2.5. Use the convolution integral to find the response $y(t)$ of an LTI system with impulse response $h(t)$ to input $x(t)$:

(a) $x(t) = \exp[-t]u(t)$ $\quad$ $h(t) = \exp[-2t]u(t)$

(b) $x(t) = t\exp[-t]u(t)$ $\quad$ $h(t) = u(t)$

(c) $x(t) = \exp[-t]u(t) + u(t)$ $\quad$ $h(t) = u(t)$

(d) $x(t) = u(t)$ $\quad$ $h(t) = \exp[-2t]u(t) + \delta(t)$

(e) $x(t) = \exp[-at]u(t)$ $\quad$ $h(t) = u(t) - \exp[-at]u(t-b)$

(f) $x(t) = \delta(t-1) + \exp[-t]u(t)$ $\quad$ $h(t) = \exp[-2t]u(t)$

2.6. The cross correlation of two different signals is defined as

$$R_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\,y(\tau - t)\,d\tau = \int_{-\infty}^{\infty} x(\tau + t)\,y(\tau)\,d\tau$$

(a) Show that

$$R_{xy}(t) = x(t) * y(-t)$$

(b) Show that the cross correlation does not obey the commutative law.

(c) Show that $R_{xy}(t)$ is symmetric ($R_{xy}(t) = R_{yx}(-t)$).

2.7. Find the cross correlation between a signal $x(t)$ and the signal $y(t) = x(t-1) + n(t)$ for $B/A = 0$, 0.1, and 1, where $x(t)$ and $n(t)$ are as shown in Figure P2.7.

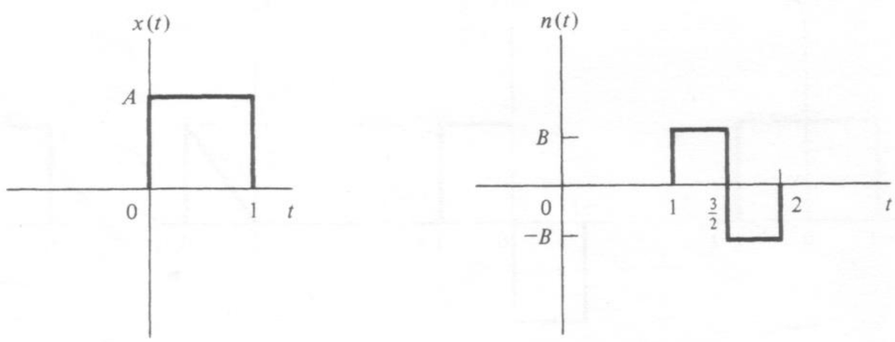

Figure P2.7

2.8. The autocorrelation is a special case of cross correlation with $y(t) = x(t)$. In this case,

$$R_x(t) = R_{xx}(t) = \int_{-\infty}^{\infty} x(\tau)x(\tau + t)\,d\tau$$

(a) Show that

$$R_x(0) = E, \qquad \text{the energy of } x(t)$$

(b) Show that

$$R_x(t) \le R_x(0) \qquad \text{(use the Schwarz inequality)}$$

(c) Show that the autocorrelation of $z(t) = x(t) + y(t)$ is

$$R_z(t) = R_x(t) + R_y(t) + R_{xy}(t) + R_{yx}(t)$$