EAS 200
EE Concepts - non-Majors

Introduction II

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EAS 200 Website

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Office Hours: Wednesdays 4:00-5:00 p.m.
1. Current, Voltage, Power and Energy
2. Kirchhoff’s Current Law
3. Kirchhoff’s Voltage Law
4. Introduction to Circuit Elements
   • Conductors
   • Voltage Sources
   • Current Sources
   • Resistors
What is Electrical Circuits?

A collection of circuit elements, connected in closed paths by conductors.
1a - Current

- Current is moving electrical charge
  \[ Q_{\text{electron}} = 1.6 \times 10^{-19} \text{ C} \quad 1 \text{C} \equiv 6 \times 10^{18} \text{electrons} \]
- Measured in Amperes (A) = 1 Coulomb/s
- Current is represented by \( I \) or \( i \)
- Constant current is called DC
- Sinusoidally varying current is called AC
- Current can vary in triangular waveform
- Current can vary in square waveform
Current is moving electrical charge

\[ i(t) = \frac{dq(t)}{dt} \]

\[ q(t_2 - t_1) = \int_{t_1}^{t_2} i(t) dt \]
The charge that passes through a circuit element is given by $q(t)=0.01 \sin(200t)$ C, in which the angle is in radians. Find the current as a function of time.

$$i(t) = \frac{dq(t)}{dt}$$

$\Rightarrow$

$$i(t) = 2 \cos(200t) \quad \text{A}$$
Reference Directions

What does it mean if you have a negative current?

Double-Subscript Notation for Current
Given: $I_2 = 1 \text{ A}$ and $I_3 = -3 \text{ A}$; Assumption: Current consist of positive charge

Find: Direction of charge moving in element C? and E?
1b - Voltage

• Electromotive force or potential - $V$ or $v$
• 1 J of energy is needed to move 1 C of charge through a 1 V potential difference.
What does it mean if you have a negative voltage?

Double-Subscript Notation for Voltage

\[ V_{ab} = -V_{ba} \]
1c - Power

*Power is the product of voltage and current:*

\[ P = I \cdot V \quad \text{or} \quad p = i \cdot v \]

*Units:*

\[ \left( \frac{C}{s} \right) \times \left( \frac{J}{C} \right) = \frac{J}{s} = W \]
Positive power is produced by positive current flowing from high to low voltage.

Negative power is produced by positive current flowing from low to high voltage.
Positive Power

\[ + \quad 8 \text{ V} \quad - \]

This circuit element is **dissipating** power:

- Resistor (light bulb, heating element)-converts electrical energy to heat energy.
- Motor-converts electrical energy to motion.
Negative Power

This circuit element is supplying power:

- Battery-converts chemical energy to electrical.
- Generator-converts mechanical energy to electrical.
Example

- Which circuit element is supplying power?
- Which is dissipating power?
- How much is supplied/dissipated?

What is wrong?
Example

- How much power is supplied/dissipated?

Power plant

\[ i(t) = 35.4 \cos(60 \ 2\pi t) \]

\[ v(t) = 170 \cos(60 \ 2\pi t) \]
\[ v(t) = 170 \cos(2\pi \times 60t) \]
\[ i(t) = 35.4 \cos(2\pi \times 60t) \]
\[ p(t) = v(t) \times i(t) = 6018 \cos^2(2\pi \times 60t) \]
\[ P_{\text{avg}} = 3.02 \times 10^3 W \]
Electric Car:
Which circuit represents accelerating?
Which circuit represents braking?
Active vs. Passive Elements

• Active elements can generate energy.
• Passive elements cannot generate energy.
Example 1.2, page 16

- Calculate power for each element.
- If each element is a battery, is it being charged or discharged?

\[
\begin{align*}
\text{Pa} &= V_a I_a = 12 \times 2 = 24 \text{ W} \\
\text{Power is positive} & \quad \text{Energy absorbed}
\end{align*}
\]

\[
\begin{align*}
\text{Pb} &= -V_b I_b = -12 \\
\text{Power is negative} & \quad \text{Energy delivered}
\end{align*}
\]

\[
\begin{align*}
\text{Pc} &= V_c I_c = -36 \\
\text{Power is negative} & \quad \text{Energy delivered}
\end{align*}
\]
1d - Energy

*Energy is transferred when charge flows through an element having a voltage across it.*

\[ w = \int_{t_1}^{t_2} p(t) \, dt \]
Example 1.3, page 17

**Power** \( P(t) = ? \)

**Energy** \( W_{t=0 \rightarrow t=\infty} = ? \)

\[ p(t) = v(t)i(t) = 12 \times 2e^{-t} = 24e^{-t} \quad W \]

\[ w = \int_{0}^{\infty} p(t)dt = \int_{0}^{\infty} 24e^{-t} dt = \left[ -24e^{-t} \right]_{0}^{\infty} = 24 \quad J \]
Switches

- Switches control the current in the circuits

- **Switch is open** → Current zero & Voltage is determined by the remainder of the circuit

- **Switch is closed** → Voltage across it is Zero & current is determined by the remainder of the circuit
Definition of Node in electrical circuit:

It is a point at which two or more circuit elements are joined together.

