Probabilistic Modeling of Academic Dishonesty via Markov Chains

Adly T. Fam  
Department of Electrical Engineering  
The State University of New York at Buffalo

Indranil Sarkar  
Department of Electrical Engineering  
The State University of New York at Buffalo

and

Khaled A. Almuhareb  
Department of Learning and Instructions  
The State University of New York at Buffalo

Abstract

Academic dishonesty is modeled via Markov chains. The case of student behavior in class assignments, quizzes and exams is analyzed in modeling examples with various levels of surveillance. The choice of modeling based on surveillance and sanctions is motivated by the research literature on the deterrence theory analysis of cheating. In addition, it is shown how surveillance and sanctions could be controlled to achieve the desired degree of intervention with the least intrusion. This could also be used to formulate optimal university policies regarding academic dishonesty.

Keywords: Academic dishonesty, Markov chains.
I. Introduction

The body of research attempting to estimate the extent of academic dishonesty among college students has produced widely varying results. Karlins [1] et. al. found that only three percent of college students engage in the act of academic dishonesty whereas Gardner [2] et. al. reported a whopping ninety-eight percent. According to McCabe and Trevino [3], this apparent disagreement in the literature on the prevalence of these incidents can be mainly attributed to the differences in the definitions of academic dishonesty, data collection methods and interpretations adopted by different authors investigating the phenomenon. Robinson [4] et. al. defined cheating as: “[the] intentional use or attempted use of unauthorized materials, information or study aids in any work submitted for academic credit.” In light of this definition, it can be argued that there is a striking evidence of a large percentage of college students actually engaging in cheating. Regardless of the type or seriousness of the cheating behavior, there is a consensus that cheating appears to be inherent to the college experience [5].

The motivation for writing this paper arose while one of the authors was teaching a junior level class on probability at a leading research university. There seemed to be a growing evidence of duplication and cheating in both the homeworks and quizzes conducted as a part of the course. There was a strong need to bring this subject up in some form to alert the students to the negative consequences of such behavior on both the professional and personal levels as well as to remind them of the university policies in this regard.
After considerable deliberation, it was decided to use the subject of the course itself to analyze the consequences of cheating and in the process, convey the moral and ethical messages to the students.

As it turned out, the resulting analysis proved to be very enlightening and could be of value in evaluating school policies that deal with cheating and ethics. This analysis could also be used to help formulate such policies. By presenting this material as a part of the course in probability, it was very well received by the students and had a very good impact.

In section II of this paper, we present a theoretical background supporting our design of a probabilistic model to represent and analyze the cheating problem. In section III, we present a simple version of the probabilistic model. Even though the model is very simplistic, it offers useful insights. In section IV, we present a more complex model using Markov chains to represent the cheating behavior. It is shown how the results can be interpreted and used to formulate policies to deal with the problem of academic dishonesty. Proposals for further research conclude the paper.

II. Theoretical support for a surveillance and sanctions-based model

The research literature on the causes of academic dishonesty could be classified into two main types. One type of research is focused on individual/personal characteristics of offending students, while the other concentrates on the effect of contextual/situational factors [3]. Individual/personal variables that were typically investigated include gender [6,7], age [8,9],
student GPA [7,10], race/social class [11,12], field of study [3], and personality type [13]. See the paper by Crown & Spiller [14] for a detailed review of all of the above.

Studies of such individual predictors of cheating have rendered mixed and conflicting findings. For example, while some studies have found males to engage in acts of academic dishonesty significantly more than females [6], many have found no significant effect of gender [15]. Yet, one study [7] found that females are significantly more likely to engage in cheating than males. Similarly, while many studies suggest that younger students are more likely to commit acts of academic cheating, some studies have shown that age is not a significant predictor of academic dishonesty [8,9]. Research on demographic background has consistently found no differences in cheating practices based on race or social class [11,12,16].

