the frequency domain. They offer a simple structure and an acceptable performance. But the method suggested in [3] cannot be applied to binary code waveforms with negative sidelobes. Further analysis has also shown that the R-G filters in [3] are not optimized from the viewpoint of minimum range sidelobes. Another way to reduce the sidelobes is to design a mismatch filter directly, as presented in [5 and 6], instead of adding a weighting network after the matched filter. S. Zoraster has shown how to get minimum peak sidelobes with a linear programming (LP) algorithm [5]. When the length of the weighting sequence of the LP filters becomes long enough, the peak sidelobes and the MSSLs can reach a satisfactory value. But the price is complexity of the filter hardware and greater loss in signal-to-noise ratio at the output, as shown in Fig. 4.

Here the author tries to bring forward a new general way which combines the advantages of the two methods introduced above. It is deduced that no matter of which polarity, negative or positive, the sidelobes of the autocorrelation function are, the transfer function of the sidelobe suppression filter can be fitted with a polynomial expansion series in the frequency domain, which consists of some unknown expansion coefficients. With these coefficients the transfer function of the filter can be approximated by a limited number of polynomial terms. Then by applying the inverse Fourier transformation and LP in the time domain, the unknown coefficients can be determined. Substituting the unknown coefficients in the filter transfer function by the determined ones, an optimized R-G filter, here called $(\text{R-G})_{\text{opt}}$ filter, is obtained. The output peak sidelobe of $(\text{R-G})_{\text{opt}}$ is much lower than in the corresponding R-G filter. For example, the peak sidelobe of $(\text{R-G-2})_{\text{opt}}$ is 2.62 dB lower than that of an R-G-2 filter. It is quite clear that this method can be applied to sidelobe suppression in other binary coding waveforms too.

II. SIDELOBE REDUCTION OPTIMIZATION IN TIME DOMAIN

A. Linear Programming Filters

Let us first consider the way to design the LP mismatched filters [5], the structural diagram of which

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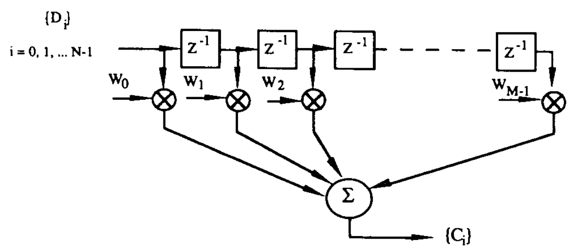

Fig. 1. Tapped delay line LP filter.

is shown in Fig. 1. The transmitted binary-coded waveform is represented by the real elements $\{D_i\}$ with $D_i = \pm 1$ and $1 \leq i \leq N$. The M filter weights are represented by W_i with $M \geq N$. For both symmetry and mathematical simplicity M is assumed to be odd if N is odd, and even if N is even. Then consider the following LP problem:

$$J = \max \sum_{i=1}^{M} W_i D_{i-(M-N/2)} \qquad (D_i = 0,\ i \leq 0 \text{ or } i > N) \tag{1a}$$

subject to the inequality:

$$\left| \sum_{i=1}^{M} W_i D_{i-k} \right| \leq 1,$$

$$1 - N \leq k \leq M - 1, \qquad k \neq \frac{(M-N)}{2}. \tag{1b}$$

The constraints above can be rewritten as

$$\sum_{i=1}^{M} W_i D_{i-k} \leq 1$$

and

$$-\sum_{i=1}^{M} W_i D_{i-k} \geq 1,$$

$$1 - N \leq k \leq M - 1, \qquad k \neq \frac{(M-N)}{2}. \tag{2}$$

So, we have a linear objective function with M variables and $2(M + N - 2)$ linear inequality constraints. It can be solved in a finite number of steps by an iterative and monotonic simplex algorithm to obtain M weights $\{W_i\}$. In order to meet the need for an at most -30 dB peak sidelobe, the tapped delay line of Fig. 1 is longer than 30 taps, resulting in a rather complicated structure.

