Problem Set 11, PS11 due Monday June 14

PS11-1 Air is stored in a 10,000 ft^3 salt dome at 500 kPa and 400 K is used at times of high electric demand to drive a turbine generator. For turbine exit conditions of 100 kPa and 300 K determine the amount of work done by the turbine during the period when the salt dome pressure drops from 500 kPa to 300 kPa. Consider both the turbine and cave to be adiabatic.

PS11-2 Waste heat is rejected at 80 F from a Carnot heat engine which receives 700 Btu/min of heat at a temperature of 1700 F. A refrigerator which receives heat at 20 F and rejects heat at 80 F uses the entire output of the heat engine. Determine a) the maximum rate at which heat is absorbed by the refrigeration machine and b) the total amount of heat rejected at 80 F.

PS11-3 .96 kg of R-134a is used to execute a Caront refrigeration cycle in the liquid-vapor region. The maximum absolute temperature of the cycle is 1.2 times the minimum absolute temperature. The net work input to the cycle is 22 kJ . During the heat rejection process the refrigerant changes from a saturated vapor to a saturated liquid. Determine the minimum pressure in the cycle.

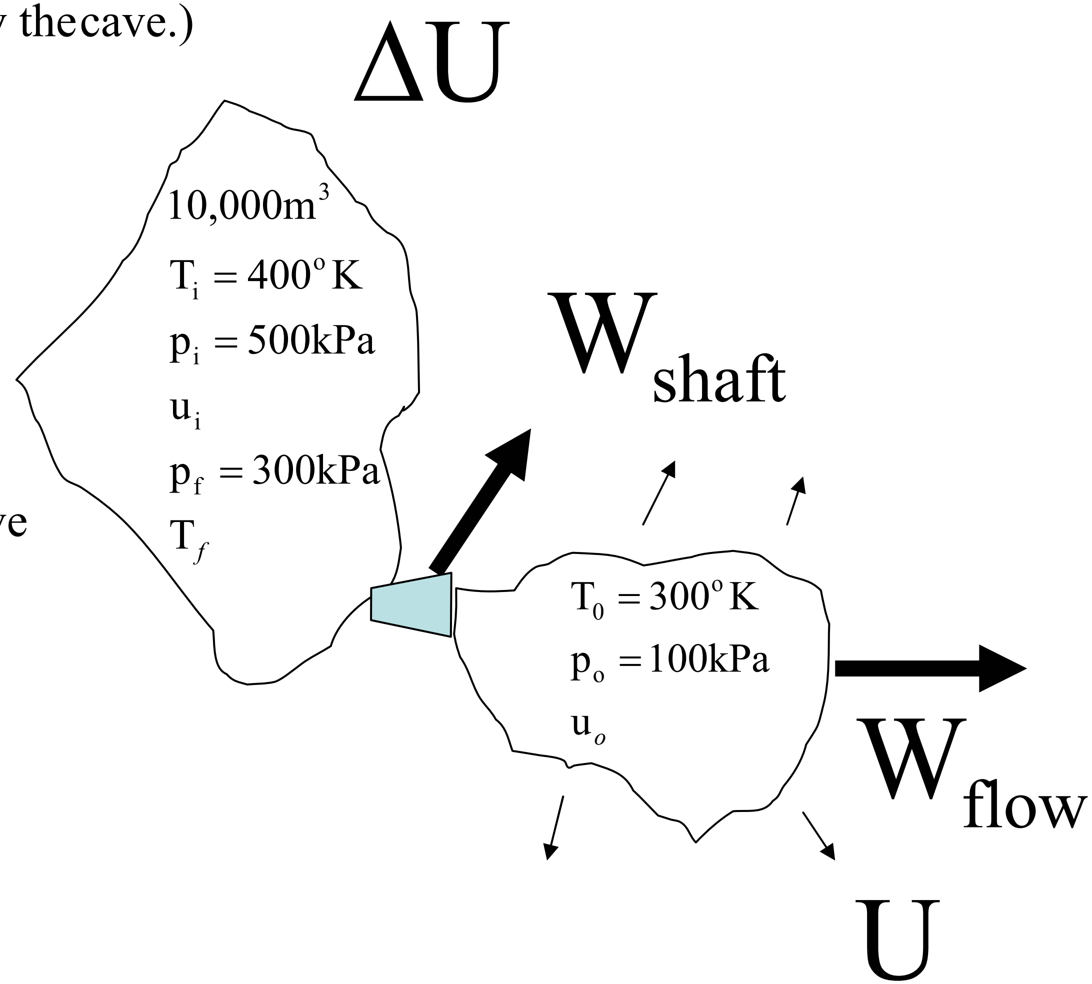
PS11-4 The efficiency of a heat engine increases as the rejection temperature decreases. It is suggested that the rejection temperature of the heat engine be reduced with a refrigeration machine driven with the additional power produced by the more efficient heat engine Is this a good idea? Support your conclusion with quantative analysis.

