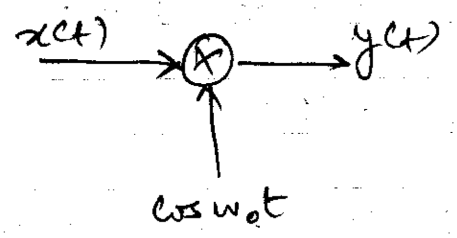
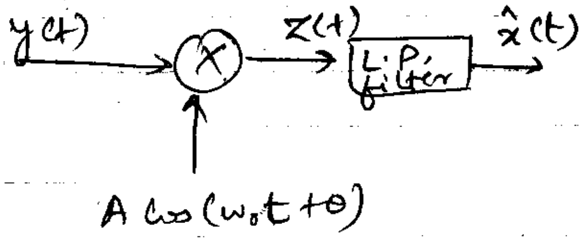
4.26 Modulation Demodulation





θ: constant unknown phase.

(a) $Z(t) = y(t) \cdot A\cos(w_0 t + \theta)$

$$Z(t) = \frac{A}{2} y(t)\, e^{j(w_0 t + \theta)} + \frac{A}{2} y(t)\, e^{-j(w_0 t + \theta)}$$

Taking F.T

$$Z(w) = \frac{A}{2} e^{j\theta} Y(w - w_0) + \frac{A}{2} e^{-j\theta} Y(w + w_0)$$

Now $Y(w) = \frac{1}{2} X(w - w_0) + \frac{1}{2} X(w + w_0)$

So,
$$Z(w) = \frac{A}{2} e^{j\theta} \left[\frac{1}{2} X(w - w_0 - w_0) + \frac{1}{2} X(w - w_0 + w_0)\right] + \frac{A}{2} e^{-j\theta}\left[\frac{1}{2} X(w - w_0 + w_0) + \frac{1}{2} X(w + w_0 + w_0)\right]$$

$$= \frac{A}{4} e^{j\theta} X(w - 2w_0) + \frac{A}{4} e^{j\theta} X(w) + \frac{A}{4} e^{-j\theta} X(w) + \frac{A}{4} e^{-j\theta} X(w + 2w_0)$$

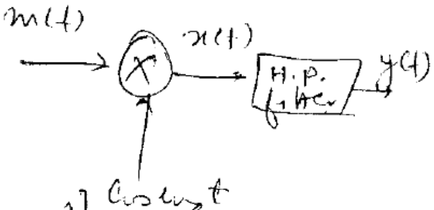
$$Z(w) = \frac{A}{4} e^{j\theta} X(w - 2w_0) + \frac{A}{4} e^{-j\theta} X(w + 2w_0) + \frac{A}{2} \cos\theta\, X(w)$$

Now, passing $z(t)$ through a low-pass filter with cut-off frequency equal to the cut-off frequency of $x(t)$ (i.e. ω_m) will result in

$$\hat{x}(t) = \frac{A}{2}\cos\theta \, x(t)$$

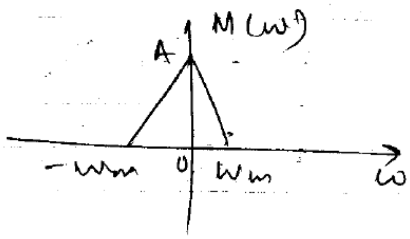
(b) So, $\hat{x}(t)$ is $\frac{A}{2}\cos\theta$ times the original input signal $x(t)$.

