EE 483 Communications Systems I
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Homework Set 3

1. (35/100) By computing the Fourier series coefficients of the periodic signal $\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ show that

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{j2\pi \frac{n}{T_s} t} \tag{1}$$

   Using this result, prove that for any signal $x(t)$ and any T_s, the following identity holds

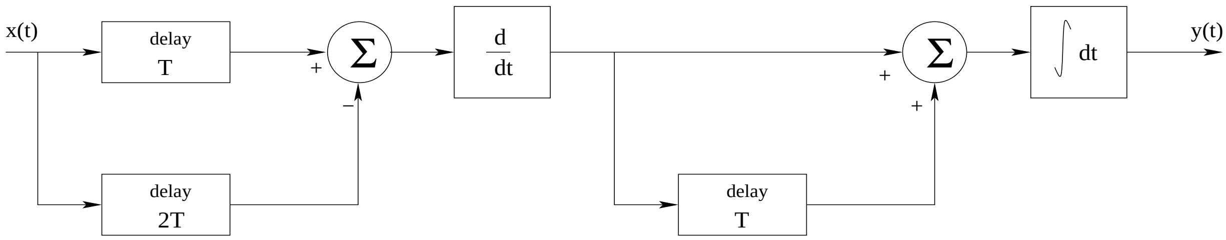
$$\sum_{n=-\infty}^{\infty} x(t - nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(\frac{n}{T_s}\right) e^{j2\pi \frac{n}{T_s} t}$$

   From this, conclude the following relation known as *Poisson's sum formula*

$$\sum_{n=-\infty}^{\infty} x(-nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(\frac{n}{T_s}\right)$$

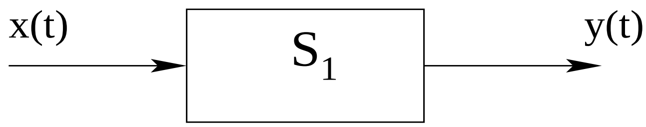
2. (35/100) The response of an LTI system to $e^{-\alpha t}u(t), (a > 0)$, is $\delta(t)$. Using frequency domain analysis techniques find the response of the system to $x(t) = e^{-\alpha t}\cos(\beta t)u(t)$.

3. (30/100) Evaluate the transfer function of a linear system represented by the block diagram shown:



Bonus Problems (Optional) (25/100, equally weighted parts)

4. Consider the system S_1 shown below with the input/output pair $x(t) = \cos(2\pi f_c t)/y(t) = \cos(4\pi f_c t)$ and $f_c = 1Hz$. Determine whether the system *(i)* could not *(ii)* could or *(iii)* must be linear time invariant (LTI). Choose the strongest statement that applies and justify your answer. If you decide that the system is LTI determine a possible impulse response for it.



5. Show that the Hilbert transform of an even signal is odd and that the Hilbert transform of an odd signal is even.