PS11-1

Unsteady System:

mass originally in cave.

(alternate: the region space occupied by the cave.)



Assume the cave is adiabatic.

$$pv^k = \text{constant}$$

$$T_f = T_i\left(\frac{p_f}{p_i}\right)^{\frac{k-1}{k}}$$

$$m = \frac{pV}{RT} \text{ for initial and final mass in the cave}$$

$$W_{flow} = \int pdv = p_o \Delta V = +(m_f - m_i)p_o v_o$$

$$Q = E_f - E_i + W_{flow} + W_{shaft}$$

$$E_i = m_i u_i$$

$$E_f = m_f u_f + (m_f - m_i)u_o$$

$$Q = E_f - E_i + W_{flow} + W_{shaft}$$

$$Q = m_f u_f + (m_f - m_i)u_o - m_i u_i + W_{flow} + W_{shaft}$$

$$0 = \underbrace{[m_f u_f - m_i u_i]}_{\Delta U_{cave}} + \underbrace{[(m_f - m_i)u_o]}_{U_{exit\ gas}} + \underbrace{[(m_f - m_i)p_o v_o]}_{W_{flow}} + \underbrace{[W_{shaft}]}_{W_{shaft}}$$

PS11-1

$$m_i = \frac{500\ \text{kPa} \times 10{,}000\ \text{m}^3}{.287\ \text{kPa m}^3/\text{kg}^{O}\text{K} \times 400^{O}\ \text{K}} = 43{,}554\ \text{kg}$$

$$T_f = T_i\left(\frac{p_f}{p_i}\right)^{\frac{k-1}{k}} = 400\left(\frac{300\ \text{kPa}}{500\ \text{kPa}}\right)^{.2857} = 345.7^{O}\ \text{K}$$

$$m_f = \frac{300\ \text{kPa} \times 10{,}000\ \text{m}^3}{.287\ \text{kPa m}^3/\ \text{kg}^{O}\text{K} \times 345.7^{O}\ \text{K}} = 30.238.8\ \text{kg}$$

$$(m_i - m_f) = 13{,}315\ \text{kg}$$

$$W_{flow} = \int p dv = p_o(V - V) = p_o\left(\frac{(m_i - m_f)RT_o}{p_o}\right)$$

$$W_{flow} = 13{,}315\ \text{kg} \times .287\ \text{kPa m}^3/\text{kg}^{O}\text{K} \times 300^{o}\ \text{K}$$

$$W_{flow} = 1{,}146{,}435\ \text{kJ}$$

$$u_i = u\ @\ 400^{o}\ \text{K} = .718 \times 400^{o}\ \text{K} = 287.2\ \text{kJ/kg}$$

$$u_f = u\ @\ 345.7^{o}\ \text{K} = .718 \times 345.7^{o}\ \text{K} = 248.2\ \text{kJ/kg}$$