Three different ways to write equations:

\[ I_1 - I_2 + I_3 = 0 \] Net current entering a node is zero

\[ -I_1 + I_2 - I_3 = 0 \] Net current leaving a node is zero

\[ I_1 + I_3 = I_2 \] Net current entering a node is equal to the sum of currents leaving a node
Nodes in a circuit that are connected directly by conductors are electrically equivalent to a single node.
Use KCL to find unknown current.
Two different ways to write equations:

1. The algebraic sum of the voltages equals zero for any closed paths (loop) in an electrical circuit.
2. KVL is a consequence of the law of energy conservation.
1- The algebraic sum of the voltages equals zero for any closed paths (loop) in an electrical circuit.

Loop 1: \(-V_1 + V_2 + V_5 = 0\)
Loop 2: \(-V_5 - V_3 + V_4 = 0\)
Loop 3: \(-V_4 + V_3 - V_2 + V_1 = 0\)
2- KVL is a consequence of the law of energy conservation.

Power for each elements

\[ P_1 + P_2 + P_5 = 0 \]

\[ -V_1 I + V_2 I + V_5 I = 0 \]

\[ -V_1 + V_2 + V_5 = 0 \]

Same as first technique

Loop 1: \[-V_1 + V_2 + V_5 = 0\]
Loop 2: \[-V_5 - V_3 + V_4 = 0\]
Loop 3: \[-V_4 + V_3 - V_2 + V_1 = 0\]
Series and parallel Circuits

**Series Circuits:** Elements A and B are connected in series when no other element is connected to the node joining A and B. All elements in a series circuits have identical currents.

**Parallel Circuits:** Elements A and B are connected in Parallel when both ends of one element are connected directly to corresponding ends of the others.
4- Introduction to Circuit Elements

- Conductors
- Voltage Sources
- Current Sources
- Resistors
Conductors

**Short Circuit:**
Shorted by a conductor

\[ V_{ab} = 0 \]
\[ I_{ab} = ? \]

**Open Circuit**

\[ I_{ab} = 0 \]
\[ V_{ab} = ? \]
Voltage Sources

• Independent

The voltage across its terminals is independent of other elements that are connected to it and of the current flowing to it.

• Dependent (or voltage-controlled voltage source)

The voltage across its terminals is a function of other voltages or currents in the circuit.

12V (or $5\cos(2\pi t)$ for AC)

$2V_x$
Voltage-controlled Voltage source

$3I_x$
Current-controlled Voltage source
Examples

Voltage-controlled Voltage source

Current-controlled Voltage source
Ideal Circuit Elements versus Reality

What is wrong with this circuit model?

Non-ideal Power supply

Non-ideal Conductor
Current Sources

• Independent

The current source forces a specified current to flow through itself. The current through the Source is independent of the elements connected to it and the voltage across it.

• Dependent

• The current flowing through the source is determined by a current or a voltage elsewhere in the circuit.
Examples

Electronic Amplifiers are good examples for current sources
Batteries are good examples for voltage sources
Resistors

\[ V = IR \quad \text{Ohm’s Law} \]

(for positive reference direction; \( V_{ab} = I_{ab} R \))
\[
\Omega \rightarrow R = \frac{\rho l}{A} \quad \text{m}^2
\]
$R = \frac{\rho l}{A}$

ρ [Ωm]

Conductors: Lowest Resistivity

Semiconductors: Modest Resistivity

Insulators: Very high Resistivity

Resistivity Values (Ωm) for selected Materials at 300 K

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$2.3 \times 10^{-8}$</td>
</tr>
<tr>
<td>Carbon</td>
<td>$3.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$1.7 \times 10^{-8}$</td>
</tr>
<tr>
<td>Gold</td>
<td>$2.3 \times 10^{-8}$</td>
</tr>
<tr>
<td>Silver</td>
<td>$1.6 \times 10^{-8}$</td>
</tr>
<tr>
<td>Tungstem</td>
<td>$5.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>Silicon</td>
<td>$10^{-5}$ to $1$</td>
</tr>
<tr>
<td>Fused Quartz</td>
<td>$&gt;10^{21}$</td>
</tr>
<tr>
<td>Glass</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Teflon</td>
<td>$10^{9}$</td>
</tr>
</tbody>
</table>
**Conductance:**

\[ G = \frac{1}{R} \quad \text{Ohm’s Law} \quad I = GV \]

**Power for Resistance:**

\[ P = V \times I \]
\[ P = R \times I^2 \]
\[ P = \frac{V^2}{R} \]

Power for resistance is always positive regardless of the sign of V and I. Power is absorbed by resistance.
What is wrong with this calculation?

\[ R = \frac{\rho l}{A} \]

\[ R = 9 \times 10^{-4} \, \Omega \]

\[ \rho = 5.4 \times 10^{-8} \, \Omega \text{m} \]

\[ 0.05 \, \text{m} \]

\[ 3.14 \times 10^{-6} \, \text{m} \]

\[
\begin{align*}
P &= \frac{V^2}{R} = \frac{120^2}{9 \times 10^{-4}} = 1.6 \times 10^7 \, \text{W} \\
I &= \frac{V}{R} = \frac{120}{9 \times 10^{-4}} = 1.3 \times 10^5 \, \text{A} \\
R &= \frac{\rho l}{P} = \frac{120^2}{9 \times 10^{-4} / 3600} = 5.2 \times 10^5 \, \Omega
\end{align*}
\]
\[ V_{rms} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = 170 \text{ V} \]
Simple Circuits

Ohm’s Law: \( I_R = \frac{V}{R} = 2 \text{ A} \)

KCL: \( I_S = I_R \)

KVL:

1. \( 10 \text{ V} \)
2. \( R = 5 \Omega \)
3. \( I_R = 2 \text{ A} \)
4. \( I_S = 2 \text{ A} \)