1.1. Find the fundamental period T of each of the following signals:

$$\cos(\pi t),\ \sin(2\pi t),\ \cos(3\pi t),\ \sin(4\pi t),\ \cos\left(\frac{\pi}{2}t\right),\ \sin\left(\frac{\pi}{3}t\right),$$

$$\cos\left(\frac{5\pi}{2}t\right),\ \sin\left(\frac{4\pi}{3}t\right),\ \cos\left(\frac{\pi}{4}t\right),\ \sin\left(\frac{2\pi}{3}t\right),\ \cos\left(\frac{3\pi}{5}t\right)$$

1.2. Sketch the following signals:

(a) $x(t) = \sin\left(\frac{\pi}{4}t + 20°\right)$

(b) $x(t) = t + e^{3t}, \quad 0 \le t \le 2$

(c) $x(t) = \begin{cases} t + 2 & t \le -2 \\ 0 & -2 \le t \le 2 \\ t - 2 & 2 \le t \end{cases}$

(d) $x(t) = 2\exp[-t], \quad 0 \le t \le 1, \quad$ and $\quad x(t+1) = x(t)$ for all t

1.11. Let

$$x(t) = \begin{cases} -t + 1, & -1 \le t < 0 \\ t, & 0 \le t < 2 \\ 2, & 2 \le t < 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch $x(t)$.

(b) Sketch $x(t-2)$, $x(t+3)$, $x(-3t-2)$, and $x\left(\frac{2}{3}t + \frac{1}{2}\right)$ and find the analytical expressions for these functions.

1.13. Sketch the following signals:

(a) $x_1(t) = u(t) + 5u(t-1) - 2u(t-2)$

(b) $x_2(t) = r(t) - r(t-1) - u(t-2)$

(c) $x_3(t) = \exp[-t]u(t)$

(d) $x_4(t) = 2u(t) + \delta(t-1)$

1.21. The probability that a random variable x is less than α is found by integrating the probability density function $f(x)$ to obtain

$$P(x \le \alpha) = \int_{-\infty}^{\alpha+} f(x)\,dx$$

Given that

$$f(x) = 0.2\delta(x+2) + 0.3\delta(x) + 0.2\delta(x-1) + 0.1[u(x-3) - u(x-6)]$$

find

(a) $P(x \le -3)$

(b) $P(x \le 1.5)$

(c) $P(x \le 4)$

(d) $P(x \le 6)$

2.1. Determine whether the systems described by the following input/output relationships are linear or nonlinear, causal or noncausal, time invariant or time variant, and memoryless or with memory.

(a) $y(t) = 2x(t) + 3$

(b) $y(t) = 2x^2(t) + 3x(t)$

(c) $y(t) = Ax(t)$

(d) $y(t) = Atx(t)$

(e) $y(t) = \begin{cases} x(t), & t \geq 0 \\ -x(t), & t < 0 \end{cases}$

(f) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(g) $y(t) = \int_{0}^{t} x(\tau)d\tau, \quad t \geq 0$