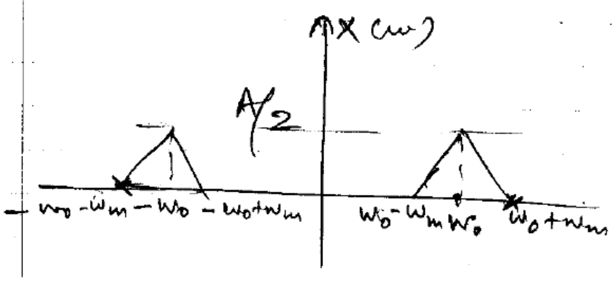
4.27 (a) After multiplication,

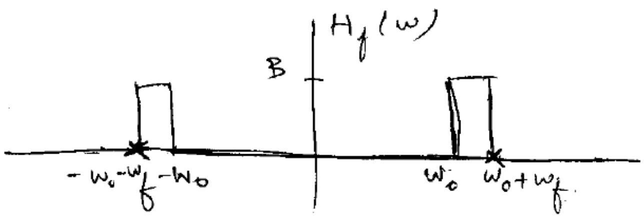


$x(t) = m(t) \cos w_0 t$

$X(w) = \frac{1}{2}\left[M(w - w_0) + M(w + w_0)\right]$







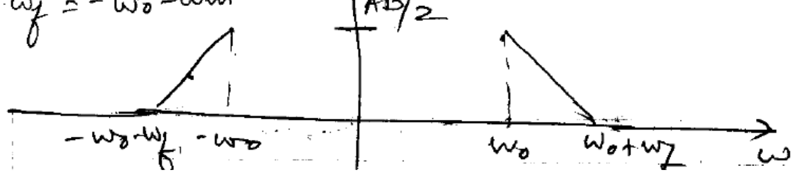
$y(t) = x(t) * h_f(t)$

$Y(w)_0 = X(w) \cdot H_f(w)$

Spectrum of $Y(w)$, for $w_f = w_m$.

$\Rightarrow W_0 + w_f = w_0 + w_m$

$\& \; -w_0 - w_f = -w_0 - w_m$



4.28a

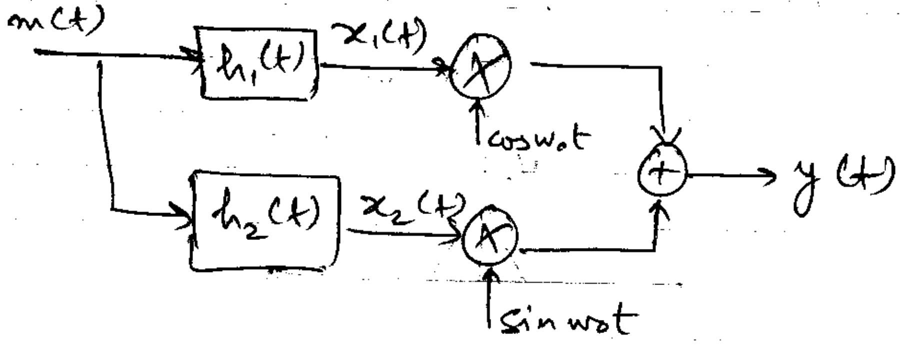
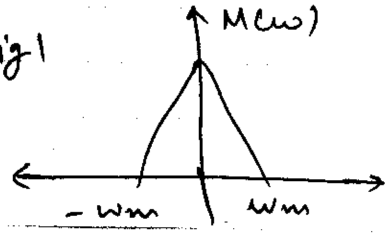


Fig 1



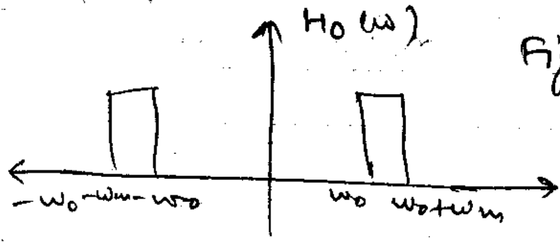


Fig 2

Spectrum of y(t)?

$$y(t) = x_1(t)\cos\omega_0 t + x_2(t)\sin\omega_0 t \quad (1)$$

Now,

$$x_1(t) = m(t) * h_1(t)$$

$$X_1(\omega) = M(\omega)\cdot H_1(\omega)$$

$$= \frac{1}{2} M(\omega)\, H_0(\omega-\omega_0) + \frac{1}{2} M(\omega)\, H_0(\omega+\omega_0)$$

