SIMPLE RANKINE CYCLE

Steady Flow, Open System - region in space

Steady Flow Energy Equation for Processes

\[ Q = m \times \Delta(u + pv + \frac{V^2}{2} + \rho gh) + W_{\text{shaft}} \]

Pump Process, \(1 \Rightarrow 2\), \(Q = 0\), \(W_{\text{in}} = m(h_2 - h_1)\)

Boiler Process, \(2 \Rightarrow 3\), \(W = 0\), \(Q_{\text{in}} = m(h_3 - h_2)\)

Expansion Process, \(3 \Rightarrow 4\), \(Q = 0\), \(W_{\text{out}} = m(h_3 - h_4)\)

Condenser Process, \(4 \Rightarrow 1\), \(W = 0\), \(Q_{\text{out}} = m(h_4 - h_1)\)

First Law for Cycles

\[ \int_{\text{cycle}} \delta Q = \int_{\text{cycle}} \delta W \]

\[ \sum_{\text{cycle}} Q = \sum_{\text{cycle}} W \]

\[ \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{\sum W}{Q_{\text{in}}} \]
FIRST LAW
Closed System $Q = \Delta E + W$

Open System $Q = m\Delta(u + pv + \frac{V^2}{2} + gz) + W$

Unsteady System
$Q = m_2u_2 - m_1u_1 - (m_2m_1)h_0 + W_{\text{boundary}}$

PROPERTIES
Ideal Gas Model
$pV = RT$
$pVmRT$

Isentropic Process
\[
\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)
\]
CARNOT CYCLE WITH WATER

\[ \eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} \]

\[ Q_{\text{in}} = T_H \Delta S \]

\[ W_{\text{net}} = \eta \times Q_{\text{in}} \]

\[ W_{\text{net}} \text{ maximum at: } \]
\[ T = 240 \, \text{C} \]

\[ \eta_{\text{Carnot}} = 41.9 \% \]

\[ W_{\text{net}} = 740. \text{kJ/kg} \]

maximum area
SUPERHEAT RANKINE CYCLE

REHEAT RANKINE CYCLE
Q = ΔU + W  \text{ First Law}

dq = du + dw
dq = Tds  \text{ Second Law}
dw = pdv  \text{ Boundary Work}

subsitiuting for dq and dw,

Tds = (du) + (pdv)
h = u + pv  \text{ h property definition,}

\text{h is an exact differential}

dh = du + pdv + vdp

subsitiuting for du,

Tds = (dh − pdv − vdp) + (pdv)

Tds = dh − vdp

for an adiabatic process, Q = Tds = 0

dh = vdp

h = \int vdp

Example: water pumped from 15 psia to 30 psia

w = h_2 − h_1 = v(p_2 − p_1)

w = \frac{(30 \text{ psia} − 15 \text{ psia}) \times 144 \text{ psi/psf}}{62.4 \text{ lb/ft}^3}

w = \frac{2160 \frac{\text{lb}_f}{\text{ft}^2}}{62.4 \frac{\text{lb}_m}{\text{ft}^2 \text{ft}}} \approx 34.6 \frac{\text{ft lb}_f}{\text{lb}_m}, \text{ (ft of fluid)}

w = 34.6 \frac{\text{ft lb}_f}{\text{lb}_m} \times \frac{1 \text{ BTU}}{778 \text{ ft lb}_f} = .044 \text{ BTU/lb}_m

Example: water pumped from 100 kPa to 300 kPa

w = v(p_2 − p_1)

w = .0010432 \frac{\text{m}^3}{\text{kg}} \times (300 \text{ kPa} − 100 \text{ kPa})

w = .2086 \frac{\text{m}^3}{\text{kg} \text{ kPa}}, \text{ kJ/kg}
A steam power plant runs on a reheat cycle and produces 80 MW. The turbine inlet conditions are 10 MPA, 500°C and 1 MPA, 500°C. The condenser operates at 10 kPa. The efficiency of the turbines is 80%. The efficiency of the pump is 95%. Determine:

a) the turbine exit conditions
b) the cycle efficiency and
c) the mass flow rate of the steam.
A steam power plant runs on a reheat cycle and produces 80 MW. The turbine inlet conditions are 10 MPa, 500°C and 1 MPa, 500°C. The condenser operates at 10 kPa. The efficiency of the turbines is 80%. The efficiency of the pump is 95%. Determine: a) the turbine exit conditions, b) the cycle efficiency and c) the mass flow rate of the steam.