(h) $y(t) = x(t - 5)$

(i) $y(t) = \exp[x(t)]$

(j) $y(t) = x(t)\,x(t - 2)$

(k) $y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)d\tau$

(l) $\frac{dy(t)}{dt} + 2y(t) = 2x^2(t)$

2.3. Evaluate the following convolutions:

(a) $\text{rect}\,(t - a/a) * \delta(t - b)$

(b) $\text{rect}\,(t/a) * \text{rect}\,(t/a)$

(c) $\text{rect}\,(t/a) * u(t)$

(d) $\text{rect}\,(t/a) * \text{sgn}(t)$

(e) $u(t) * u(t)$

(f) $t[u(t) - u(t - 1)] * u(t)$

(g) $\text{rect}\,(t/a) * r(t)$

(h) $r(t) * [\text{sgn}(t) + u(-t - 1)]$

(i) $[u(t + 1) - u(t - 1)]\text{sgn}(t) * u(t)$

(j) $u(t) * \delta'(t)$

2.4. Graphically determine the convolution of the pairs of signals shown in Figure P2.4.

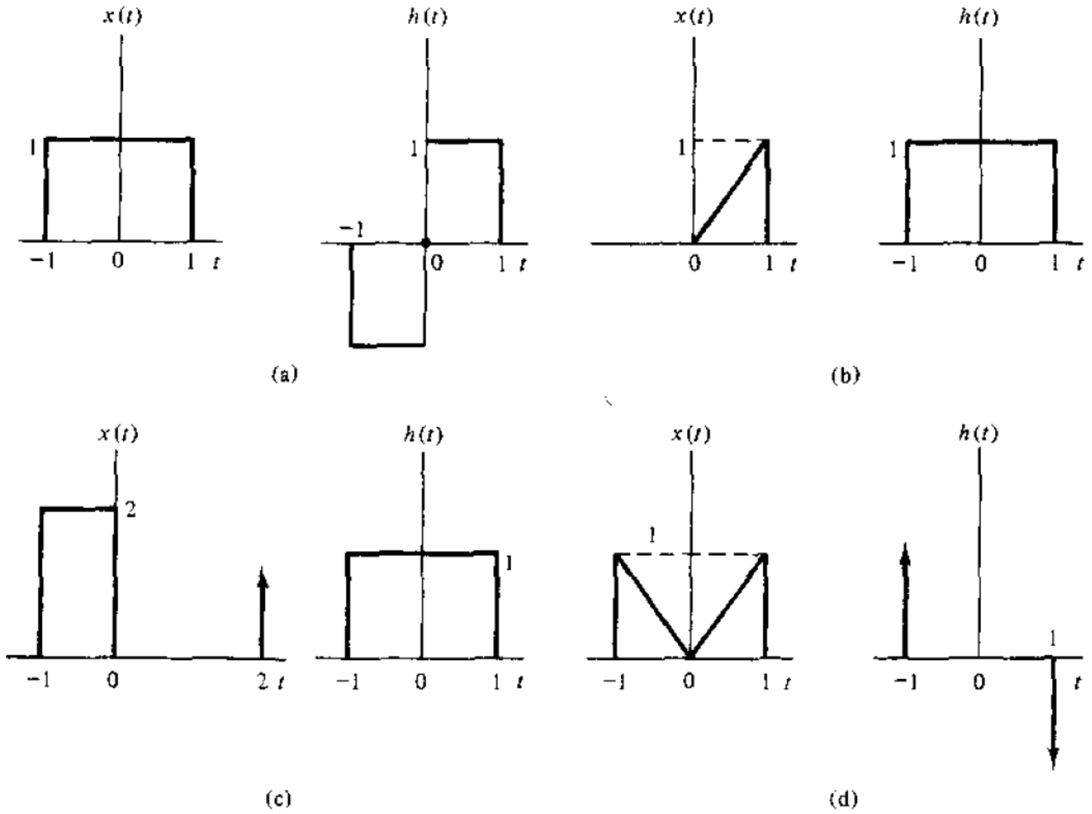


Figure P2.4

2.6. The cross correlation of two different signals is defined as

$$R_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\, y(\tau - t)\, d\tau = \int_{-\infty}^{\infty} x(\tau + t)\, y(\tau)\, d\tau$$

(a) Show that

$$R_{xy}(t) = x(t) * y(-t)$$

(b) Show that the cross correlation does not obey the commutative law.

(c) Show that $R_{xy}(t)$ is symmetric ($R_{xy}(t) = R_{yx}(-t)$).

2.10. The input to an LTI system with impulse response $h(t)$ is the complex exponential $\exp[j\omega t]$. Show that the corresponding output is

$$y(t) = \exp[j\omega t]\, H(\omega)$$

where

$$H(\omega) = \int_{-\infty}^{\infty} h(t) \exp[-j\omega t]\, dt$$

3.8. The signal shown in Figure P3.8 is created when a cosine voltage or current waveform is rectified by a single diode, a process known as half-wave rectification. Deduce the exponential Fourier-series expansion for the half-wave rectified signal.

3.10. The signal shown in Figure P3.10 is created when a sine voltage or current waveform is rectified by a a circuit with two diodes, a process known as full-wave rectification. Deduce the exponential Fourier-series expansion for the full-wave rectified signal.