Fig 3

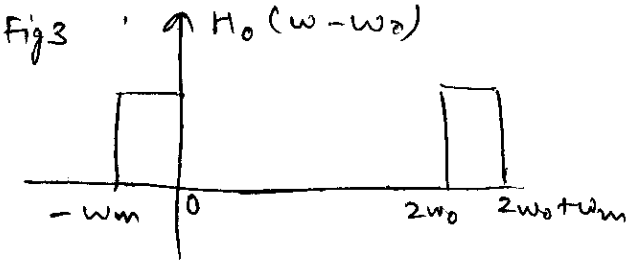
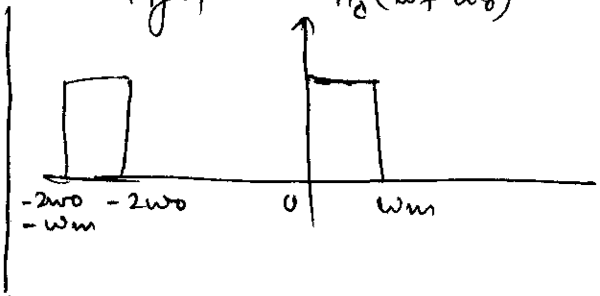


Fig 4



Using Figs 1, 3 and 4, $X_1(\omega)$ is

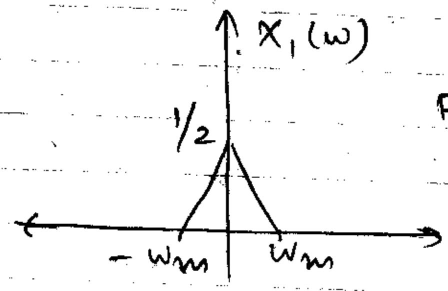


Fig 5

$$x_2(t) = m(t) * h_2(t)$$
$$X_2(\omega) = M(\omega) \cdot H_2(\omega)$$

$$= \frac{-1}{2j}\left[\underbrace{M(\omega)\,H_0(\omega-\omega_0) - M(\omega)\,H_0(\omega+\omega_0)}\right]$$

$$X_2(\omega) = -\frac{1}{2j}\,F(\omega)$$

Using Figs 1, 3 and 4, $F(\omega)$ is

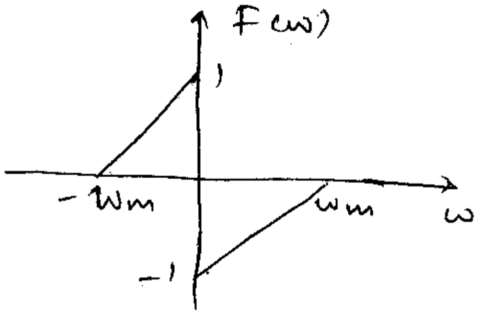


Fig 6

Taking F.T. of eqn. (1)

$$Y(\omega) = \frac{1}{2}\left[X_1(\omega-\omega_0) + X_1(\omega+\omega_0)\right]$$
$$+\frac{1}{2j}\left[X_2(\omega-\omega_0) - X_2(\omega+\omega_0)\right]$$

Using Fig 5, $\frac{1}{2}\left[X_1(\omega-\omega_0)+X_1(\omega+\omega_0)\right]$ is

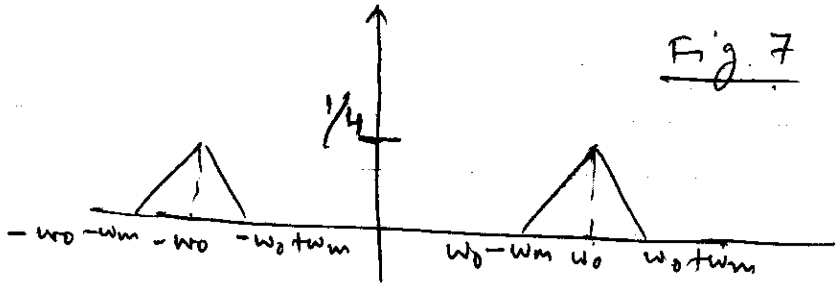


Fig. 7

Using Fig 6, $\frac{1}{2j}\left[\frac{-1}{2j}F(\omega-\omega_0)-\left(\frac{-1}{2j}\right)F(\omega+\omega_0)\right]$