<table>
<thead>
<tr>
<th>Pt</th>
<th>T</th>
<th>p</th>
<th>h</th>
<th>s</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 kPa</td>
<td>191.81</td>
<td>.6492</td>
<td>.001010</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10 MPa</td>
<td>6492</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>500 °C</td>
<td>3375.1</td>
<td>6.5995</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1 MPa</td>
<td>6.5995</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>500 °C</td>
<td>3479.1</td>
<td>7.7642</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10 kPa</td>
<td>7.7642</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10 kPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
compressed liquid water, Table A-7, \( p_2 = 10 \text{MPa} \)

<table>
<thead>
<tr>
<th>T</th>
<th>h</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>259.43</td>
<td>.8260</td>
</tr>
<tr>
<td></td>
<td>( s_2 = s_1 = .6492 )</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>176.37</td>
<td>.5685</td>
</tr>
</tbody>
</table>

\( h_2 = 202.39 \text{kJ/kg} \)

\[
W_{\text{pump actual}} = \frac{W_{\text{pump ideal}}}{.95} = \frac{h_2 - h_1}{.95}
\]

\[
W_{\text{pump actual}} = \frac{202.39 - 191.81}{.95} = 11.14 \text{kJ/kg}
\]

\( h_3 = h_1 + W_{\text{pump actual}} = 191.81 + 11.14 \)

\( h_3 = 203.95 \text{kJ/kg} \)

\[
W_{\text{pump}} = \frac{v \times \Delta p}{\eta} = \frac{.001010 \times (10,000 - 10)}{.95}
\]

\( W_{\text{pump}} = 10.71 \text{kJ/kg} \)

@ 1 MPa, \( s_6 = s_5 \)

<table>
<thead>
<tr>
<th>T</th>
<th>h</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>2828.3</td>
<td>6.6956</td>
</tr>
<tr>
<td></td>
<td>( s_6 = s_5 = 6.5995 )</td>
<td></td>
</tr>
<tr>
<td>179.88</td>
<td>2777.1</td>
<td>6.5850</td>
</tr>
</tbody>
</table>

\( h_6 = 2784.11 \text{J/kg} \)

\[
\eta = \frac{h_5 - h_7}{h_5 - h_6}
\]

\[
h_7 = h_5 - .8 \times (h_5 - h_6)
\]

\[
h_7 = 3373.7 - .8 \times (3373.7 - 2784.11)
\]

\( h_7 = 2902.03 \text{kJ/kg} \)

@ 10 kPa, \( s_9 = s_8 \)

\[
x = \frac{s_8 - s_f}{s_{fg}} = \frac{7.7642 - .6492}{7.4996} = .9487
\]

\[
h_9 = 191.81 + x \times 2392.8 = 2460.86 \text{kJ/kg}
\]

\[
\eta = \frac{h_8 - h_{10}}{h_8 - h_9} = .8
\]

\[
h_{10} = h_8 - .8 \times (h_8 - h_9)
\]

\[
h_{10} = 3479.1 - .8 \times (3479.1 - 2460.86)
\]

\( h_{10} = 2664.51 \text{kJ/kg} \quad (h_{\text{sat}} = 2519.8, \approx 90^\circ \text{C}) \)

9-29 ex
\[ Q_{\text{in}} = (h_8 - h_7) + (h_5 - h_3) \]
\[ Q_{\text{in}} = (3479.1 - 2902.03) + (3375.1 - 203.95) \]
\[ Q_{\text{in}} = 3748.22 \text{ kJ/kg} \]
\[ Q_{\text{out}} = (h_{10} - h_1) = (2664.51 - 191.81) = 2472.7 \text{ kJ/kg} \]
\[ W = (h_5 - h_7) + (h_8 - h_{10}) - W_{\text{pump}} \]
\[ W = (3375.1 - 2902.03) + (3479.1 - 2664.51) - 11.28 \]
\[ W = 472.74 + 814.11 - 11.28 = 1276.38 \text{ kJ/kg} \]
\[ \int dQ = \int dW \]
\[ Q_{\text{in}} - Q_{\text{out}} = W_{\text{net}} \]
\[ 3748.22 - 2472.7 = 1275.52 \]
\[ \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{1276.38}{3748.22} = 34.05\% \]
\[ m = \frac{\text{Total Work}}{\text{Specific Work}} = \frac{80,000\text{kJ/sec}}{1275.527} = 62.72 \text{ kg/sec} \]
EES Model