Challenging the premise that cheating behavior is predisposed by individual student characteristics, the contextual/situational research suggests that certain social contexts inspire or reduce the occurrence of cheating [17]. Crown and Spiller [14] discuss surveillance, honor codes, sanctions, and value counseling as the main situational factors studied in the academic dishonesty literature. One focus that appears to be common to many of the different studies in the contextual/situational research relates to students perceptions of the effectiveness of the cheating countermeasures in place. The effectiveness of those measures obviously pertains to the student perceived chance of being caught cheating. Student perception about being caught was shown by previous research to be one of the most important determinants of the decision to cheat [18]. In an early study, Tittle and Rowe [19] went as far as concluding that cheating could only be reduced by a credible threat of being caught and punished.
As the chance of being caught cheating is logically linked to the amount of surveillance present in a situation, surveillance as a variable was found to have a strong effect on cheating behavior. Surveillance was examined as a situational variable in many studies and was operationalized in different ways, including opportunity to cheat [20], student-proctor ratio [21], chance of success [3], and high risk versus low risk situations [22]. All of these studies have found significant results supporting an inverse relationship between surveillance and cheating.

Students’ perceived chance of being caught and penalized for cheating was similarly inversely correlated with cheating in honor code settings. It is argued that the effectiveness of honor code schemes in reducing academic dishonesty is actually dependent upon the likelihood that another student would report the misconduct. The increased likelihood of reporting, in turn, creates a perceived stronger chance of being caught, and thus reduces cheating [18,20]. McCabe and Trevino [3] comment that because academic dishonesty may often be concealed from faculty members, peer reporting could play an important role in shaping students’ perception about the certainty of being caught in acts of academic dishonesty. In this sense, peer reporting, which is part of explicit honor codes in many universities, could be viewed as another form of surveillance.

The contextual research in academic dishonesty showed little effect of value counseling on reducing collegiate cheating (see [14] for a review). This and the research results showing the significant role of students’ perception of the probability of being caught in shaping the decision to cheat, are supportive of the deterrence theory explanation of cheating as a deviant behavior
The theory suggests that cheating occurs within the boundaries of expected costs and benefits. As such, individuals would be less likely to cheat because of their expectation of negative consequences [23,24]. Studies that used this theory suggested that the opportunities to cheat and the fear of external sanctions were the most significant factors in reducing cheating.

Building upon the research literature presented above, and the deterrence theory explanation of cheating behavior, we argue that the factors of surveillance and threat of sanctions are the most reliable in analyzing college students’ cheating behavior. We therefore elected to build our probabilistic model based on the deterrence model taking into account the factors of surveillance and threat of sanctions. The choice to use this deterrence model was also driven by a perception that contextual factors such as surveillance and sanctions, unlike individual/personal predispositions for cheating, are open to administrative influence. Controlling these factors could offer the faculty and administrators a means to effectively respond to the problem of academic dishonesty.

Despite the apparent theoretical support for the use of surveillance to deter deviant behavior, in an academic setting, deterrent benefits of surveillance must be weighted against the possible unfavorable effects on the academic atmosphere. Unnecessarily increased surveillance can create a sense of diminished trust and loss of privacy [14]. The proposed model, as will be shown later in this paper, can be applied to accurately estimate the optimum amount of intervention to intercept acts of cheating. It may therefore provide to be a viable tool for achieving the desired control over cheating with minimal intrusion.
III. A simple probabilistic model

In this section, we develop a simple model to calculate the probability of getting caught after a given number of cheating incidents. A cheating incident here could mean cheating in a quiz in a particular class. The analysis then would reflect the probability of getting caught after cheating incidents in quizzes only in this particular class. On the other hand, the incidents could be counted based on cheating activities in all classes that a particular student is taking and in all types of activities such as homeworks, quizzes etc. The counting of the incidents could also be somewhere in between the above extremes.

Let $P_c(n)$ = probability of getting caught at least once in $n$ cheating incidents. $P_c(1)$ denote the probability of being caught in any given incident. This probability depends on the degree of surveillance in the given environment. Henceforth, we will denote this probability as just $P_c$.

One of the results of the analysis is to determine the minimum needed $P_c$ to achieve a desired effective control on cheating without being too intrusive. An unnecessarily large $P_c$ could interfere with the healthy academic atmosphere in the class without necessarily leading to significant increase in control over cheating, as will be evident from the following analysis.