B. Optimized R-G Filters

Next we try to find a more practical alternative way to implement Barker code sidelobe reduction filters. Because the R-G filters presented in [3] have simpler hardware structures, we use this advantage and try to enhance their performance by an optimization algorithm to meet our needs. For simplicity, we consider an 11 bit Barker code, which possesses the autocorrelation function $R(t)$. Note that $R(t)$ can also be rewritten as two subfunctions, $R_m(t)$ and $R_s(t)$, representing the contributions of the main lobe and the sidelobes, respectively. Each subfunction has its own known Fourier transform pair. The convolution of the subfunctions yields $R(t)$:

$$R(t) = R_m(t) * R_s(t). \tag{3}$$

By using the Fourier transformation in (3), we obtain the energy density spectrum of an 11 bit Barker code:

$$E(f) = E_m(f) E_s(f) \tag{4a}$$

where

$$E_m(f) = \frac{\sin^2(\pi f T)}{(\pi f T)^2} \tag{4b}$$

$$E_s(f) = N + 1 - \frac{\sin(2\pi f N T)}{2\pi f T} \tag{4c}$$

and $E_m(f)$ and $E_s(f)$ represent the spectrum contribution of the main lobe and the sidelobes, respectively. It is obvious that if we could find a network which has a transfer function $1/E_s(f)$, then the sidelobes in every range cell would vanish. Often it is rather difficult to synthesize a filter with a transfer function exactly equal to $1/E_s(f)$. But the closer the transfer function of the filter approximates $1/E_s(f)$, the lower the peak output sidelobe will be. From (4c) we obtain for the required transfer function:

$$H(f) = \frac{1}{E_s(f)} = \frac{1}{N + 1 - \dfrac{\sin(2\pi f N T)}{2\pi f T}}. \tag{5}$$

For any f, the following inequality is always true,

$$\frac{1}{N+1} \frac{\sin(2\pi f N T)}{2\pi f T} < 1. \tag{6}$$

So (5) can be expanded into a convergent exponential series. For simplicity, only the first four terms of the series are retained in $H(f)$. That is,

$$H(f) \approx A + B \frac{\sin(2\pi f N T)}{2\pi f T} + C \left(\frac{\sin(2\pi f N T)}{2\pi f T} \right)^2 + D \left(\frac{\sin(2\pi f N T)}{2\pi f T} \right)^3 \tag{7}$$

where A, B, C, and D are unknown coefficients and are determined later. If we let $A, B \neq 0$, $C = D = 0$, then $H(f)$ will be a first-degree polynomial approximation, resulting in an $(\text{R-G-1})_{\text{opt}}$ filter; if $A, B, C \neq 0$, and only $D = 0$, then $H(f)$ will be a second-degree approximation, resulting in an $(\text{R-G-2})_{\text{opt}}$ filter, and so forth. Obviously, the higher the degree of the approximating polynomial, the lower the peak output sidelobe will be, and, of course, the more complicated the filter structure will be.

In order to utilize the LP algorithm to solve the unknown coefficients A, B, C, and D above, we transform $H(f)$ to its impulse response by an inverse Fourier transformation. We note that

$$\frac{\sin(2\pi f NT)}{2\pi f T} \Leftrightarrow \sum_{n=-(N-1)/2}^{(N-1)/2} \delta(t-2nT). \tag{8}$$

So the impulse response corresponding to $H(f)$ is of the form

$$\begin{aligned} h(t) \approx\ & A\delta(t) + B \sum_{n=-(N-1)/2}^{(N-1)/2} \delta(t-2nT) \\ & + C \sum_{n=-N+1}^{N-1} (N-|n|)\delta(t-2nT) \\ & + D\left(\sum_{n=-N+1}^{N-1} (N-|n|)\delta(t-2nT)\right) \\ & * \left(\sum_{m=-(N-1)/2}^{(N-1)/2} \delta(t-2mT)\right). \end{aligned} \tag{9}$$

We see that $h(t)$ is a delta function sequence and can be sampled every T seconds. The sampled sequence is a discrete set $\{h_i\}$ $(i = 0,\pm1,\pm2,\ldots)$. We let the input signal be the discrete autocorrelation sequence of the Barker code $\{R_i\}$. It is for an 11 bit Barker code:

$$R_i = \begin{cases} N & i = 0 \\ -1 & i = \pm2,\pm4,\ldots,\pm10 \\ 0 & \text{e. w.} \end{cases} \tag{10}$$

The discrete convolution between $\{h_i\}$ and $\{R_i\}$

$$\{y_i\} = \{R_i\}*\{h_i\}, \qquad i = 0,\pm1,\pm2,\ldots \tag{11}$$

is the output waveform of the sidelobe suppression filter. For the sake of convenience, we discard the fourth term in (9), i.e., let $D = 0$. By (11) we will get an LP model for an (R-G-2)$_{opt}$ filter. The output waveform of (11) is shown in Fig. 2, in which Fig. 2(a) shows the convolution between the first term in (9) and $R(t)$, Fig. 2(b) that between the second term in (9) and $R(t)$, and Fig. 2(c) that between the third term and $R(t)$.

