PROJECT for EE 483

COMMUNICATIONS SYSTEMS I - Fall 2004

Computer Assignment 2:
Fourier Transform and Fourier Series

Part I : Fourier Transform and Properties

Exercise 1 : Plotting the Fourier Transform of signals

In MATLAB you can find the Fourier transform of a signal using the built-in function `fft`. For example, to plot the amplitude of the Fourier transform $S(f)$ of the signal $s(t) = \cos(20\pi t)$ where t is in the range -5 to 5 using increments of 0.01, you can do the following:

```
Dt=0.01;
t=-5:Dt:5;
s=cos(20*pi*t);
S=fftshift(fft(fftshift(s)));
Df=(1/((length(s)-1)*Dt));
f=(-0.5/Dt):Df:(0.5/Dt);
plot(f,abs(S));
```

We note that the use of the function `fftshift` is required due to the particular algorithm MATLAB uses to implement the function `fft`. Alternatively, the Fourier Transform can be evaluated by using the function `fouriert` that is included in the appendix. In this case the previous example is given by the following commands:

```
t=-5:0.01:5;
s=cos(20*pi*t);
[S,f]=fouriert(s,t);
plot(f,abs(S));
```

(a) Create an M-file that plots the sinusoidal waveform $s(t) = \cos(2\pi f_c t - \pi/2)$ from $t = -2$ to $t = 2$ (seconds) in 0.01 intervals, as well as its amplitude spectrum (amplitude of the Fourier Transform). Use a frequency $f_c = 5Hz$.

(b) Repeat (a) for a frequency $f_c = 10Hz$.

(c) Create an M-file that plots the sinc function $\text{sinc}(t) = \sin(\pi t)/\pi t$, where t is in the range -5 to 5 using increments of 0.05, as well as the amplitude spectrum. NOTE: The function `sinc` is available in MATLAB.

Exercise 2 : Fourier Transform properties

In this exercise we will see examples that illustrate four important properties of the Fourier Transform, namely the linearity property, the time scaling property, the time shifting property and the frequency shifting property (or modulation theorem).

(a) *Linearity.*
Create an M-file to:

1. Plot the waveforms $s_1(t) = \cos(10\pi t)$, $s_2(t) = \cos(20\pi t)$, and $s(t) = s_1(t) + s_2(t)$ from $t = -1$ to $t = 1$ (seconds) in 0.01 increments.
2. Plot the amplitude of the Fourier transforms $S_1(f)$, $S_2(f)$ and $S(f)$ of the signals $s_1(t)$, $s_2(t)$ and $s(t)$, respectively.
3. Plot the amplitude of $S_1(f) + S_2(f)$. How does this compare to the amplitude of $S(f)$?

(b) *Time Scaling.*
Create an M-file to:

1. Plot the pulse $s(t) = \text{rectpuls}(\alpha t/0.5)$ where $\alpha = 1$ and t is in the range -5 to 5 (use increments of 0.05).
2. Plot the amplitude spectrum of $s(t)$.
3. Repeat (b.1) and (b.2) for $\alpha = 2$. Compare the bandwidth for those two cases.

(c) *Time Shifting.*
Create an M-file to:

1. Plot the triangular pulses $s_1(t)$ and $s_2(t)$ given by:

$$s_1(t) = \begin{cases} t+1, & -1 \leq t \leq 0 \\ -t+1, & 0 < t \leq 1 \\ 0, & otherwise \end{cases}$$

$$s_2(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ -t+2, & 1 < t \leq 2 \\ 0, & otherwise \end{cases}$$

   for t in the range -5 to 5 using increments of 0.05.

2. Plot the amplitude spectrum of $s_1(t)$ and $s_2(t)$. How do they compare?

3. Plot the phase of the Fourier transform of $s_1(t)$ and $s_2(t)$ in degrees.

(d) *Modulation Theorem.*
Create an M-file to:

1. Plot the pulse $s(t)$ = rectpuls($t/0.5$) (i.e. a rectangular pulse with a duration $T = 0.5$), where t is in the range -5 to 5 using increments of 0.025.

