EE 483 Communications Systems I  
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Homework Set 7

1. Consider transmitting a three digit message over a noisy channel having error probability 
   \( P(E) = \frac{2}{5} \) per digit, so that the probability of a correct digit is 
   \( P(C) = 1 - \frac{2}{5} = \frac{3}{5} \). We assume that the 3 digits are statistically independent.

   (i) What are the error patterns that constitute the sample space and what are their probabilities?

   (ii) What is the probability of one error? What is the probability of two errors?

2. Derive the chain rule: 
   \( P(XYZ) = P(X)P(Y|X)P(Z|XY) \)

3. Let \( X = \frac{1}{2}N^2 \), where \( N \) is a random integer whose value is equally likely to be any in the range \( -1 \leq N \leq 3 \). Plot the CDF of \( X \) and use it to evaluate the probabilities of the following events: \( X \leq 0, 2 < X \leq 3, X < 2, X \geq 2 \).

4. Suppose that a certain random variable has the CDF

   \[
   F(x) = \begin{cases} 
   0 & x \leq 0 \\ 
   kx^2 & 0 < x \leq 10 \\ 
   100k & x > 10 
   \end{cases}
   \]

   Evaluate \( k \), find the values of \( P(X \leq 5) \) and \( P(5 < X \leq 7) \), and plot the corresponding PDF.