From the sum of the three waveforms in Fig. 2 we obtain the following LP model, in which the objective function is

$$J = \max(11A + B + 41C) \tag{12a}$$

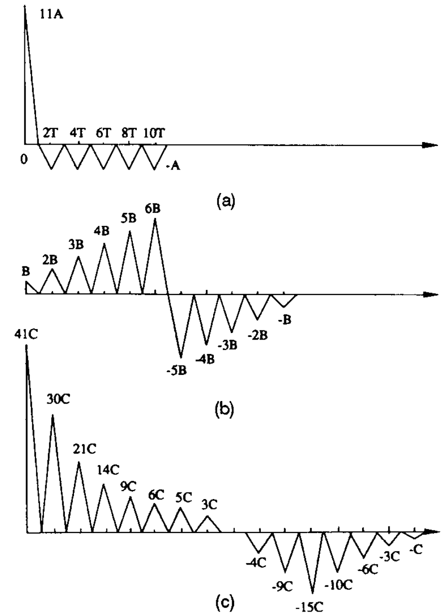

Fig. 2. (R-G-2)$_{opt}$ filter output waveform for 11 bit Barker code (only right half plotted because of symmetry).

subject to the constraints

$$\begin{aligned} |-A+2B+30C| &\le 15 \\ |-A+3B+21C| &\le 15 \\ |-A+4B+14C| &\le 15 \\ |-A+5B+9C| &\le 15 \\ |-A+6B+6C| &\le 15 \\ |-5B+5C| &\le 15 \\ |-4B+3C| &\le 15 \\ |-3B| &\le 15 \\ |-2B-4C| &\le 15 \\ |-B-9C| &\le 15 \\ |-15C| &\le 15, \qquad A,B,C \ge 0. \end{aligned} \tag{12b}$$

It can again be solved in a limited number of successive steps by an iterative monotonic simplex algorithm to get the coefficients A, B, and C. Inserting the resulting A, B, and C into (7), the optimized sidelobe reduction filter (R-G-2)$_{opt}$ is obtained. Fig. 3 shows its structural diagram, from which it can be seen that there are only two independent weights, B/A and C/A, and six delay elements. As compared with Fig. 1, it is much easier to implement.

III. PERFORMANCE ANALYSIS OF OPTIMIZED FILTERS

Using the algorithm above, we obtain the coefficients for (R-G-1)$_{opt}$, (R-G-2)$_{opt}$, and (R-G-3)$_{opt}$,

TABLE I
Optimized Coefficients of $(R\text{-}G)_{opt}$ Filters for 11 and 13 Bit Barker Codes

	$(R\text{-}G\text{-}1)_{opt}$		$(R\text{-}G\text{-}2)_{opt}$		$(R\text{-}G\text{-}3)_{opt}$	
	11	13	11	13	11	13
A	7.0	25.0	44.0	366.6	172.1	4953.0
B	1.0	−1.0	4.0	−27.4	8.75	−420.3
C	0.0	0.0	1.0	1.0	0.0	28.48
D	0.0	0.0	0.0	0.0	0.729	−0.88

TABLE II
Performances of $(R\text{-}G)_{opt}$ Filters for 11 and 13 Bit Barker Codes

	Peak sidelobe	FZPL*	Mean sidelobe	Mean square sidelobe	LSNR**
$(R\text{-}G\text{-}1)_{opt11}$	−24.50	−79.75	−35.07	−32.40	−0.34
$(R\text{-}G\text{-}1)_{opt13}$	−33.95	−76.49	−44.42	−42.13	−0.15
$(R\text{-}G\text{-}2)_{opt11}$	−31.05	−68.85	−39.76	−37.54	−0.59
$(R\text{-}G\text{-}2)_{opt13}$	−46.42	−77.86	−58.59	−55.42	−0.14
$(R\text{-}G\text{-}3)_{opt11}$	−35.75	−60.80	−45.63	−43.13	−1.06
$(R\text{-}G\text{-}3)_{opt13}$	−53.90	−61.32	−69.38	−65.35	−1.90

Note: *FZPL, the sidelobe Level on First Zero Point beside the mainlobe, which will affect radar ranging resolution.
**LSNR, the Loss in Signal-to-Noise Ratio.

