1. Prove the following Fourier Transform pairs (from p. 764):

\[
F\{\delta(t-t_0)\} = e^{-j2\pi ft_0}
\]

\[
F\{e^{j2\pi ft_0}\} = \delta(f-f_0)
\]

\[
F\{\cos(2\pi ft)\} = 0.5\{\delta(f-f_c) + \delta(f+f_c)\}
\]

\[
F\{\sin(2\pi ft)\} = -0.5j\{\delta(f-f_c) - \delta(f+f_c)\}
\]

\[
F\{\text{sgn}(t)\} = \frac{1}{j\pi f}
\]

\[
F\{u(t)\} = 0.5\delta(f) + \frac{1}{j2\pi f}
\]

2. A signal \(x(t)\) of finite energy is applied to a square-law device whose output \(y(t)\) is defined by

\[y(t) = x^2(t)\]

The spectrum of \(x(t)\) is limited to the frequency interval \(-W \leq f \leq W\). Hence, show that the spectrum of \(y(t)\) is limited to \(-2W \leq f \leq 2W\). \textit{Hint:} Express \(y(t)\) as \(x(t)\) multiplied by itself.

3. Prove that if \(g(t)\) is a real-valued function of time \(t\), then \(G^*(f) = G(-f)\). \textit{Hint:} Use the definition for \(G(f)\) and the Euler equation.

3. The Fourier Transform \(G(f)\) of a signal \(g(t)\) is defined by

\[
G(f) = \begin{cases} 1, & f > 0 \\ \frac{1}{2}, & f = 0 \\ 0, & f < 0 \end{cases}.
\]

Determine the signal \(g(t)\). \textit{Hint:} Use the Fourier transform of \(u(t)\) and the duality property.