Multiplicative Mismatched Filters for Optimum Range Sidelobe Suppression in Barker Code Reception

Adly Fam and Indranil Sarkar

Department of Electrical Engineering,
332, Bonner Hall,
The State University of New York at Buffalo,
Buffalo, NY-14260, USA.
{afam,isarkar}@eng.buffalo.edu

Abstract. Very efficient sidelobe suppression of Barker codes of length 13 is achieved through the use of a novel mismatched filter. The mismatched filter is comprised of a conventional matched filter cascaded with a computationally efficient filter based on multiplicative expansion. Several constant parameters are introduced in the terms of the expansion and they are optimized to improve the performance of the filter. The filter is shown to achieve a mainlobe to peak sidelobe ratio of almost 61.9 dB. This is achieved at the cost of some deterioration in the signal to noise ratio at the filter output. The loss in signal to noise ratio (SNR) is shown to be only 0.2 dB.
It is suggested that by applying the technique to compound Barker codes, the mainlobe to peak sidelobe ratio could be maintained and the signal to noise ratio performance could be improved.

Confidential Document

Provisional patent application has been filed for this work. Patent pending. The entire document is to be treated as confidential. The contact details about licensing and technical questions and queries are provided in table 1:

Table 1. Contact information

For licensing information, contact:	For technical information, contact:
Rupal Desai, MS Engg, MBA	Adly T. Fam, Ph.D
Commercialization Manager	Professor
Intellectual Property Division	Department of Electrical Engineering
Science, Technology Transfer and Economic Outreach	332, Bonner Hall
UB Technology Incubator, Suite 111	The State University of New York at Buffalo
1576 Sweet Home Road	Buffalo, NY - 14260-1920
Amherst, NY 14228	
(716) 645-5500 ext. 15	Email: afam@eng.buffalo.edu
FAX: (716) 645 3436	(716) 645-3115 ext. 2128
www.stor.buffalo.edu	FAX: (716) 645-3656

References Cited

U.S. Patent Documents

4,095,225, 6/1978 Erikmats
4,156,876 5/1979 Debuisser
4,313,170 1/1982 Lewis, et al.
4,379,295 4/1983 Lewis, et al.
4,384,291 5/1983 Lewis, et al.
4,566,010 1/1986 Collins
5,070,337, 12/1991 Chen et al.
5,760,732 6,1998 Marmarelis, et al.
5,786,788, 7/1998 Schober et al.

Other References

1. A. W. Rihaczek and R. M. Golden, *Range Sidelobe Suppression for Barker Codes*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-7, No. 6, Nov 1971, pp 1087-1092.

2. Chen Xiao Hua and Juhani Oskman, *A New Algorithm to Optimize Barker Code Sidelobe and Suppression Filter*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-26, No. 4, July 1990, pp 673-677.

3. M. H. Ackroyd and F. Ghani, *Optimum Mismatched Filters for Sidelobe Suppression*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-9, No. 2, March 1973, pp 214-218.

4. S. Zoraster, *Minimum Peak Range Sidelobe Filters for Binary Phase Coded Waveforms*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-16, No. 1, Jan 1980, pp 112-115.

5. M. H. Ackroyd, *Economical Filters for Range Sidelobe Reduction with Combined Codes*, The Radio and Electronic Engineer, Vol 52, No. 6, June 1982, pp. 309-310.

6. Hermann Rohling and Wilfried Plagge, *Mismatched-Filter Design for Periodical Binary Phased Signals*, IEEE Transactions on Aerospace and Electronic Systems, Vol 25, No. 6, November 1989, pp. 890-897.

Field of the invention

The invention relates to mismatched filters for sidelobe suppression in radar systems and wireless communication systems using Barker codes.

Background of the invention

Bi-phase codes are widely used for pulse compression in radar systems. Barker codes are especially preferred in these applications because it achieves the best possible ratio between the mainlobe and sidelobe peaks. However, the largest known is 13. It has been proved that there is no barker code of odd length greater than 13. On the other hand, even though there is no rigorous proof about the non-existence of Barker codes of even length greater than 4, published research based on simulations show that they do not exist for lengths up to a few thousands.