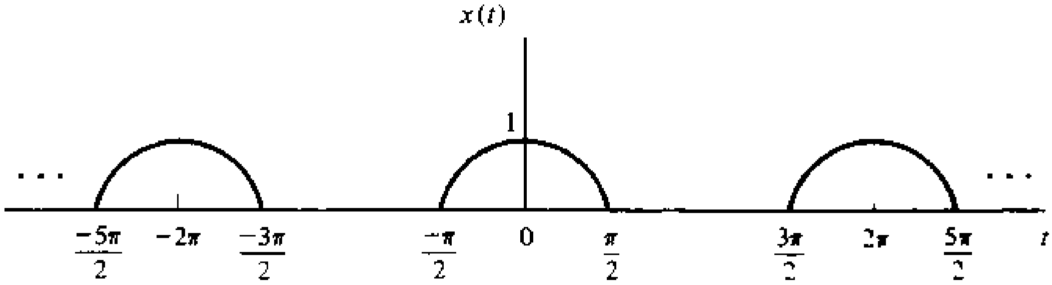


Figure P3.8

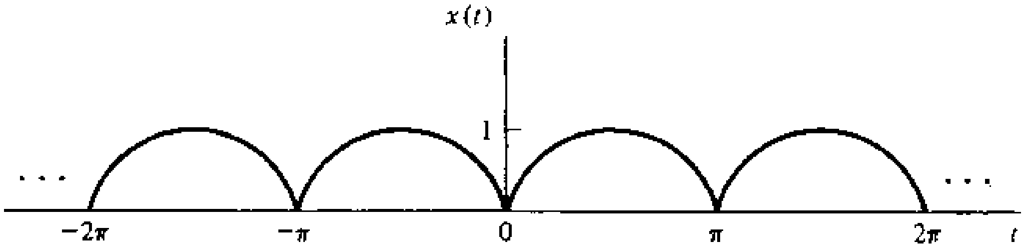


Figure P3.10

3.12. Find the exponential Fourier-series representations of the signals shown in Figure P3.12. Plot the magnitude and phase spectrum for each case.

