PROJECT for EE 483

COMMUNICATIONS SYSTEMS I - Fall 2004

Computer Assignment 7: Random Processes and Applications

Exercise 1: Cross-correlation and Auto-correlation

In MATLAB you can calculate the cross-correlation between two random signals using the function `ccorr` given in the appendix. As an example, to plot the cross-correlation function $R_{xy}(\tau)$ of the random signals $x(t)$, $y(t)$ where both $x(t)$ and $y(t)$ are white Gaussian random processes with zero mean and unit variance you can type the following:

```
t=-10:0.01:10;
x=randn(1,length(t));
y=randn(1,length(t));
[cc,tau]=ccorr(x,y,t);
plot(tau,cc);
```

Similarly, you can plot the auto-correlation of `x` as follows:

```
[ac,tau]=ccorr(x,x,t);
plot(tau,ac);
```

Assume that $y(t)$ is the output of a linear filter with impulse response

$$h(t) = |sinc(t-2)| \tag{1}$$

when the input is the white Gaussian random process $x(t)$ with zero mean and unit variance.

Create an M-file to:

(a) Plot the impulse response $h(t)$ where t ranges from -10 to 10, using 0.01 increments.

(b) Plot the processes $x(t)$ and $y(t)$ where t ranges from -10 to 10, using 0.01 increments. To calculate $y(t)$ you may use the function **linfilt** included in the Appendix of Project 3.

(c) Plot the cross-correlation function $R_{xy}(\tau)$. (Note that to calculate $R_{xy}(\tau)$ you must use the same support for both $x(t)$ and $y(t)$)

(d) Plot the auto-correlations $R_{xx}(\tau)$, $R_{yy}(\tau)$ of the processes $x(t)$ and $y(t)$, respectively.

Exercise 2: *Distance estimation in radar systems*

A typical radar system is depicted in Fig. 1.

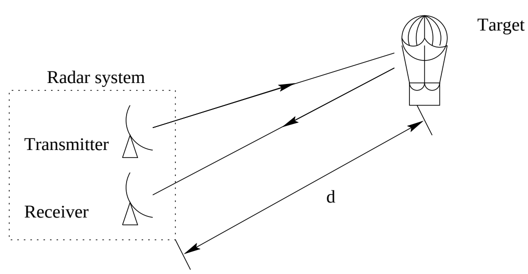

Fig. 1

It consists of a transmitter and a receiver. The transmitter transmits a pulse $p(t)$ which is reflected by the target back to the receiver. If d is the distance from the target to the radar system the pulse will reach the receiver after $T = 2d/c$ secs, where c is the propagation speed of the pulse. Therefore, the received signal $r(t)$ is

$$r(t) = p(t - T). \tag{2}$$

In reality, the received signal is a *noisy* version of $p(t-T)$, that is

$$r(t) = p(t-T) + n(t), \tag{3}$$

where $n(t)$ is white gaussian noise process (for simplicity we assume that the process has zero mean and unit variance). Since,

$$d = Tc/2, \tag{4}$$

we can calculate the target distance d if we can measure the time delay between the received signal $r(t)$ and the transmitted pulse $p(t)$. This can be done by means of the structure shown in Fig. 2.

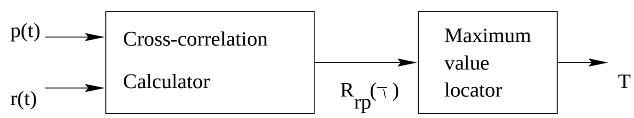

Fig. 2

First, the cross-correlation

$$R_{rp}(\tau) = E\{r(t)p(t+\tau)\} \tag{5}$$

between the received signal and the transmitted pulse is calculated. The cross-correlation function will peak at the actual value of the time delay, so T is the value of τ that maximizes $R_{rp}(\tau)$.
Assume that the transmitted pulse $p(t)$ is given by

$$p(t) = \begin{cases} 5\sin(2\pi 10t + \phi), & 0 \le t \le 1 \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

where ϕ is the random phase of $p(t)$. ϕ assumes a uniform distribution in $[-\pi/2, \pi/2]$. The propagation speed of the pulse is assumed to be $c = 1,000$m/sec and the target is located $d = 1,500$m away from the radar system. Create an M-file to:

(a) Fix ϕ to a particular value and plot the pulse $p(t)$ where t ranges from -5sec to 5sec, using 0.01 increments.

(b) Fix ϕ to a particular value and plot the received signal $r(t)$ where t ranges from -5sec to 5sec, using 0.01 increments.

(c) Plot the cross-correlation function $R_{rp}(\tau)$ between $p(t)$ and $r(t)$. Verify that $R_{rp}(\tau)$ peaks at $\tau = 2d/c$.