We assume that the probability of getting caught in a given activity (e.g. a quiz or homework) is independent of the probability of getting caught in any other activity. Therefore, the probability of getting caught at least once in $n$ incidents is given by:
This is plotted in figure 1 for various values of $P_c$ for a range of $n$ up to 40. It is interesting to note that each of the curves has a knee before which it rises steeply and after which the rise is more gradual. This represents the point after which we have a region of diminishing returns. This observation could be used to formulate a policy that is not unnecessarily intrusive. The procedure is explained next with the help of figure 2. The increment in $P_c(n)$ as $n$ is increased to the next value of $n+1$ is represented by:

$$\Delta P_c(n) = P_c(n+1) - P_c(n)$$

(2)

Using (1) to substitute for both $P_c(n+1)$ and $P_c(n)$, we have:
From (3), we find that the ratio of $\Delta P_c(n)$ and $\Delta P_c(n+1)$ is given by:

$$\frac{\Delta P_c(n)}{\Delta P_c(n+1)} = \frac{1}{1 - P_c} > 1$$

(5)

where it is assumed that $P_c > 0$. We observe that since this ratio is always greater than 1, $\Delta P_c(n)$ decreases monotonically with $n$ and the largest value is at $n=0$. This largest value is denoted as:

$$\Delta P_c(0) = P_c$$

(6)

Let us consider the steep part of the piece-wise linear curve to be up to the point $\hat{n}$ such that:

$$\Delta P_c(\hat{n}) = \alpha P_c(0) = \alpha P_c$$

(7)

where $0 < \alpha < 1$. Therefore, $\hat{n}$ is the point when the rate of increment becomes less than a certain fraction of the initial rate of increment given by (6). This implies that:

$$\Delta P_c(\hat{n}) = \alpha P_c = P_c(1 - P_c(\hat{n}))$$

(8)

$$\Rightarrow \alpha = 1 - P_c(\hat{n})$$

(9)

$$\Rightarrow P_c(\hat{n}) = 1 - \alpha$$

(10)

It is of particular interest to note that neither $\alpha$ nor $P_c(\hat{n})$ depend on $P_c$. Also, $P_c(\hat{n})$ denotes the probability that the cheating students would be caught at least once in $\hat{n}$ attempts. If the school policy decides to catch the cheating students with a certain probability, this is taken as $P_c(\hat{n})$ and $\alpha$ is found using (9). The required number of surveillance occasions $\hat{n}$ is then calculated as:

$$\hat{n} = \frac{\log(\alpha)}{\log(1 - P_c)}$$

(11)
The \( \hat{n} \) found in this way will give us the number of surveillance occasions required to catch the cheating students for the given value of \( P_c \). On the other hand \( P_c(\hat{n}) \) will be the probability with which the cheating students would be intercepted. It is intuitive to note from figure (1) that higher the value of \( P_c \), the lesser will be the number of surveillance occasions required.

The \( \hat{n} \) surveillance occasions can be distributed as desired. We recommend the surveillance to be done as a random selection of \( \hat{n} \) occasions thus achieving the desired probability of interception with minimal intrusion. This is an example of how such analysis could lead to policy recommendations.

![Figure 2: Determining \( \hat{n} \)](image)

### IV. Modeling via Markov chains

The previous section is useful as a first look at the utilization of probability analysis to model academic dishonesty. However, it is too simplistic. It does not provide for modeling the change of behavior of a student who has been caught cheating once. Such a student might, for example,
be more cautious and cheat less frequently after such an event. It also does not accommodate the school policy that the student might get an $F$ in the quiz or even the course if caught cheating a certain number of times. Markov chains have been used in behavioral modeling in many areas. In this work, we use Markov chains to analyze the phenomenon of cheating.

In this section, we propose a Markov chain model for the analysis of cheating. The different states of the Markov model are defined as follows.

i) $F$: The state where the student receives a failing grade.

ii) $ce$: The state where the student cheats and gets caught for the first time.

iii) $cnc_1$: The state where the student cheats and does not get caught.

iv) $nc_1$: The state where the student does not cheat initially.

v) $cnc_2$: The state where the student cheats but does not get caught after having been caught once.

vi) $nc_2$: The state where the student does not cheat after having been caught once.

The probabilities associated with these states are defined as $P_F$, $P_{ce}$, $P_{cnc_1}$, $P_{nc_1}$, $P_{cnc_2}$, and $P_{nc_2}$, respectively. The initial values that these probabilities should ideally be determined from statistical studies via questionnaires and other appropriate data collection methods. Of course, these values will be a function of the school concerned, the department of interest and even the course in question. They will also be affected by the school policy on cheating. They could also differ from year to year based on the student population of interest.
In the illustrative examples to follow, these probabilities are assigned hypothetical but judiciously chosen values just to illustrate how the model functions. Of course, after obtaining the actual values for a particular situation, the analysis has to be repeated.

