For SSB-AM modulation we need to find the Hilbert transform of the message signal. The Hilbert transform of the message signal can be computed using the Hilbert transform m-file of MATLAB, that is hilbert.m. It should be noted, however, that this function returns a complex sequence whose real part is the original signal and its imaginary part is the desired Hilbert transform. Therefore, the Hilbert transform of a sequence m is obtained by using the command imag(hilbert(m)).

**Exercise 1: AM**

Let the information bearing signal \( m(t) \) be given by

\[
m(t) = \text{sinc}(2t).
\]

Let also the carrier \( c(t) \) be given by

\[
c(t) = A_c \cos(2\pi f_c t),
\]

where the carrier frequency is \( f_c = 50 \) and the carrier amplitude is \( A_c = 1 \).

(a) Plot the signal \( m(t) \), where \( t \) ranges from -4 to 4, using increments of 0.01.

(b) Find the maximum value \( k_{a,\text{max}} \) of the amplitude sensitivity \( k_a \) needed to be used in AM modulation that ensures no phase reversals of the modulated carrier. Plot the modulated carrier for \( k_a = \frac{k_{a,\text{max}}}{2}, k_a = k_{a,\text{max}}, k_a = 2k_{a,\text{max}} \).
Exercise 2: DSB and SSB

Let the information bearing signal \( m(t) \) be given by (1). In the case of DSB and SSB modulation the modulated carrier \( v(t) \) is:

\[
\begin{align*}
    v(t) &= A_c m(t) \cos(2\pi f_c t) \quad \text{DSB modulation.} \\
    v(t) &= \frac{A_c}{2} m(t) \cos(2\pi f_c t) - \frac{A_c}{2} \dot{m}(t) \sin(2\pi f_c t) \quad \text{SSB (Upper sideband transmitted).} \\
    v(t) &= \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \dot{m}(t) \sin(2\pi f_c t) \quad \text{SSB (Lower sideband transmitted).}
\end{align*}
\]

Let the carrier frequency be \( f_c = 50 \) and the carrier amplitude \( A_c = 1 \).

(a) Plot the signal \( m(t) \), where \( t \) ranges from -4 to 4, using increments of 0.01.

(b) 1. Plot the DSB modulated carrier.
   2. Plot the USSB (Upper Sideband) modulated carrier.
   3. Plot the LSSB (Lower Sideband) modulated carrier.

(c) For each one of the above methods demodulate the signal using the coherent detector (multiply the modulated signal with \( \cos(2\pi f_c t) \) and pass the product through a low pass filter to recover the original signal \( m(t) \)). Plot the output of the detector. Assume that the phase difference \( \phi \) is 0. For the lowpass filter use a cutoff frequency equal to 50Hz. Hints for lowpass filtering are given in the Appendix at the end of this assignment.

(d) Suppose that the local oscillator is not completely synchronized with the carrier frequency. Let \( \Delta f = 2 \text{Hz} \) be an error in the carrier frequency of the detector measured with respect to the carrier frequency of the incoming signal. Repeat step 3 by taking \( \Delta f \) into account.
Appendix

In Exercise 2 you must lowpass a signal. You can do that by using the following function:

\[
\text{function } [s\text{lp}, t\text{slp}] = \text{lowpass}(s, t\text{s}, f\text{cut})
\]

\[
B = f\text{cut};
\]
\[
h = 2 * B * \text{sinc}(2 * B * t\text{s});
\]
\[
s\text{lp} = \text{conv}(h, s);
\]
\[
T = t\text{s}(2) - t\text{s}(1);
\]
\[
N = \text{length}(t\text{s});
\]
\[
t\text{slp} = -((N - 1) / 2) * T : T : ((N - 1) / 2) * T;
\]
\[
s\text{lp} = s\text{lp}([((N - 1) / 2 : (N - 1) / 2 + N - 1])];
\]

where \( s \) is the signal to be lowpass filtered, \( t\text{s} \) is the time axis of the signal and \( f\text{cut} \) is the cutoff frequency of the filter. The output \( s\text{lp} \) is the lowpass filtered signal, while \( t\text{slp} \) is the time axis of \( s\text{lp} \).

Note

Your report should include all plots and M-files you are asked to create in Exercises 1 and 2.