When using the Barker codes for pulse compression, the sidelobes need to be suppressed as much as possible. This results in improved resolution in radar and wireless systems. The code of length 13 achieves a mainlobe to peak sidelobe ratio of only 22.28 dB which is much less than the practical requirement of most practical radar systems. Typically, the ratio is required to be at least 30 dB in most radar applications.A peak sidelobe from a strong target echo can sometimes weaken or even completely mask the mainlobe of a smaller target echo. This motivates the need for sidelobe suppression filters or mismatched filters at the receiver in order to improve the performance of the radar system. Mismatched filters have been a subject of research in the the literature for a long time. These filters achieve improved mainlobe to sidelobe ratio at the cost of some deterioration in the signal to noise ratio at the filter output.

Prior research in the area of sidelobe suppression filters can be broadly classified into two general methods. In the first method, a matched filter is first used to perform the pulse compression correlation. A mismatched filter is then used in cascade with the matched filter to suppress the sidelobes. Rihaczek and Golden introduced the R-G filters which follow this methodology and have been a subject of active research ever since their introduction in 1971. The R-G filters are presented in the publication: A. W. Rihaczek and R. M. Golden, *Range Sidelobe Suppression for Barker Codes*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-7, No. 6, Nov 1971, pp 1087-1092. The R-G filters were further improved by Hua and Oskman who proposed a new algorithm to optimize the filter coefficients of the R-G filters. This work is presented in the publication: Chen Xiao Hua and Juhani Oskman, *A New Algorithm to Optimize Barker Code Sidelobe and Suppression Filter*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-26, No. 4, July 1990, pp 673-677. It is also described in the U.S. Pat. No. 5,070,337 by the same authors.

The second method involves directly designing the mismatched filter for the Barker code without first passing it through a mismatched filter. These filters have been designed using the Least Mean Square (LMS) or Linear Programming (LP) algorithms. The LMS approach can be found in the publication: M. H. Ackroyd and F. Ghani, *Optimum Mismatched Filters for Sidelobe Suppression*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-9, No. 2, March 1973, pp 214-218. In the LP approach, linear programming methods are used to optimize the coefficients of the filter being designed to minimize the peak range sidelobe of the Barker coded waveform. The LP filters were found to be more effective in peak sidelobe suppression. This technique was introduced in the publication: S. Zoraster, *Minimum Peak Range Sidelobe Filters for Binary Phase Coded Waveforms*, IEEE Transactions on Aerospace and Electronic Systems, Vol AES-16, No. 1, Jan 1980, pp 112-115.

In this work, we design the mismatched filter in cascade with a matched filter. The mismatched filter is basically an implementation of an inverse filter to suppress the sidelobes produced at the matched filter output. The inverse filter should not be directly implemented due to its instability and unacceptable deterioration in the SNR performance. Rihaczek and Golden approximated the inverse filter by expanding the transfer function as a Fourier series and then taking finite terms from the expansion. The proposed filter is fundamentally different from this approach because we use a multiplicative expansion to approximate the transfer function of the inverse filter.

Claims

We claim:

1. A novel mismatched filter to be used in radar equipments and applications. The filter is used in cascade with a matched filter and used to suppress the sidelobes produced at the matched filter output. The mismatched filter is based on a multiplicative expansion and approximation of the transfer function of the inverse filter $\frac{1}{R(z)}$ where $R(z)$ denotes the matched filter output when the incoming Barker code is matched with itself. The method and design of the filter is described in details in the next section.
2. The method and filtering technique as described in claim 1, when applied to Barker code of any other length.
3. The method and filtering technique as described in claim 1, when applied to compound Barker codes.
4. The method and filtering technique as described in claim 1, when applied to sonar applications. This could include applications such as submarine detection using sonar beams and imaging of the ocean floor.
5. The method and filtering technique as described in claim 1, when applied to other applications such as wireless communication, biomedical applications and geophysical exploration.

Detailed Description of the Invention

The proposed filter is designed as a sidelobe suppression filter placed in cascade with a matched filter. The proposed filter aims to suppress the sidelobes produced at the output of the matched filter. Let us consider the z-transform of the incoming Barker code to be $X(z)$. Hence the matched filter transfer function is $X(z^{-1})$ and the matched filter output is given by:

$$R(z) = X(z)X(z^{-1}) \tag{1}$$

$R(z)$ is composed of a mainlobe of height N and sidelobes of height 1. In general, we denote the sidelobes as $\sum S_n(z^n + z^{-n})$. Hence $R(z)$ can also be denoted by:

$$R(z) = N + \sum S_n(z^n + z^{-n}) \tag{2}$$

$$= N[1 + \frac{1}{N} \sum S_n(z^n + z^{-n})] \tag{3}$$

Evidently, the ideal mismatched filter should be the inverse of $R(z)$. However, such a filter would be unstable and cannot be implemented in practice. We propose a multiplicative expansion of this inverse filter as shown. Since the mainlobe to peak sidelobe ratio is what matters to us, we ignore N in the following derivation to allow for any desired scale factor.