NOTE The expectation in (5) has to be carried out with respect to all random quantities, that is the noise $n(t)$ and the random phase ϕ. To do that we use the conditional cross-correlation function $R_{rp|\phi}(\tau)$ (i.e. the cross-correlation of $r(t)$ and $p(t)$ for a fixed value of ϕ). Then $R_{rp}(\tau) = E_{\phi}\{R_{rp|\phi}(\tau)\}$. Thus, we may evaluate the cross-correlation between $r(t)$ and $p(t)$ by averaging $R_{rp|\phi}(\tau)$ over a large number of independently drawn values of the phase from a uniform distribution. For this experiment you may average over 100 independent values of ϕ.

Exercise 3: *Direction finding*

If a radar system employs a *pair* of receivers then in addition to distance estimation we can perform direction estimation.

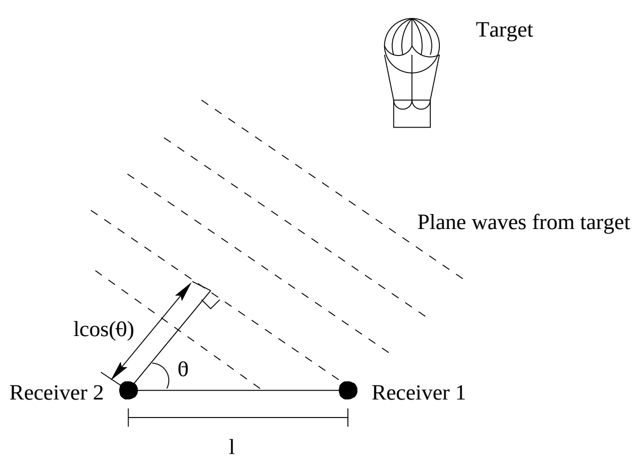

Fig. 3

Referring to Fig. 3, if the target is located at an angle θ with respect to the second receiver then the reflected pulse from the target reaches receiver 2 T secs after it reached receiver 1. So, if l is the distance between the two receivers, we have

$$T = l\cos(\theta)/c. \tag{7}$$

If we denote the signal received by the first receiver $r_1(t)$ and the signal received by the second $r_2(t)$ then the following relation holds

$$r_2(t) = r_1(t-T). \tag{8}$$

However, in real systems the signal $r_2(t)$ is a noisy version of $r_1(t-T)$, i.e.

$$r_2(t) = r_1(t-T) + n(t), \tag{9}$$

where $n(t)$ is white gaussian noise process. Again, we assume for simplicity that the process has zero mean and unit variance. Since

$$\theta = \cos^{-1}\left(\frac{Tc}{l}\right) \tag{10}$$

in order to calculate the target direction θ it suffices to measure the time delay T between the two received signals. This is done using the same structure as in Fig. 2 with $r_1(t)$, $r_2(t)$ in place of $r(t)$ and $p(t)$, respectively. Assume that the signal $r_1(t)$ is given by

$$r_1(t) = \begin{cases} 5\sin(2\pi 10(t-1)+\phi), & 0 \leq t \leq 1 \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

where ϕ is the random phase of $p(t)$ assumed to be uniformly distributed in $[-\pi/2, \pi/2]$. For this case and for illustrative purposes assume that the propagation speed of the pulse is $c = 100$m/sec and the target is located at a 60^o angle with respect to receiver 2, which is $l = 500$m away from receiver 1.

Create an M-file to:

(a) Fix ϕ at a particular value and plot the signal $r_1(t)$ where t ranges from -5sec to 5sec, using 0.01 increments.

(b) Fix ϕ at a particular value and plot the signal $r_2(t)$ where t ranges from -5sec to 5sec, using 0.01 increments.

(c) Plot the cross-correlation function $R_{1,2}(\tau)$ between $r_1(t)$ and $r_2(t)$.

Verify that $R_{1,2}(\tau)$ peaks at $\tau = l\cos(\theta)/c$. The cross-correlation should be evaluated as in Exercise 2.

Appendix

In this project you must calculate the cross-correlation of two random signals. You can do that by using the following function:

```
function [cc,tau]=ccorr(x,y,t)



Dt=t(2)-t(1);

if length(x)~=length(y)

   error('Vectors must have the same length\n');

end

cc=xcorr(x,y);

tau=Dt*(-length(x)+1:length(x)-1);
```

where `x`, `y` are the signals to be cross-correlated, `t` is the time axis of these signals. The output `cc` contains the cross-correlation of the two signals, while `tau` is the time axis of `cc`.