2. Plot the amplitude spectrum of $s(t)$.

3. Plot the amplitude of the Fourier transform of the pulse $s_m(t) = s(t)\exp(j2\pi f_c t)$ for $f_c = 5Hz$ and $f_c = 15Hz$.

(e) *Radio Frequency (RF) pulse.*
Create an M-file to:

1. Plot the pulse $s(t)$ = rectpuls($t/0.5$) (i.e. a rectangular pulse with a duration $T = 0.5$), where t is in the range -5 to 5 using increments of 0.01.

2. Plot its amplitude spectrum.

3. Plot the signal $s_m(t) = s(t)\cos(2\pi f_c t)$, for $f_c = 5Hz$ and $f_c = 15Hz$.

4. Plot the amplitude spectrum of the signal $s_m(t)$ for $f_c = 5Hz$ and $f_c = 15Hz$.

Part II : Fourier Series

Exercise 3 : Fourier Coefficients and Signal Reconstruction

Any periodic real signal $g(t)$ with period T_0 can be written as an infinite weighted sum of cosine and sine functions, i.e.

$$g(t) = a_0 + 2\sum_{n=1}^{\infty}[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)],$$

where $f_0 = 1/T_0$ is the fundamental frequency and

$$\begin{aligned} a_n &= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\cos(2\pi n f_0 t), \\ b_n &= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\sin(2\pi n f_0 t). \end{aligned}$$

If the signal $g(t)$ is even then $b_n = 0, n = 1, \ldots \infty$. For example, if $g(t)$ is a periodic train of rectangular pulses of duration T then

$$g(t) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t), \tag{1}$$

with

$$a_n = \frac{1}{\pi n}\sin\frac{n\pi T}{T_0}, \text{ and } a_0 = \frac{T}{T_0}.$$

(a) Let $g(t)$ be a pulse train of amplitude A and period T_0 sec where the pulse is on (not zero) only for $T \leq T_0/2$ sec. The function can be described analytically over one period $-\frac{T_0}{2}$ to $\frac{T_0}{2}$ as follows:

$$g(t) = \begin{cases} A, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases}$$

Create an M-file that approximates a unit amplitude train (with period $T_0 = 4$sec) of rectangular pulses of duration $T = 2$sec, $g(t)$, where t

ranges from -5 to 5 (seconds). Use increments of 0.01. This approximation is done by adding up the first $N = 10$ terms in (1), i.e. $g(t)$ is approximated by the sum:

$$g(t) = a_0 + 2\sum_{n=1}^{N} a_n \cos(2\pi n f_0 t), \text{ for } N = 10.$$

Plot the approximation.

(b) Repeat question (a) for $N = 1$, $N = 3$, $N = 20$ and $N = 50$. Plot the approximations.

(c) Plot the discrete spectrum of the signal for n=[-20, 20] with steps of 1 (use the command `stem` instead of `plot`).

Appendix

You can evaluate the Fourier transform of a signal by using the following function:

```
function [S,f]=fouriert(s,t);

Dt=t(2)-t(1);
Df=(1/((length(s)-1)*Dt));
f=(-0.5/Dt):Df:(0.5/Dt);
S=fftshift(fft(fftshift(s)));
```

The inputs to this function are the signal `s` whose Fourier transform we want to evaluate, and the time axis `t` of the signal. The outputs are the vector `S` containing the values of the Fourier transform of `s` evaluated at the points contained in the output vector `f`. So, `plot(f,angle(S))` will plot the phase of the Fourier transform of `s` in radians. Don't forget to name the M-file that contains the above function `fouriert.m`.

Note

Your report should include all plots and M-files you are asked to create in Exercises 1, 2 and 3.