# Sample Points and Sample Spaces

## Random Experiments and Outcomes:

Define an experiment and specify all its possible outcomes.

## Sample Space:

The mutually exclusive and exhaustive set of all outcomes of a random experiment.

### Examples of Experiments, Outcomes, and Sample Spaces:

**TABLE 1.**

| Experiment | Sample Space |
| --- | --- |
| Toss Coin | $S = \{H, T\}$ or $S = \{H, T, E\}$ |
| Toss Coin Twice | $S = \{(H, H), (H, T), (T, H), (T, T)\}$ |
| Toss Coin until Head Comes Up | $S = \{H, (T, H), (T, T, H), \ldots, (T, T, \ldots, T, H)\}$ or |
| | $S = \{1, 2, 3, \ldots\}$, the number of times until head is recorded |
| Toss Die | $S = \{1, 2, 3, 4, 5, 6\}$ |
| Sample Fraction of CO in Air Sample | $S = \{x: 0 \le x \le 1\}$ |
| Measure Ampl. and Phase of Sin. Wave | $S = \{(a, \phi): 0 \le a < \infty, 0 \le \phi < 2\pi\}$ |

# Events:

An event is a subset of a sample space.

**Examples:**

Ex. 1: $S = \{1, 2, 3, 4, 5, 6\}$. Events: $A = \{2\}$, elementary event or sample point, $B = \{2, 3, 4\}$, $C = \{2, 4, 6\}$.

*Set Notation:* $s \in S$, $s \in B$, $B \subset S$. If $X \subset S$ and $s \subset S$, then we say $X$ *occurred* if $s \subset X$.

Trivial events: Certain event $A = S$, Impossible event $A = \{\varnothing\}$ = empty set.

Ex. 2: Toss pair of dice. $S = \{(1, 1), (1, 2), \ldots, (6, 6)\}$. Events: $A_2 = \{(1, 1)\}$, $A_3 = \{(1, 2), (2, 1)\}$, $A_n =$ sample points or outcomes that add to n each, $A_{12} = \{(6, 6)\}$.

Ex. 3: Take samples of water and count Ecoli cells. $S = \{0, 1, 2, 3, \ldots\}$, $A = \{n: n \geq 10\}$

Ex. 4: Measure amplitude and phase of sin. wave. $S = \{(a, \phi):0 \leq a < \infty , 0 \leq \phi < 2\pi\}$.

# Algebra of Sets (Boolean Algebra)

$S$ = sample space (universal set).

$A \subset S$ $A$ is a subset of $S$.

$x \in S$ $x$ is a member of $S$.

$x \in A$ $x$ is a member of $A$.

$A \subset B$ $A$ is a subset of $B$. If $A$ occurs then $B$ occurs. An event in $A$ is also an event in $B$.

*Set Union:* $A \cup B$ *or* $A + B$.

*Set Intersection:* $A \cap B$ *or* $AB$ .

*Empty Set: If A and B have no common element (disjoint) then* $A \cap B = \varnothing$.

*Compliment: If* $A \subset S$, *then* $A^c = \{x : x \notin A\}$ . $A^c$ *is the event that* $A$ did not occur.

$A \cup A^c = S$

$A \cap A^c = \varnothing$

$S^c = \varnothing$ and $\varnothing^c = S$

$A^{cc} = A$

*De Morgan's Rules:*

A. $(A \cup B)^c = A^c \cap B^c$

B. $(A \cap B)^c = A^c \cup B^c$

Ex: $(A^c \cup B)^c = A \cap B^c$

*C. Principle of Duality:*

If the following symbols are interchanged in an expression, provided equality and complementation are left unchanged, then a new correct, *dual* expression results.

$\cup \Leftrightarrow \cap$

$\subset \Leftrightarrow \supset$

$S \Leftrightarrow \varnothing$.

# Partitioning Sample Space

Divide $S$ into a collection of mutually exclusive and exhaustive subsets:

$A_1 \cup A_2 \cup A_3 \ldots \cup A_N = S$, $A_i \cap A_j = \varnothing$, $i \neq j$