$$H(z) = \frac{1}{1 + \frac{1}{N}\sum S_n(z^n + z^{-n})} \tag{4}$$

$$= \frac{1 - \frac{1}{N}\sum S_n(z^n + z^{-n})}{1 - \frac{1}{N^2}[\sum S_n(z^n + z^{-n})]^2} \tag{5}$$

$$= \frac{[1 - \frac{1}{N}\sum S_n(z^n + z^{-n})][1 + \frac{1}{N^2}[\sum S_n(z^n + z^{-n})]^2]}{1 - \frac{1}{N^4}[\sum S_n(z^n + z^{-n})]^4} \tag{6}$$

$$= \frac{[1 - \frac{1}{N}\sum S_n(z^n + z^{-n})][1 + \frac{1}{N^2}\{\sum S_n(z^n + z^{-n})\}^2][1 + \frac{1}{N^4}\{\sum S_n(z^n + z^{-n})\}^4]}{1 - \frac{1}{N^8}\{\sum S_n(z^n + z^{-n})\}^8} \tag{7}$$

The filter transfer function can be approximated to higher order terms if desired. For our present purposes, we will use only three terms in the multiplicative expansion. Also since

$1 - \frac{1}{N^8}\{\sum S_n(z^n + z^{-n})\}^8 \approx 1$

The filter transfer function is redefined as:

$$H(z) = \left[1 - \frac{1}{N}\sum S_n(z^n + z^{-n})\right]\left[1 + \frac{1}{N^2}\left\{\sum S_n(z^n + z^{-n})\right\}^2\right]\left[1 + \frac{1}{N^4}\left\{\sum S_n(z^n + z^{-n})\right\}^4\right] \tag{8}$$

In order to improve the performance of the filter, we introduce some parameters in this transfer function. The parameters either replace the constant terms in the derived equation or work in conjunction with them. The transfer function is parameterized in a way such that these parameters add flexibility in optimizing the performance of the filter. The parameterized transfer function of the proposed filter is as given below:

$$H(z) = \left[C_1 - \frac{1}{N + A_1}\left\{\sum S_n(z^n + z^{-n}) - A_1\right]\left[C_2 + \frac{1}{(N + A_2)^2}\left\{\sum S_n(z^n + z^{-n}) - A_2\right\}^2\right] \left[C_3 + \frac{1}{(N + A_3)^4}\left\{\sum S_n(z^n + z^{-n}) - A_3\right\}^4\right] \tag{9}$$

For notation purposes, we define the three terms separately as:

$$T_1(z) = \left[C_1 - \frac{1}{N + A_1}\left\{\sum S_n(z^n + z^{-n}) - A_1\right] \tag{10}$$

$$T_2(z) = \left[C_2 + \frac{1}{(N + A_2)^2}\left\{\sum S_n(z^n + z^{-n}) - A_2\right\}^2\right] \tag{11}$$

$$T_3(z) = \left[C_3 + \frac{1}{(N + A_3)^4}\left\{\sum S_n(z^n + z^{-n}) - A_3\right\}^4\right] \tag{12}$$

This filter can be implemented by connecting the three filters $T_1(z)$, $T_2(z)$ and $T_3(z)$ in cascade. To facilitate even more flexibility, we have incorporated three multipliers in our implementation of the filters. This allows us to optimize these multipliers as well in order to improve the performance of the filter. The filter structure is shown in figure 1. MF denotes the matched filter.

