1. (35/100) By computing the Fourier series coefficients of the periodic signal $\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ show that

$$
\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{j2\pi \frac{n}{T_s} t}
$$

(1)

Using this result, prove that for any signal $x(t)$ and any $T_s$, the following identity holds

$$
\sum_{n=-\infty}^{\infty} x(t - nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(\frac{n}{T_s}\right) e^{j2\pi \frac{n}{T_s} t}
$$

From this, conclude the following relation known as Poisson's sum formula

$$
\sum_{n=-\infty}^{\infty} x(-nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(\frac{n}{T_s}\right)
$$

2. (35/100) The response of an LTI system to $e^{-at}u(t), (a > 0)$, is $\delta(t)$. Using frequency domain analysis techniques find the response of the system to $x(t) = e^{-at} \cos(\beta t)u(t)$.

3. (30/100) Evaluate the transfer function of a linear system represented by the block diagram shown:
**Bonus Problems (Optional) (25/100, equally weighted parts)**

4. Consider the system $S_1$ shown below with the input/output pair $x(t) = \cos(2\pi f_c t)$ and $y(t) = \cos(4\pi f_c t)$ and $f_c = 1\text{Hz}$. Determine whether the system (i) could not (ii) could or (iii) must be linear time invariant (LTI). Choose the strongest statement that applies and justify your answer. If you decide that the system is LTI determine a possible impulse response for it.

![System diagram]

5. Show that the Hilbert transform of an even signal is odd and that the Hilbert transform of an odd signal is even.