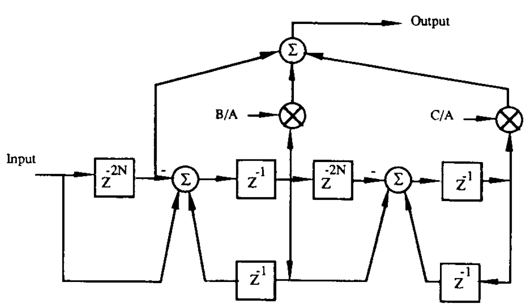

Fig. 3. Block diagram for $(R\text{-}G\text{-}2)_{opt}$ filter.

shown in Table I. It should be emphasized that what is important in Table I is the ratio between the coefficients, not their absolute values.

The improved performances of $(R\text{-}G)_{opt}$ are shown in Table II, in which we can see that, as compared with the results given in [3], the filters obtained here achieve an optimum performance in sidelobe reduction. For example, $(R\text{-}G\text{-}2)_{opt}$ filter for a 13 bit Barker code can give a peak sidelobe output of −46.4 dB, 2.62 dB better than the R-G-2 filter suggested in [3] with the same complexity of the structure. To compare the LP filters with the $(R\text{-}G)_{opt}$ filters, the performance of the LP filters versus the length of the filter is plotted in Fig. 4, from which we can conclude that with the length of the filter increased, the peak output sidelobe and mean square sidelobe will decrease, but the LSNR will increase. From Fig. 4, we also note that with a much simpler hardware

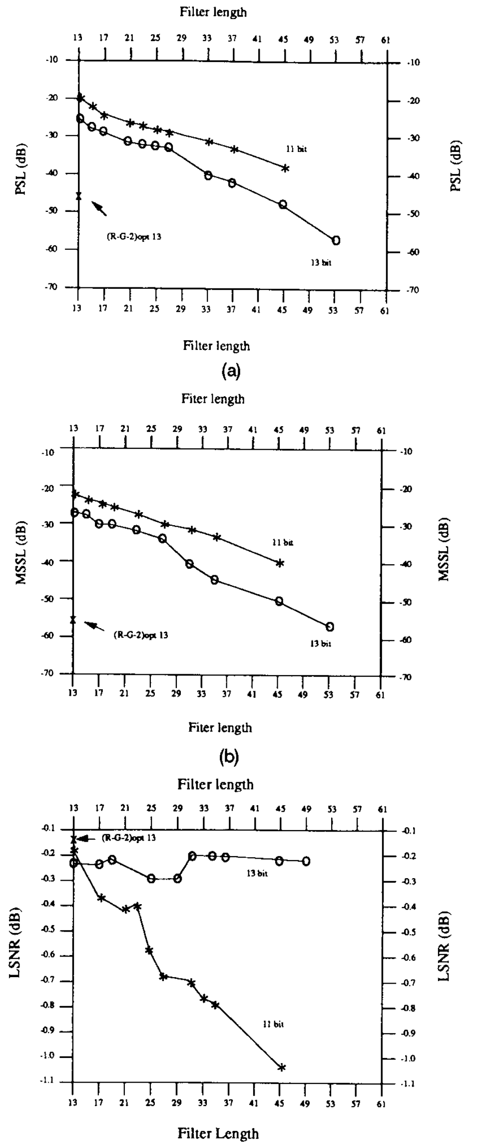

Fig. 4. LP filter performances versus length of filter. (a) Peak sidelobe versus length. (b) Mean square sidelobe versus length. (c) Loss in S/N ratio versus length.

(shown in Fig. 3) the $(R\text{-}G\text{-}2)_{opt}$ filter for a 13 bit Barker code is almost equal in performance to a 45 taps long LP filter, which has a far more complicated structure than the $(R\text{-}G\text{-}2)_{opt}$ filter.