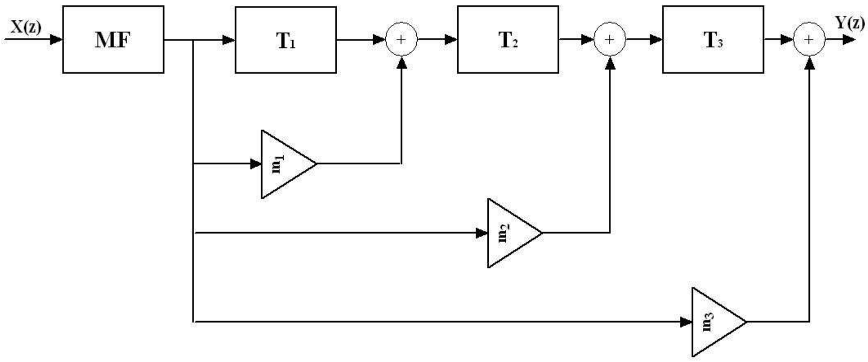

Fig. 1. Structure of the proposed filter

To find the optimum filter for the given structure, we optimize the parameters jointly. The parameters optimized are A_1, A_2, A_3, C_1, C_2, C_3, m_1, m_2 and m_3. To reduce the complexity of the optimization process, we employ a multi-pass scanning technique. In the first pass, we use large step sizes for all the parameters to determine the approximate region of optimum performance. In the subsequent passes, finer step sizes are used to find the joint optimum values of all the parameters. The parameters are optimized to achieve the maximum sidelobe suppression. The sidelobe suppression is measured as the ratio of the magnitude of the mainlobe to the sidelobe (MSR). The optimum values of the parameters and the corresponding MSR obtained is as follows:
$MSR = 1241.9391$ (61.88 dB)
$A_1 = 0.0216$
$A_2 = 0.003$
$A_3 = 0.046$
$C_1 = 1.229$
$C_2 = 1.565$
$C_3 = 2.037$
$m_1 = 0.094$
$m_2 = 0.008$
$m_3 = -1.106$

We now show the performance of the filter for Barker code of length 13. The matched filter output is shown in figure 2. As expected, the mainlobe is seen to be of height 13

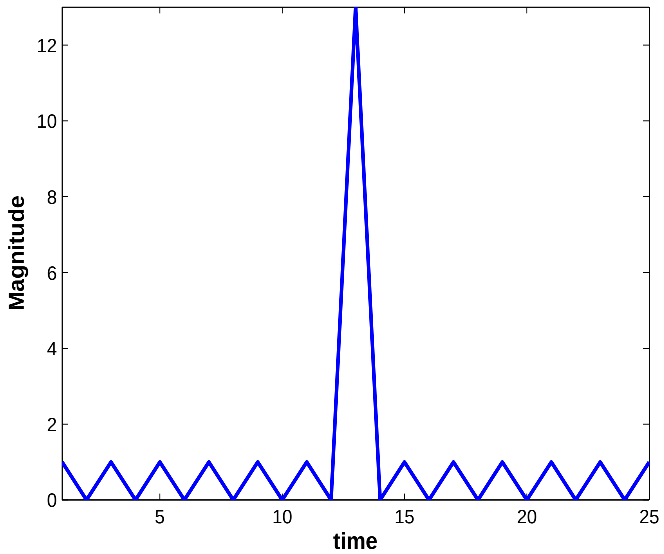

Fig. 2. Output of the matched filter

while the sidelobes are of height 1 and 0. The output of the matched filter is fed to the proposed filter to suppress the sidelobes. The output of the proposed filter is shown in the normal and logarithmic scales in figures 3 and 4, respectively. The sidelobes are found to be suppressed and the mainlobe is enhanced. Table 2 gives the comparative performance of the different sidelobe suppression filters reported in the literature. R-G denotes the original filters reported by Rihaczek and Golden. $(R-G-1)_{opt13}$, $(R-G-2)_{opt13}$ and $(R-G-3)_{opt13}$ denote the improved R-G filters as reported by Hua and Oskman.

Table 2. Comparative performance of different filters

	Peak Sidelobe	LSNR
R-G	-43.8 dB	-
$(R-G-1)_{opt13}$	-33.95 dB	0.15 dB
$(R-G-2)_{opt13}$	-46.42 dB	0.14 dB
$(R-G-3)_{opt13}$	-53.90 dB	1.90 dB
proposed filter	-61.88 dB	0.2 dB

The large amount of sidelobe suppression comes at the cost of a deterioration in the signal to noise ratio (SNR) performance as compared to the matched filter output. This is evident from the results shown in table 2. Hua and Oskman achieved a sidelobe suppression of -53.90 dB with a 1.9 dB deterioration in the SNR performance. The proposed filter not only achieves a higher sidelobe suppression of almost -61.9 dB, but also the loss in SNR is found to be only 0.2 dB. In the current age of ultra-fast processors and ultra large scale