We consider two different representative scenarios. In both cases we assume that the amount of cheating prevalent within the student population is low. In the first case it is assumed that the surveillance is loose while in the second case we assume the surveillance to be strict. In both the cases, the school policy is to give a zero score in the quiz or homework in which the student is caught cheating the first time and an \( F \) in the course if he is caught cheating a second time. This policy is the one currently used in the probability course mentioned earlier.

![Figure 3: State diagram for the Markov model. (Loose surveillance and low cheating)](image)
A. Loose surveillance and low cheating

In this section we choose the initial probabilities of the different states and the transitional probabilities assuming that the surveillance policies are loose in a student population where the amount of cheating is low. The initial probability vector is chosen as:

\[
P_{\text{init}} = \begin{bmatrix}
P_{cc}(0) \\ P_{nc1}(0) \\ P_{cnc1}(0) \\ P_{nc}(0) \\ P_{cnc2}(0) \\ P_{nc2}(0)
\end{bmatrix} = \begin{bmatrix}
0.01 \\ 0.9 \\ 0.09 \\ 0 \\ 0 \\ 0
\end{bmatrix}
\]  

(12)

The initial probabilities for the states \(F, cnc_2\) and \(nc_2\) are taken as zeros because a student can only enter these states under certain conditions and initially these states are empty. Due to the low amount of cheating considered, 90 percent of students are assumed not to cheat which is reflected in \(P_{nc1}\) being equal to 0.9. Of the remaining 10 percent, 90 percent get away with the cheating because of the loose surveillance policy. This condition manifests itself in the values chosen for \(P_{cc}\) and \(P_{cnc1}\).

The transitional probability matrix is then constructed for this case from the given state diagram in fig 3.

\[
P_{\text{trans}} = \begin{bmatrix}
0 & 0.01 & 0.09 & 0 & 0 & 0 \\ 0 & 0.9 & 0.05 & 0 & 0 & 0 \\ 0 & 0.09 & 0.86 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 1 & 0.1 & 0.05 \\ 0.6 & 0 & 0 & 0.7 & 0.25 \\ 0.3 & 0 & 0 & 0.2 & 0.7
\end{bmatrix}
\]  

(13)

We define as an event every time a quiz or homework is set, thus presenting an opportunity for transition between the states of the Markov model. The probabilities of the different states after \(n\) events are thus given by:
\[ P_n = P_{\text{init}}^T (P_{\text{tran}})^n \]  

(14)

where \( \{.\}^T \) denotes the transpose operator.

Next, we plot the variation of the probabilities of the different states as a function of \( n \). This is shown in fig. 4.

![Figure 4: Probabilities of different states as a function of \( n \) in presence of loose surveillance.](image)

B. Strict surveillance and low cheating

Now we consider the scenario when a strict surveillance policy is present to prevent cheating in the quizzes and homeworks. This could mean a much stricter vigilance during the quizzes and more careful scrutiny of the homeworks to find evidence of copying. In this case, both the initial probability vector and the transitional probability matrix are modified to reflect the change in policy. The initial probability vector for this case is given by:
Note that due to the higher level of surveillance, the initial probability that a student does not cheat is taken as 0.95. Of the remaining 5 percent that cheat, only 1 percent is assumed to get away with it and 4 percent are intercepted. The transitional probabilities also change due to the higher level of surveillance and is given by:

\[
P_{init} = \begin{bmatrix}
P_{cc}(0) \\
P_{nc1}(0) \\
P_{cnc1}(0) \\
P_{F}(0) \\
P_{cnc2}(0) \\
P_{nc2}(0)
\end{bmatrix} = \begin{bmatrix}
0.04 \\
0.95 \\
0.01 \\
0 \\
0 \\
0
\end{bmatrix}
\] (15)

Figure 5: State diagram for the Markov model. (Strict surveillance and low cheating)
The new Markov chain diagram for this case is shown in figure 5. The probability of the different states are given by (14) as in the previous example. These probabilities are plotted in fig. 6 as a function of $n$.