IV. CONCLUSIONS

The analysis above has revealed the fact that the transfer function of a Barker code sidelobe reduction filter can be further optimized in the time domain with an LP algorithm, in which the benefits of the methods suggested in [3 and 5] are combined.

Obviously this new approach can readily be applied to sidelobe reduction filter design for other binary coding waveforms, such as truncated PN sequences, concatenated codes, etc., which often find their applications in radar systems and spread spectrum communication systems [1, 2].

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Additional Results on "Reducing Geometric Dilution of Precision Using Ridge Regression"

Reference [1] presented preliminary results on the feasibility of using ridge regression to reduce the effects of geometric dilution of precision (GDOP) error inflation in position-fix navigation systems. Recent results indicate that the ridge technique will *not* reduce bias inflation due to the effects of GDOP in applications where bias-like measurement errors persist for time periods which greatly exceed the response time of the aircraft's guidance loop. This conclusion will preclude the use of ridge regression on navigation systems whose dominate error sources are bias like. This applies in particular to the Global Positioning System (GPS) selective availability error source. All the simulation results, however, given in [1] are valid for the conditions defined.

Although ridge regression has not yielded a satisfactory solution to the general GDOP problem it has illuminated the role that multicollinearity plays in navigation signal processors such as the Kalman filter. After a background discussion, four topics are discussed: bias inflation, initial position guess errors, ridge parameter selection methodology and the recursive ridge filter.

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BACKGROUND

Hoerl [2] in 1970 developed ridge regression to reduce variance inflation when the predictor matrix is nearly collinear. This author [1] has proposed using his technique to combat the effects of GDOP in navigation systems and also extended Hoerl's results to include bias inflation. The purpose of this correspondence is to document recent results for the case when the model has unknown bias measurement errors.

Following the notation in [1] let the measurement model have two error components, a bias term and a random term given by

$$\mathbf{e} = \mathbf{e}_0 + \Delta\mathbf{B} \tag{1}$$

where $E[\mathbf{e}] = \Delta\mathbf{B}$, the unknown $n \times 1$ bias vector. Also, $E[\mathbf{e}_0] = 0$ and $\text{cov}[\mathbf{e}] = E[\mathbf{e}_0\mathbf{e}_0^{\mathrm{T}}] = \sigma^2 I$ where I is the $n \times n$ identity matrix. The linear model is

$$\mathbf{Y} = H\beta + \mathbf{e}_0 + \Delta\mathbf{B}. \tag{2}$$

The least squares (LS) estimate of (1) is

$$\hat{\beta}_{\mathrm{OLS}} = \beta + (H^{\mathrm{T}}H)^{-1}H^{\mathrm{T}}\Delta\beta + (H^{\mathrm{T}}H)^{-1}H^{\mathrm{T}}\mathbf{e}_0. \tag{3}$$

The expectation of $\hat{\beta}_{\mathrm{OLS}}$ is

$$E[\hat{\beta}_{\mathrm{OLS}}] = \beta + (H^{\mathrm{T}}H)^{-1}H^{\mathrm{T}}\Delta\mathbf{B}$$

while the bias of $\hat{\beta}_{\mathrm{OLS}}$ is

$$\text{bias}[\hat{\beta}_{\mathrm{OLS}}] = (H^{\mathrm{T}}H)^{-1}H^{\mathrm{T}}\Delta\mathbf{B} \tag{4}$$

and its variance is

$$\text{var}[\hat{\beta}_{\mathrm{OLS}}] = \sigma^2(H^{\mathrm{T}}H)^{-1}. \tag{5}$$

Thus, geometric dilution of precision (GDOP) as exhibited by $(H^{\mathrm{T}}H)^{-1}$ can amplify the bias given by (4) when the angle γ between the lines of position is small, i.e., $\gamma \simeq 1°$ as given in [1, Fig. 2].

The ridge estimate of (2) is

$$\hat{\beta}_R = G_\kappa H^{\mathrm{T}}H\beta + G_\kappa H^{\mathrm{T}}\Delta\mathbf{B} + G_\kappa H^{\mathrm{T}}\mathbf{e}_0 \tag{6}$$

where $G_\kappa = (P_R + H^{\mathrm{T}}H)^{-1}$.