\[ p_1 = 10 \]
\[ p_2 = 10000 \]
\[ \text{effp} = 0.95 \]
\[ \text{efft1} = 0.80 \]
\[ \text{efft2} = 0.8 \]
\[ T_5 = 500 \]
\[ p_5 = p_2 \]
\[ p_6 = 1000 \]
\[ p_8 = p_6 \]
\[ p_9 = p_1 \]
\[ p_{10} = p_9 \]
\[ T_8 = 500 \]

\[ h_1 = \text{enthalpy}(\text{STEAM}, p=p_1, x=0) \]
\[ s_1 = \text{entropy}(\text{STEAM}, p=p_1, x=0) \]
\[ h_2 = \text{enthalpy}(\text{STEAM}, p=p_2, s=s_1) \]
\[ h_3 = h_1 + (h_2-h_1)/\text{effp} \]
\[ h_5 = \text{enthalpy}(\text{STEAM}, T=T_5, p=p_5) \]
\[ s_5 = \text{entropy}(\text{STEAM}, T=T_5, p=p_5) \]
\[ h_6 = \text{enthalpy}(\text{STEAM}, p=p_6, s=s_5) \]
\[ h_7 = h_5 - (h_5-h_6)*\text{efft1} \]
\[ h_8 = \text{enthalpy}(\text{STEAM}, p=p_8, T=T_8) \]
\[ s_8 = \text{entropy}(\text{STEAM}, p=p_8, T=T_8) \]
\[ h_9 = \text{enthalpy}(\text{STEAM}, p=p_9, s=s_8) \]
\[ h_{10} = h_8 - (h_8-h_9)*\text{efft2} \]
\[ T_{10} = \text{temperature}(\text{STEAM}, p=p_{10}, h=h_{10}) \]
\[ Q_{\text{in}} = (h_5-h_3) + (h_8-h_7) \]
\[ Q_{\text{out}} = (h_{10}-h_1) \]
\[ \text{Eff} = 1 - (Q_{\text{out}}/Q_{\text{in}}) \]
\[ \text{wnet} = Q_{\text{in}} - Q_{\text{out}} \]
\[ m = 80000/\text{wnet} \]
**Solution**

**Unit Settings:** [kJ]/[CJ]/[kJ]/[kg]/[degrees]

Eff1 = 0.3406  effp = 0.85  efft1 = 0.8  efft2 = 0.8  h1 = 191.7  h10 = 2664  h2 = 201.8  h3 = 202.3  h5 = 3374

h6 = 2783  h7 = 2901  h8 = 3479  h9 = 2460  m = 62.65  p1 = 10  p10 = 10  p2 = 10000  p5 = 10000

p6 = 1000  p8 = 1000  p9 = 10  Qin = 3749  Qout = 2472  s1 = 0.6489  s5 = 6.587  s8 = 7.762  T10 = 87.97

T5 = 500  T8 = 500  wnet = 1277

10 potential unit problems were detected. Check Units

Calculation time = 0.0 sec
Q and W dependent on path
\[ W = \int pdv \]

**FIRST LAW**
\[ \oint \delta Q = \oint \delta W \quad \text{Cycle} \]

CLOSED SYSTEM quantity of mass
\[ Q = \Delta E + W \quad \text{Process} \]
\[ \delta q = du + pdv \]
Processes
Q and W for \( p = c, v = c, T = c, \text{adiabatic, polytropic} \)

OPEN SYSTEM region in space
\[ Q = m\Delta(u + pv + \frac{V^2}{2} + gz) + W \]
Processes
compression, expansion, heat exchanger, throttling, diffuser, nozzle

