

## HW#6 Solutions

HW#6

$$4.1 (a) \mathcal{F}\{\operatorname{Re}\{x(t)\}\} = \mathcal{F}\left\{\frac{x(t) + x^*(t)}{2}\right\}$$

$$= \frac{1}{2}\mathcal{F}\{x(t)\} + \frac{1}{2}\mathcal{F}\{x^*(t)\}$$

$$= \frac{1}{2}X(\omega) + \frac{1}{2}X^*(-\omega) \quad [4.1 (a)]$$

4.5 (b)  $x(\omega) = \operatorname{rect}[(\omega-1)/2]$

~~$\mathcal{F}\{x(t+4)\} = \mathcal{F}\{x(t-2)\}$~~

(i)  $\mathcal{F}\{x(t)e^{-j2t}\}$

Property:  $x(t)e^{j\omega_0 t} \longleftrightarrow X(\omega - \omega_0)$

$$\begin{aligned} \mathcal{F}\{x(t)e^{-j2t}\} &= X(\omega+2) \\ &= \operatorname{rect}\left[\frac{(\omega+2)-1}{2}\right] \\ &= \operatorname{rect}\left(\frac{\omega+1}{2}\right) \end{aligned}$$

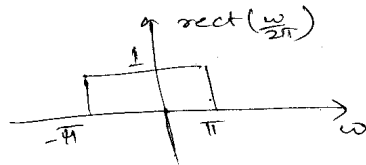
$$\underline{4.9(d)} \quad x(t) = \frac{\sin(\pi t)}{\pi t}$$

F.T. pair :-

$$\frac{\sin \omega_B t}{\pi t} \longleftrightarrow \text{rect} \left( \frac{\omega}{2\omega_B} \right)$$

Here  $\omega_B = \pi$

$$\Rightarrow \mathcal{F}\{x(t)\} = \text{rect} \left( \frac{\omega}{2\pi} \right)$$



$$\begin{aligned} E &= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} 1^2 d\omega = 1 \end{aligned}$$

$$\underline{4.15} \quad X(\omega) = \frac{\omega^2 + j4\omega + 2}{-\omega^2 + j4\omega + 3}$$

$$(b) \quad \mathcal{F}\{x(t) e^{-jt}\}$$

Property

$$\mathcal{F}\{x(t) e^{j\omega_0 t}\} = X(\omega - \omega_0)$$

$$\mathcal{F}\{x(t)e^{-jt}\} = X(\omega+1)$$

$$= \frac{(\omega+1)^2 + j4(\omega+1) + 2}{-(\omega+1)^2 + j4(\omega+1) + 3}$$

$$(c) \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega X(\omega) = j\omega \cdot \frac{\omega^2 + j4\omega + 2}{-\omega^2 + j4\omega + 3}$$

$$\mathcal{F}\left\{\frac{d^n x(t)}{dt^n}\right\} = (j\omega)^n X(\omega)$$

$$(d) \mathcal{F}\{x(t) * \delta(t-1)\} = X(\omega) \cdot \mathcal{F}\{\delta(t-1)\}$$

$$= X(\omega) e^{-j\omega}$$

$$= e^{-j\omega} \cdot \frac{\omega^2 + j4\omega + 2}{-\omega^2 + j4\omega + 3}$$