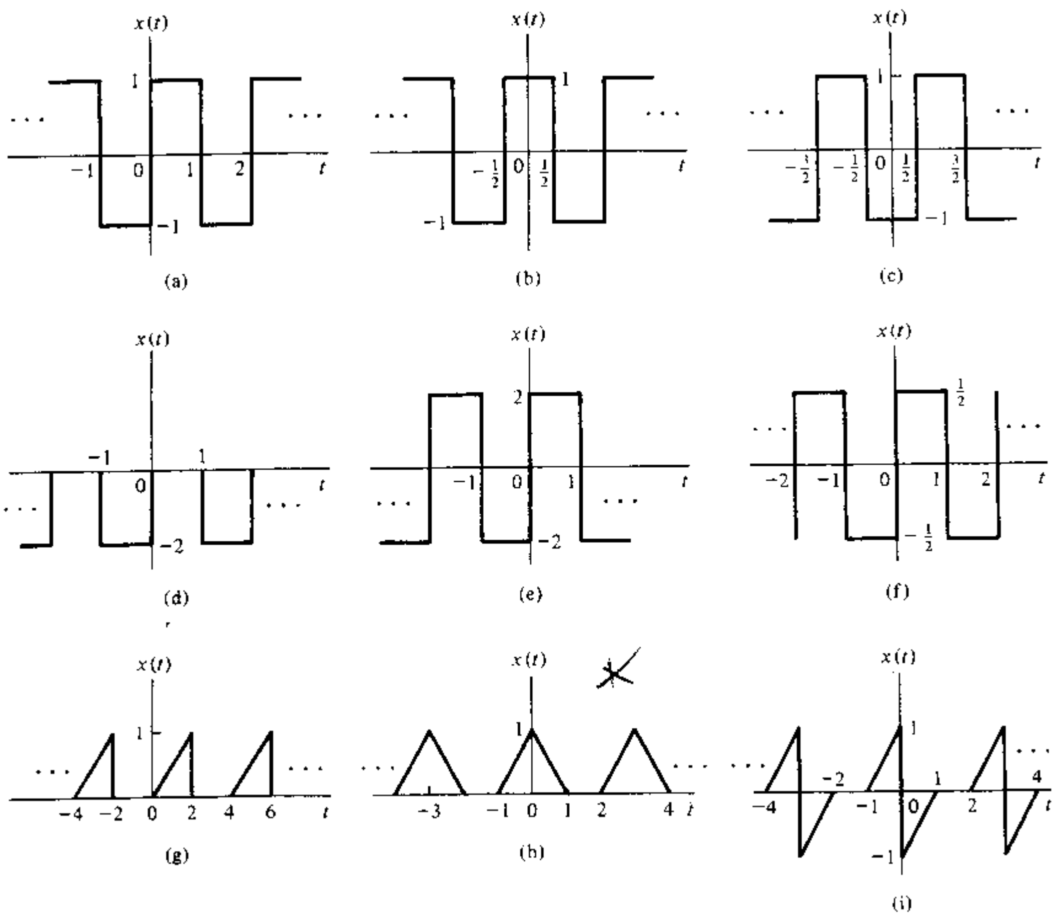


Figure P3.12

4.1. Find the Fourier transform of the following signals in terms of $X(\omega)$, the Fourier transforrn of $x(t)$.

(a) $x(-t)$

(b) $x_e(t) = \frac{x(t) + x(-t)}{2}$

(c) $x_o(t) = \frac{x(t) - x(-t)}{2}$

(d) $x^*(t)$

(e) $\text{Re}\{x(t)\} = \frac{x(t) + x^*(t)}{2}$

(f) $\text{Im}\{x(t)\} = \frac{x(t) - x^*(t)}{2j}$

4.5. Let $X(\omega) = \text{rect}[(\omega - 1)/2]$. Find the transform of the following functions, using the properties of the Fourier transform:

(a) $x(-t)$

(b) $tx(t)$

(c) $x(t+1)$

(d) $x(-2t+4)$

(e) $(t - 1)x(t + 1)$

(f) $\dfrac{dx(t)}{dt}$

(g) $t\dfrac{dx(t)}{dt}$

(h) $x(2t - 1)\exp[-j2t]$

(i) $x(t)\exp[-j2t]$

(j) $tx(t)\exp[-j2t]$

4.6. Let $x(t) = \exp[-2t]u(t)$ and let $y(t) = x(t + 1) + x(t - 1)$. Find $Y(\omega)$.

4.9. Find the energy of the following signals, using Parseval's theorem.

(a) $x(t) = \exp[-2t]u(t)$

(b) $x(t) = u(t) - u(t - 5)$

(c) $x(t) = \Delta(t/4)$

(d) $x(t) = \dfrac{\sin(\pi t)}{\pi t}$

4.15. A signal has Fourier transform

$$X(\omega) = \frac{\omega^2 + j4\omega + 2}{-\omega^2 + j4\omega + 3}$$

Find the transforms of each of the following signals:

(a) $x(-2t + 1)$

(b) $x(t)\exp[-jt]$

(c) $\dfrac{dx(t)}{dt}$

(d) $x(t)\sin(\pi t)$

(e) $x(t) * \delta(t - 1)$

(f) $x(t) * x(t - 1)$

4.26. As discussed in Section 4.4.1, AM demodulation consists of multiplying the received signal $y(t)$ by a replica, $A\cos\omega_0 t$, of the carrier and low-pass filtering the resulting signal $z(t)$. Such a scheme is called synchronous demodulation and assumes that the phase of the carrier is known at the receiver. If the carrier phase is not known, $z(t)$ becomes

$$z(t) = y(t)A\cos(\omega_0 t + \theta)$$

where θ is the assumed phase of the carrier.

(a) Assume that the signal $x(t)$ is band limited to ω_m, and find the output $\hat{x}(t)$ of the demodulator.

(b) How does $\hat{x}(t)$ compare with the desired output $x(t)$?

4.27. A single-sideband, amplitude-modulated signal is generated using the system shown in Figure P4.27.

(a) Sketch the spectrum of $y(t)$ for $\omega_f = \omega_m$.

