# Least-Square Curve Fitting

**Statement of Problem:**

***Given*** $m$ data points $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ .

***find*** coefficients $a_0, a_1, a_2, a_3$ of cubic expression $\hat{y}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$, such that the *square error*

$\varepsilon = \sum_{i=1}^{m} \left|\hat{y}(x_i) - y_i\right|^2 w_i$ is *minimized.*

**Solution via Pseudo Inverse:**

Let $\underline{a} = col\left[a_0\ a_1\ a_2\ a_3\right], [\hat{y}] = col\left[\hat{y}(x_1)\ \hat{y}(x_2)\ \ldots\ \hat{y}(x_m)\right], \underline{y} = col\left[y_1\ y_2\ \ldots\ y_m\right]$ , then

$$\begin{bmatrix}\hat{y}(x_1)\\ \hat{y}(x_2)\\ \ldots\\ \hat{y}(x_m)\end{bmatrix} = \begin{bmatrix}1 & x_1 & x_1^2 & x_1^3\\ 1 & x_2 & x_2^2 & x_2^3\\ & \vdots & & \\ 1 & x_m & x_m^2 & x_m^3\end{bmatrix}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}$$

, or $[\hat{y}] = [x_i^j]\underline{a}$ . Assume $[x_i^j]$ is of full rank, and define its ***pseudo inverse*** by $[x_i^j]^{\ddagger} = [[x_i^j]^{\dagger}[x_i^j]]^{-1}[x_i^j]^{\dagger}$ , then $\underline{a} = [[x_i^j]^{\dagger}[x_i^j]]^{-1}[x_i^j]^{\dagger}\underline{y}$