C. Discussion

From the figures 4 and 6, we note a few important and interesting points.

- The curve for $P_{\text{cne1}}$ rises in the case of loose surveillance to a value of around 0.3087 at about 8 events before beginning to drop. On the other hand, in the strict surveillance case, the peak is much smaller at about 0.01 and decreases monotonically. This is intuitive because the probability of cheating and not getting caught would obviously be lower when the surveillance is strict.

- The curve for $P_{cc}$ reaches a low peak of about 0.032 in figure 4, while it drops monotonically in figure 6. This is because more and more of the cheaters are intercepted and removed from the class in the more strict surveillance scheme.
Figure 4: Probabilities of different states as a function of $n$ in presence of strict surveillance.

- The probability of cheating and not getting caught after having been caught once, $P_{\text{cnt2}}$ increases to a value of around 0.1397 in figure 4. The corresponding value in figure 6 is seen to be only around 0.0525. This is because students are more cautious after having been caught once especially when the surveillance is strict. Also, under this condition, more and more of the cheaters are removed from the class after getting caught a second time. Hence there are fewer students remaining in the class and they happen to be the more honest ones.

- The probability of getting an $F$ increases with a higher slope in figure 6 than in figure 4. Even though the sanctions remain the same in both these cases, a higher level of surveillance ensures faster intervention thus producing a higher slope for the curve.

- The probability of not cheating is found to increase much faster in figure 6 as compared to figure 4. This corroborates the fact that students are deterred from cheating when a more strict surveillance is in effect.
V. Conclusions and further research

A simple probabilistic model, as well as a more sophisticated model based on Markov chains have been proposed and used to analyze academic dishonesty. Two representative scenarios were examined via Markov chain modeling. The results indicate that this approach could lead to significant advances in analyzing cheating in actual situations. This could be used to judiciously choose the needed level of surveillance required to achieve the desired degree of intervention without being intrusive. It could also be used in formulating school policies on cheating.

The future research should include formulating experiments to gather actual data to determine the initial and transitional probabilities used in the Markov model. Collecting actual data to evaluate the actual probabilities is an important step in applying this work to actual situations. Hence, the data collection methods should be both reliable and efficient. In fact, Markov chains have also been used very recently [25] to make the data collection process foolproof and dependable. This technique, based on randomized item response theory, could be used in tandem to the analysis introduced in this paper. The Markov chain modeling could also be made more sophisticated as needed. This modeling approach could also be applied to other investigations such as predicting the level of student retention in a department or school.

References


Biographies

Dr. Adly T. Fam is a Professor in the Department of Electrical Engineering, The State University of New York at Buffalo. He received the BSEE degree from Cairo University, Egypt in 1968 and MS and PhD degrees in electrical engineering from the University of California, Irvine, in 1975 and 1977 respectively. He has over 85 journal and conference publications in the areas of system theory, digital control, geometry of polynomials in their coefficient space, digital signal processing, non-linear filters, and arithmetic intensive and high throughput computing. Dr. Fam is a member of the ASEE, a senior member of the IEEE, with affiliation to several societies. He received the Hughes Aircraft Company Division Invention Award in 1984.

Address: 332, Bonner Hall, The State University of New York at Buffalo, Buffalo, NY-14260.
Telephone: 716-645-2422; e-mail: afam@eng.buffalo.edu

Indranil Sarkar is a doctoral candidate at the Department of Electrical Engineering, The State University of New York at Buffalo. He received a Bachelor of Engineering degree from the
Visveswaraiah Technological University, India and a MS degree from the State University of New York at Buffalo in 2002 and 2004, respectively.

Address: 332, Bonner Hall, The State University of New York at Buffalo, Buffalo, NY-14260.
Telephone: 716-645-2422; e-mail: isarkar@eng.buffalo.edu

Khaled Almuhareb is a doctoral candidate at the department of Learning and Instructions, The State University of New York at Buffalo. He has taught different ESP courses to undergraduate engineering students at Kuwait University for seven years. He has also chaired the English Language Unit of the College of Engineering, Kuwait University for three years. His current research focuses on theories of learning and behavior as they pertain to various educational settings.

Address: 332, Bonner Hall, The State University of New York at Buffalo, Buffalo, NY-14260.
Telephone: 716-645-2422; e-mail: khaledaa@buffalo.edu