(b) Write a mathematical expression for $h_f(t)$. Is it a realizable filter?

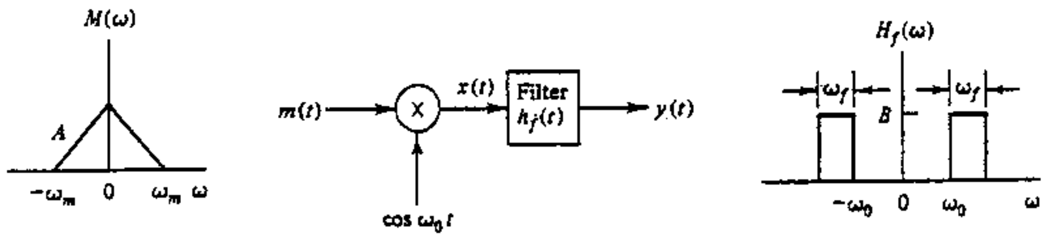
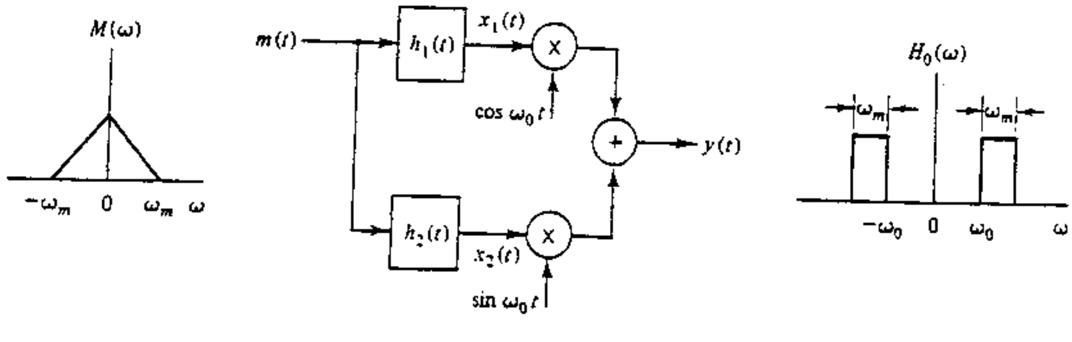


Figure P4.27

4.28. Consider the system shown in Figure P4.28(a). The systems $h_1(t)$ and $h_2(t)$ respectively have frequency responses



(a)

Figure P4.28(a)

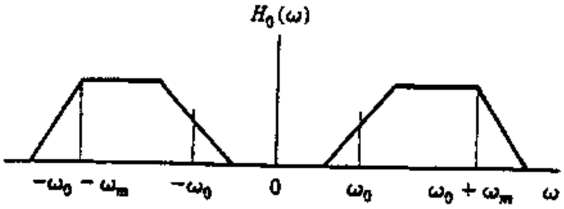
$$H_1(\omega) = \frac{1}{2}[H_0(\omega - \omega_0) + H_0(\omega + \omega_0)]$$

and

$$H_2(\omega) = \frac{-1}{2j}[H_0(\omega - \omega_0) - H_0(\omega + \omega_0)]$$

(a) Sketch the spectrum of $y(t)$.

(b) Repeat part (a) for the $H_0(\omega)$ shown in Figure P4.28(b).



(b)

Figure P4.28(b)

4.30. In natural sampling, the signal $x(t)$ is multiplied by a train of rectangular pulses, as shown in Figure P4.30.

(a) Find and sketch the spectrum of $x_s(t)$.

(b) Can $x(t)$ be recovered without any distortion?