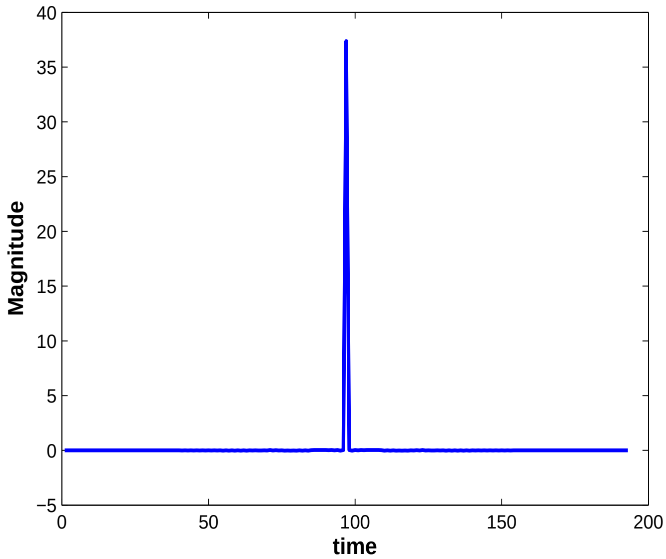

Fig. 3. Output of the proposed filter

integration in hardware chips, filter length is a factor of less concern than the degradation of the SNR performance due to the filter. The degradation in SNR performance is excellent for the proposed filter. This clearly indicates that even higher order terms can be incorporated in the multiplicative expansion of the transfer function. This will reduce the peak sidelobe level even further without degrading the SNR performance significantly.

Even though we have optimized the filter for only 3 stages, more stages can be easily added to achieve better sidelobe suppression. Apart from further deteriorating the SNR performance, the incorporation of more stages would increase the order of the filter and hence the hardware requirement. In table 3, we show these variations as functions of the number of stages. The lengths of the mismatched filter and the whole filter including the matched filter are also plotted in figure 5 as a function of the number of stages.

Let us denote the length of the ith stage as L_i. For notational purposes, the length of the matched filter is denoted as L_0. Evidently L_0 will be the length of the Barker code that is being used. The length of each stage of the filter is approximately double that of the preceding section and is given as:

$$L_i = 2L_i - 1 \tag{13}$$

In terms of the matched filter length L_0, the length of the ith stage of the filter can be shown to be:

$$L_i = 2^i(L_0 - 1) + 1 \tag{14}$$

Next, we enumerate the number of adders and multipliers required for the proposed filter. The matched filter requires 12 adders irrespective of the number of stages in the mismatched filter. As evident from (9), the number of adders for the first stage is $(1 + 12)$. Similarly the number of adders from the second and third stages are $(1 + 12 \times 2)$ and $(1 + 12 \times 4)$, respectively. It can easily be observed that the number of adders from the ith stage is given

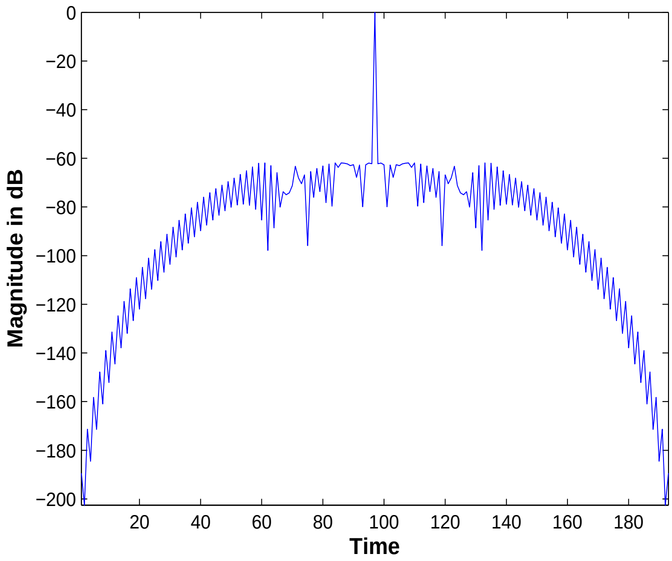

Fig. 4. Output of the proposed filter in logarithmic scale

by:

$$a(i) = 1 + 12 \times 2^{i-1} \tag{15}$$

Therefore, the total number of adders required for n stages of the filter is given by:

$$\begin{aligned} A(n) &= 12 + n + \sum_{i=1}^{n} 12 \times 2^{i-1} && (16)\\ &= 12 + n + 12(2^n - 1) && (17)\\ &= n + 12 \times 2^n && (18) \end{aligned}$$

This includes the 12 adders from the matched filter placed before the mismatched filter. The variation in the required number of adders is shown in figure 6 as a function of the number of stages.