UNSTEADY SYSTEM unequal mass flow
\[ Q = m_2u_2 - m_1u_1 - (m_2 - m_1)h_o + W_{\text{boundary}} \]

First Law is an Energy Balance
PROPERTIES

Ideal Gas Model

\[ pv = RT + \text{room temperature } c_p \text{ and } c_v \]

\[ R = \frac{\text{universal gas constant}}{\text{molecular weight}} = (8.314 \text{ metric, 1545.25 English}) \]

\[ R = c_p - c_v \text{ in the same units} \]

Isentropic process \( pv^k = \text{constant, } Q = 0, \Delta s = 0 \)

\[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = \left( \frac{v_1}{v_2} \right)^{k-1} \]

\[ \Delta s = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right) \]

\[ \Delta s = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right) \]

Ideal Gas with Temperature dependent \( c_p \) and \( c_v \)

TableA – 17

\[ \left( \frac{Pr_1}{Pr_2} \right)_{\text{TableA–17}} = \left( \frac{p_1}{p_2} \right)_{\text{problem}} \]

Real Gases – Steam, R – 134a TABLES

\[ x = \frac{\text{mass gas}}{\text{mass mixture}} \]

\[ v = v_f = x \times v_{fg} \quad \text{(also } u, h, s) \]

\[ x = \frac{v - v_f}{v_{fg}} \quad \text{(also } u, h, s) \]

\[ v_{\text{sub cooled}} = v_f @T \]
SECOND LAW

\[ Q_{in} \neq W \Rightarrow \text{Heat Engine} \]

\[ \eta = \frac{\text{benefit}}{\text{effort}} = \frac{\text{Work}}{Q_{in}} \]

by First Law \[ \eta = \frac{Q_{in} - Q_{out}}{Q_{in}} \]

\[ \eta = \text{function} \left( T_H, T_L \right) \]

\[ \frac{Q_H}{Q_L} = \frac{T_H}{T_L} \]

\[ \int \frac{\delta Q}{T} \text{ independent of path} \Rightarrow \text{property} \]

\[ \Delta s = \int \frac{\delta Q}{T} \]

\[ \Delta s = 0 \text{ for reversible processes} \]
\[ \Delta s > 0 \text{ for irreversible processes} \]
\[ \Delta S = m \times \Delta s > 0 \text{ isolated systems, irreversible processes} \]

\[ \eta_{\text{CARNOT}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{T_H - T_L}{T_H} = \frac{W}{Q_{in}} \]

\[ \text{COP}_{\text{refrigerator}} = \frac{Q_{in}}{Q_{in} - Q_{out}} = \frac{T_L}{T_H - T_L} = \frac{Q_{in}}{W} \]

\[ \text{COP}_{\text{heat pump}} = \frac{Q_{out}}{Q_{in} - Q_{out}} = \frac{T_H}{T_H - T_L} = \frac{Q_{out}}{W} \]

PROCESS EFFICIENCY

\[ \eta_{\text{expansion process}} = \frac{h_1 - h_2}{h_1 - h_{\text{issentropic}}} = \frac{W_{\text{actual}}}{W_{\text{ideal}}} \]

\[ \eta_{\text{compression process}} = \frac{h_1 - h_2}{h_1 - h_{\text{issentropic}}} = \frac{W_{\text{ideal}}}{W_{\text{actual}}} \]

CYCLE EFFICIENCY

\[ \eta_{\text{CYCLE}} = \frac{W}{Q_{in}} \]

FIRST AND SECOND LAWS COMBINED

\[ T ds = du + pdv \]
1. **Problem Statement**
   Carbon dioxide is contained in a cylinder with a piston. The carbon dioxide is compressed with heat removal from T1,p1 to T2,p2. The gas is then heated from T2, p2 to T3, p3 at constant volume and then expanded without heat transfer to the original state point.

2. **Schematic**

3. **Select Thermodynamic System**
   - open - closed - control volume
   - a closed thermodynamic system composed to the mass of carbon dioxide in the cylinder

4. **Property Diagram**
   - state points - processes - cycle

5. **Property Determination**

6. **Laws of Thermodynamics**
   - Q=?
   - W=?
   - E=?
   - material flows=?