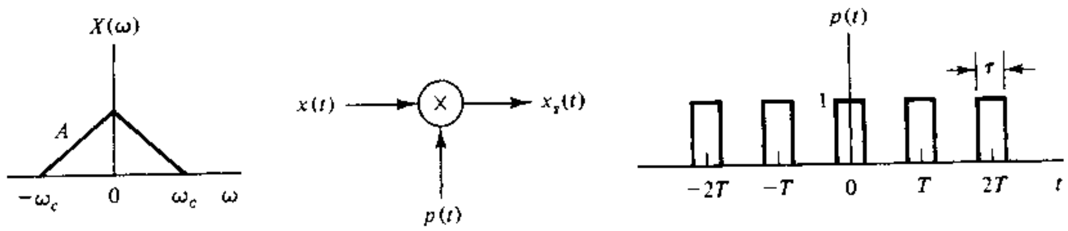


Figure P4.30

Example 5.5.1

Suppose we want to find the Laplace transform of

$$(A + B \exp[-bt])u(t)$$

From Table 5.1, we have the transform pair

$$u(t) \leftrightarrow \frac{1}{s} \qquad \text{and} \qquad \exp[-bt]u(t) \leftrightarrow \frac{1}{s+b}$$

Thus, using linearity, we obtain the transform pair

$$Au(t) + B\exp[-bt]u(t) \leftrightarrow \frac{A}{s} + \frac{B}{s+b} = \frac{(A+B)s + Ab}{s(s+b)}$$

The ROC is the intersection of $\mathrm{Re}\{s\} > -b$ and $\mathrm{Re}\{s\} > 0$, and, hence, is given by $\mathrm{Re}\{s\} > \max(-b, 0)$.

Example 5.5.2

Consider the rectangular pulse $x(t) = \mathrm{rect}((t-a)/2a)$. This signal can be written as

$$\mathrm{rect}((t-a)/2a) = u(t) - u(t-2a)$$

Using linearity and time shifting, we find that the Laplace transform of $x(t)$ is

$$X(s) = \frac{1}{s} - \exp[-2as]\frac{1}{s} = \frac{1 - \exp[-2as]}{s}, \qquad \mathrm{Re}\{s\} > 0$$

It should be clear that the time shifting property holds for a right shift only. For example, the Laplace transform of $x(t + t_0)$, for $t_0 > 0$, cannot be expressed in terms of the Laplace transform of $x(t)$. (Why?)

Example 5.5.3

From entry 8 in Table 5.1 and Equation (5.5.3), the Laplace transform of

$$x(t) = A\exp[-at]\cos(\omega_0 t + \theta)u(t)$$

is

$$\begin{aligned}
X(s) &= \mathscr{L}\{A\exp[-at](\cos\omega_0 t\cos\theta - \sin\omega_0 t\sin\theta)u(t)\} \\
&= \mathscr{L}\{A\exp[-at]\cos\omega_0 t\cos\theta\, u(t)\} - \mathscr{L}\{A\exp[-at]\sin\omega_0 t\sin\theta\, u(t)\} \\
&= \frac{A(s+a)\cos\theta}{(s+a)^2 + \omega_0^2} - \frac{A\,\omega_0\sin\theta}{(s+a)^2 + \omega_0^2} \\
&= \frac{A[(s+a)\cos\theta - \omega_0\sin\theta]}{(s+a)^2 + \omega_0^2}, \qquad \mathrm{Re}\{s\} > -a
\end{aligned}$$

Example 5.5.13

Suppose that the input $x(t) = \exp[-2t]u(t)$ is applied to a relaxed (zero initial conditions) LTI system. The output of the system is

$$y(t) = \frac{2}{3}(\exp[-t] + \exp[-2t] - \exp[-3t])u(t)$$

Then

$$X(s) = \frac{1}{s+2}$$

and

$$Y(s) = \frac{2}{3(s+1)} + \frac{2}{3(s+2)} - \frac{2}{3(s+3)}$$

Using Equation (5.5.14), we conclude that the transfer function $H(s)$ of the system is

$$\begin{aligned} H(s) &= \frac{2}{3} + \frac{2(s+2)}{3(s+1)} - \frac{2(s+2)}{3(s+3)} \\ &= \frac{2(s^2+6s+7)}{3(s+1)(s+3)} \\ &= \frac{2}{3}\left[1 + \frac{1}{s+1} + \frac{1}{s+3}\right] \end{aligned}$$

from which it follows that

$$h(t) = \frac{2}{3}\delta(t) + \frac{2}{3}[\exp[-t] + \exp[-3t]]u(t)$$