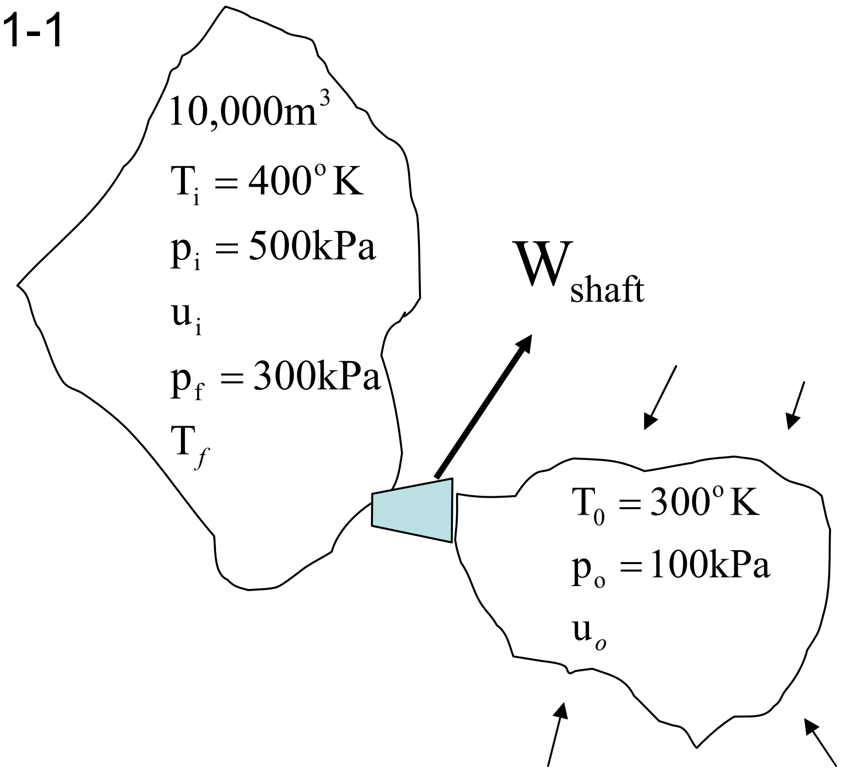
$$u_o = u\ @\ 300^{o}\ \text{K} = .718 \times 300^{o}\ \text{K} = 215.4\ \text{kJ/kg}$$

$$Q = E_f - E_i + W_{flow} + W_{shaft}$$

$$E_i = m_i u_i$$

$$E_f = m_f u_f + (m_f - m_i)u_o$$

$$0 = m_f u_f + (m_f - m_i)u_o - m_i u_i + W_{flow} + W_{shaft}$$



$$W_{shaft} = m_i u_i - m_f u_f + (m_f - m_i)u_o - W_{flow}$$

$$W_{shaft} = 43{,}554\ \text{kg} \times 287.2\ \text{kJ/kg}$$

$$-30{,}238.8\ \text{kg} \times 248.2\ \text{kJ/kg}$$

$$-13315.\ \text{kg} \times 215.4\ \text{kJ/kg}$$

$$-1{,}146{,}435\ \text{kJ}$$

$$W_{shaft} = 988{,}952\text{kJ}$$

PS11-2

$$\eta = \frac{W}{Q_{in}} = \frac{T_H - T_L}{T_H} = \frac{1620}{2160} = .75$$

$$W = Q_{in} \times \eta = 700 \times .75 = 525 \text{ BTU}$$

$$Q_{out} = Q_{in} - W = 700 - 525 = 175 \text{ BTU}$$

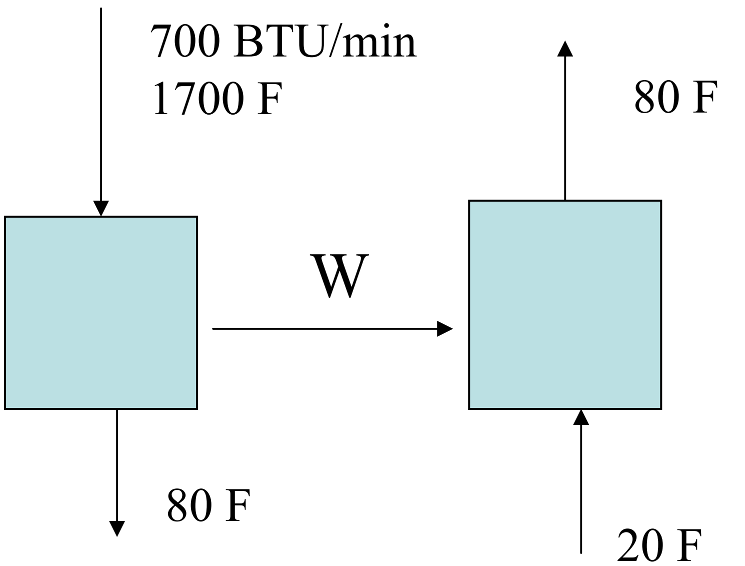
$$COP_{ref} = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{460 + 20}{80 - 20} = 8.$$