Similar calculations can be done to enumerate the number of multipliers required for implementing the filter. Proceeding similarly as in the case of adders, it can be shown from (9) that the number of multipliers required for the ith stage is given by:

$$m(i) = 2 + 2^{i-1} \tag{19}$$

Therefore, the total number of multipliers required for a filter with n stages is given by:

$$M(n) = 2n + 2^n - 1 \tag{20}$$

The variation in the number of multipliers as a function of the number of stages is also shown in figure 6.

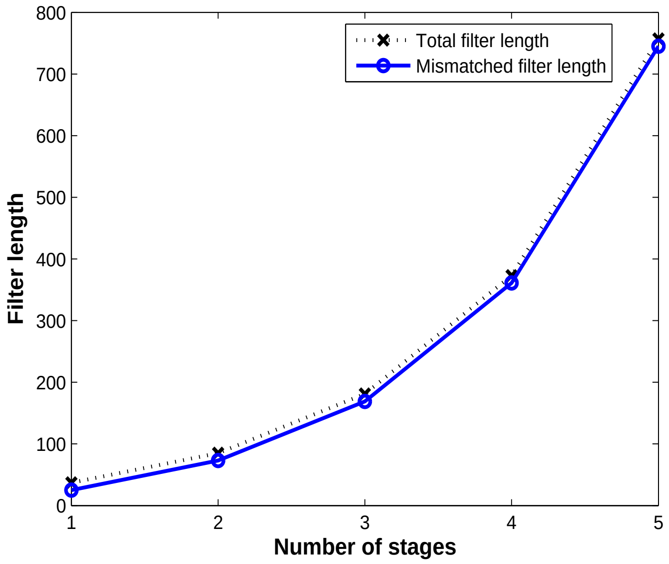

Fig. 5. Filter lengths as functions of the number of stages

Table 3. Filter lengths and hardware requirements as a function of number of stages

Stages	Mismatched filter length	Total filter length including the MF	# of adds per output	# of mult. per output
1	25	37	25	3
2	73	85	50	7
3	169	181	99	13
4	361	373	196	23
5	745	757	389	41

The proposed filter with only 3 multiplicative expansion terms, cascaded with the matched filter, results in a total filter length of 181. However, the number of non zero coefficients is much smaller resulting in 99 additions and 13 multiplications per output.

In this work, we have achieved high sidelobe suppression of length 13 Barker code through the use of a novel mismatched filter based on multiplicative expansion. The loss in SNR to achieve the high sidelobe suppression has been shown to be only 0.2 dB. The idea behind this filter is immediately extensible to compound Barker codes. As in the case of a regular matched filter, the use of compound Barker codes is expected to improve the SNR performance while preserving the mainlobe to peak sidelobe ratio obtained for the length 13 Barker code.

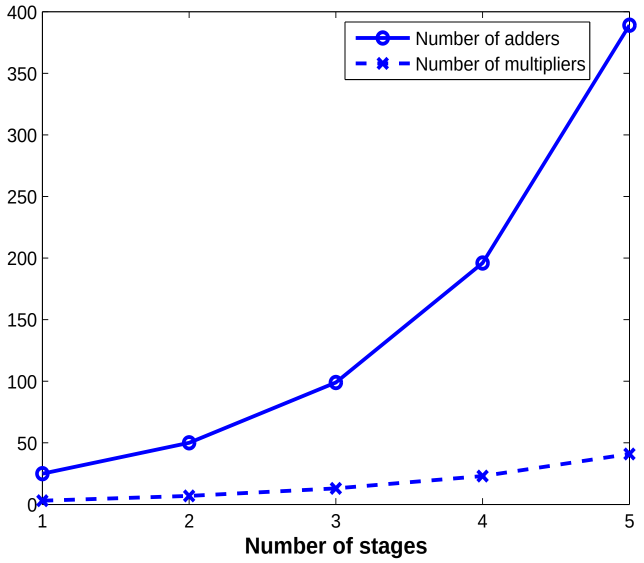

Fig. 6. Number of adders and multipliers required for different number of stages