$$=\frac{1}{4}\left[F(\omega-\omega_0)-F(\omega+\omega_0)\right]$$

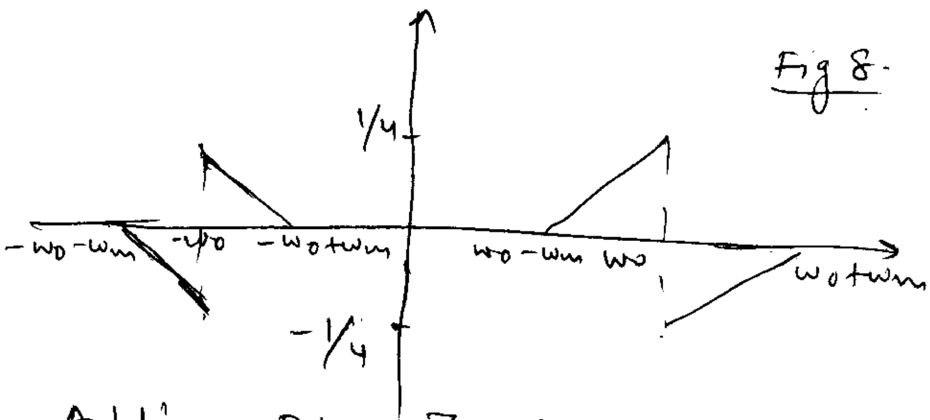
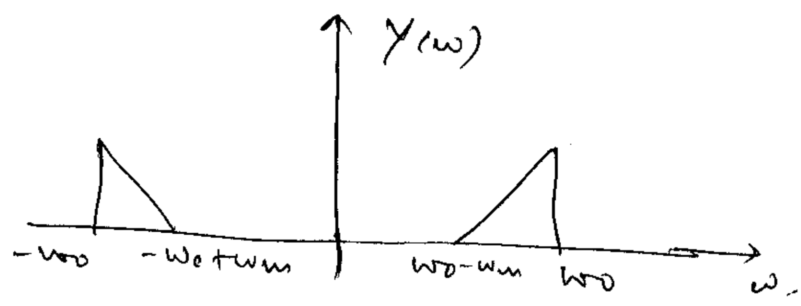
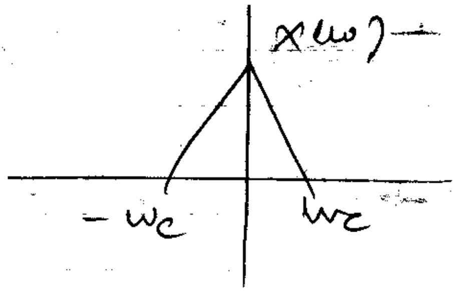


Fig 8.

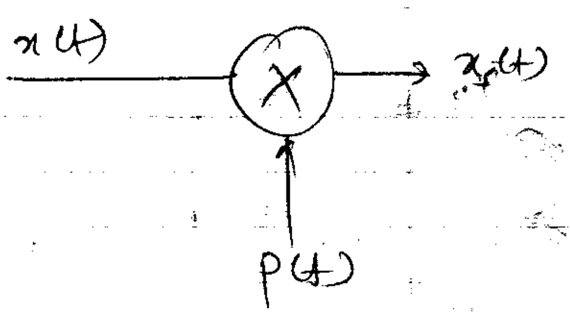
Adding Figs 7 & 8, to get $Y(\omega)$



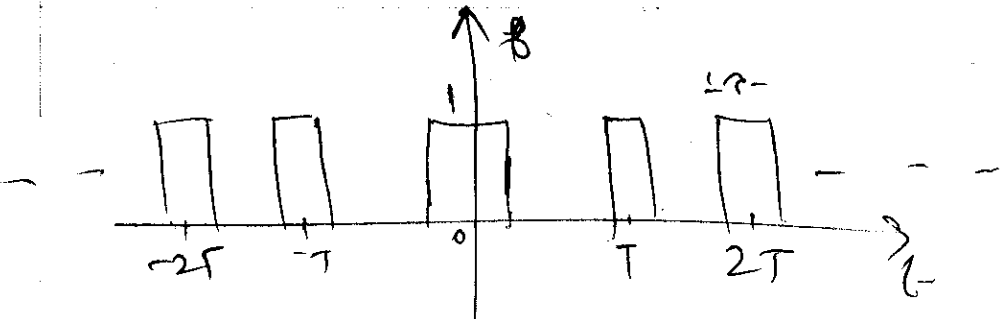
4.30



$$x_s = x(t)\, p(t)$$



$$X_s(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$$



$$p(t) = \sum_{n=-\infty}^{\infty} \mathrm{rect}\left(\frac{t - nT}{\tau}\right)$$

p(t) is periodic & hence we can find it's Fourier Series expansion

(Fourier series co-efficients)