a) $Q_L = 8 \times W = 8 \times 525 = 4200 \text{ BTU/min}$

b) $Q_H = Q_L + W = 4725 \text{ BTU}$

$\text{Total rejected} = \text{enigne } Q_{out} + \text{heat pump } Q_H$

$\text{Total rejected} = 175 + 4725 = 4900 \text{ BTU}$



PS11-3

$$COP_{ref} = \frac{Q_{in}}{W} = \frac{T_L}{T_H - T_L}$$

$$COP_{ref} = \frac{T_L}{1.2\,T_L - T_L} = 5$$

$$Q_{in} = 5 \times W = 5 \times 22 = 110\,kJ$$

$$Q_{out} = Q_{in} + W = 110 + 22 = 132\,kJ$$

$$\text{closed system} \Rightarrow Q_{out} = \Delta U + W$$

$$Q_{out} = m_2 u_2 - m_1 u_1 + p_2 (V_2 - V_1)$$

$$\text{since } p_2 = p_1$$

$$Q_{out} = m_2 u_2 - m_1 u_1 + (m_2 p_2 v_2 - m_1 p_1 v_1)$$

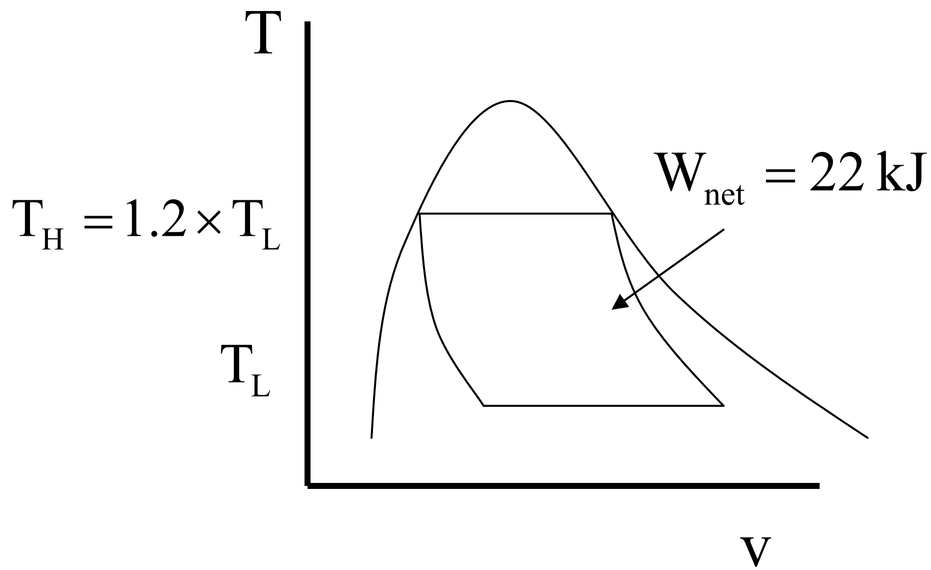
$$Q_{out} = m_2 h_2 - m_1 h_1 = m h_{fg}$$

$$h_{fg} = \frac{Q_{out}}{m} = \frac{132}{.96\,kg} = 137.5 \;\; kJ/kg$$

$$@h_{fg} = 137.5 \;\; kJ/kg, \quad T_h = 61^o C = 334^o K$$

$$T_L = T_h / 1.2 = 267.3^o K = 5.3^o C$$

$$p = p_{sat} @ 5.2^o C = .354 \;\; MPa$$



PS11-4

$$\eta = \frac{W_{E2}}{Q_{in}} = \frac{T_2 - T_1}{T_2}$$

$$W_{E1} = Q_2\left(\frac{T_2 - T_1}{T_2}\right)$$

$$COP_{HP} = \frac{Q_o}{W_{HP}} = \frac{T_2}{T_2 - T_3}$$

$$W_{HP} = Q_o\left(\frac{T_2 - T_1}{T_2}\right)$$

for $W_{E1} = W_{hp}$

$$Q_2\left(\frac{T_2 - T_1}{T_2}\right) = Q_o\left(\frac{T_2 - T_1}{T_2}\right)$$

$Q_2 = Q_o$ and there is no bebefit with reversible equipment and a loss with real irreversible equipment