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} \mathrm{rect}\left(\frac{t}{\tau}\right) e^{-j\frac{2\pi n t}{T}}\, dt$$

$$= \frac{1}{T} \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j\frac{2\pi n t}{T}}\, dt$$

$$= \frac{\cancel{T}}{\cancel{T}} \left. \frac{e^{-j\frac{2\pi n}{T}t}}{-j2\pi n} \right|_{-\tau/2}^{\tau/2}$$

$$= \frac{j}{2\pi n}\left[e^{-j\frac{2\pi n}{T}\cdot\frac{\tau}{2}} - e^{j\frac{2\pi n}{T}\frac{\tau}{2}} \right]$$

$$= \frac{1}{\pi n} \sin \frac{\cancel{2}\pi n\tau}{\cancel{2}T}$$

$$= \frac{1}{\pi n} \sin \frac{\pi n \tau}{T} = \frac{1}{\pi n} \sin \frac{\omega_s n \tau}{2}$$

$$c_n = \frac{\tau}{T} \cdot \frac{\sin(\pi n \tau/T)}{(\pi n \tau/T)}$$

$$\Rightarrow \quad p(t) = \frac{\tau}{T}\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n\tau/T)}{(\pi n \tau/T)} \underbrace{e^{j\frac{2\pi n t}{T}}}$$

↳ only term with 't'

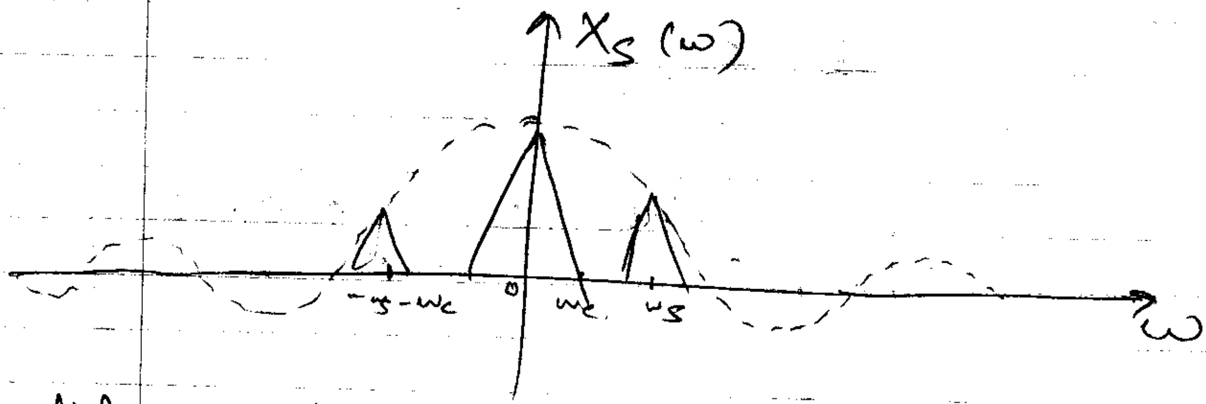
$$P(\omega) = \frac{2\pi\tau}{T} \sum_{n=-\infty}^{\infty} \frac{\sin(\pi n\tau/T)}{(\pi n\tau/T)} \delta\left(\omega - \frac{2\pi n}{T}\right)$$

$$X_s(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$$

$$= X(\omega) * \left[\frac{\tau}{T} \sum_{n=-\infty}^{\infty} \frac{\sin(\pi n\tau/T)}{\pi n\tau/T} \underbrace{\delta\left(\omega - \frac{2\pi n}{T}\right)} \right.$$

only fn with 'ω'

$$X_s(\omega) = \frac{\tau}{T} \sum_{n=-\infty}^{\infty} \frac{\sin(\pi n\tau/T)}{(\pi n\tau/T)} X\left(\omega - \frac{2\pi n}{T}\right)$$



(b) As long as $\omega_s \geq 2\omega_c$, we can recover $x(t)$ without distortion using a low-pass filter.