Evaluation of Methods for Measuring the

Volume of Blocks

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Executive Summary

The corporation has recently installed facilities for manufacturing blocks. A customer has specified that a fraction of production blocks must be tested to make sure their volume is within given tolerances. This project was undertaken to identify methods that could be used on the production line to perform the required test. Three possible methods were identified and evaluated. One method was based upon measuring the three dimensions of a block. The two other methods were based upon displacing a fluid. The methods were compared on the basis of their accuracy and precision. Consideration was also given to the ease of implementation on the production line with respect to the time required per test and the potential for operator error. None of the methods was fully satisfactory as tested in the lab, but the limitations of the laboratory equipment can be eliminated when equipment is designed for use on the production line. The most attractive approach involves measuring the volume of a fluid, immersing a test block in the fluid and measuring the resultant volume of the fluid and block. The difference between these two measured volumes is the desired volume of the test block. A test apparatus should be designed specifically for use on the production line. Assuming proper design of the apparatus this method will provide the desired information in the least amount of time per sample and with fewer possibilities for operator error.
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Introduction

The corporation has recently installed facilities for the manufacture of blocks. One of our customers has expressed an interest in these products. The customer would require us to perform quality assurance checks on a specified fraction of blocks produced under any contract. At present we do not have any apparatus on the production line for making this kind of test. This project was undertaken to identify methods than might be used for production line testing and to evaluate the methods. The criteria of importance in the evaluation are the precision and accuracy of the method, the time required per test, and the possibility of operator error.

The blocks to be tested are sealed on their outer surface so that they are impermeable to common fluids like water. This property suggested that methods based upon fluid displacement should be considered. Internally they do contain voids, and consequently the blocks do not have uniform density. For this reason, it will not be possible to use test methods based upon block density. It is not known whether or not the production blocks are true (that is whether all corners are exact $90^\circ$ angles). Assuming that they are, another method for finding a block’s volume is by measuring length, width and volume and then finding the product of these measured values. This project considered three methods for measuring the volume of a block: two based on fluid displacement and one based on measurement of dimensions.

Theory

The first method tested involved measuring the length, $l$, width, $w$, and depth, $d$, of the block. Having recorded these dimensions, the volume of the block, $V$, can be calculated using equation (1) if it is assumed that all the corners of the block are square.

$$V = lwd$$ (1)

The second method involved displacing an amount of fluid, $V_{dis}$, from a full container by dropping the block into the container. This fluid was collected and its volume was measured. If the block is impermeable, if no displaced fluid is lost and if the measurement is performed under isothermal conditions, then the displaced fluid volume equals the volume of the block, $V$, as given in equation (2).

$$V = V_{dis}$$ (2)
The third method also involved displacing a fluid. In this case a container was initially partially filled with a volume of fluid, $V_i$. The block was dropped into the container, and the resultant volume was $V_f$. If the block is impermeable, if no fluid escapes from the container and if the experiment is performed under isothermal conditions, the volume of the block is found from equation (3).

$$V = V_f - V_i$$  

(3)

A block of known volume was needed in order to test the precision of the different methods. For this purpose a rough block of pure aluminum was machined into a true block of the same nominal size as the production blocks. Since this block was fabricated from pure aluminum, its volume could be found with high accuracy by weighing it. The mass, $m$, is related to the volume, $V$, through the density, $\rho$, as given in equation (4).

$$V = \frac{m}{\rho}$$  

(4)

Procedure

A rough block of pure aluminum was machined into a true block of appropriate size as mentioned in the previous section. The mass of this block was then measured using an analytical balance. The density of aluminum is known (1) to equal 2.70 g cm$^{-3}$ at 20°C (the temperature at which the block was weighed). The dimensions of the block were next measured using a standard ruler.

A beaker of water was clamped to a ringstand and filled to the very top with water. A funnel was located under the beaker to collect any displaced fluid and direct it into a graduated cylinder. This equipment is shown schematically in Figure 1. The block was then dropped into the beaker, and once all the displaced fluid had drained into the graduated cylinder, the volume of the displaced fluid was read from the markings on the graduated cylinder.

The same beaker was then partially filled with water so that the initial water level was exactly even with one of the volume scale markings on its side. The block was dropped into the beaker being careful that no water splashed out of the beaker. The final volume, corresponding to the water plus the block, was read from the volume scale marked on the side of the beaker.
Results and Discussion

The volume of the aluminum test block was calculated using each of the procedures described previously. The results are summarized in Table 1 which also includes an estimate of the uncertainty in the results. Appendix 1 presents samples of all the calculations involved, including the estimate of uncertainty.

Table 1. Volume as Measured using Different Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Volume (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>3.83</td>
</tr>
<tr>
<td>Dimensions</td>
<td>4.03 ± 0.15</td>
</tr>
<tr>
<td>Volume of Displaced Water</td>
<td>3.6 ± 0.05</td>
</tr>
<tr>
<td>Volume Change</td>
<td>3 ± 1.4</td>
</tr>
</tbody>
</table>

The method wherein each of the three dimensions is measured is not sufficiently precise. That is, the volume measured using this method does not agree with the true volume to within the experimental uncertainty. The reasons for this are not immediately clear. There may have been a systematic error (poorly calibrated ruler) or the technician may have read the ruler incorrectly. Nevertheless, there are other reasons for not selecting this method. It is the most time-consuming method, and as such will be more expensive per test on the production line. It also involves the most steps. Each step represents a possible operator error, and in this case the uncertainty associated with each of the steps is about the same. Thus this method has the greatest chance of an operator error generating a false result.

The method wherein the displaced fluid is collected is also imprecise as implemented in the laboratory. The measured volume is less than the true volume; this is believed to be a result of poor laboratory procedure. Specifically, it seems that some of the water in the beaker spilled out as the funnel was being put in position. This water was not replaced before the block was dropped into the beaker. Therefore part of the displaced fluid did not leave the beaker, but instead filled up the empty space. This experimental mishap displays the awkwardness of this method and argues against its selection for the production measurements. Like the previous method it is a relatively lengthy procedure, and hence will be costly. While the measurement is direct, the procedure itself affords several opportunities for operator error. In addition to the one experienced in the lab, the operator might not completely drain the graduated cylinder before starting another measurement, or some of fluid might splash outside the funnel and not be recovered.
The method of choice is the fluid displacement method wherein the volume change is measured. In the experimental system this method was not sufficiently accurate, but this can be corrected in the design of the production line apparatus. The problem with the experimental equipment was that the beaker was very large and had a scale that was too coarsely calibrated. The production equipment should feature a container that has a cross-section only slightly larger than the nominal size of the blocks. This container should then be calibrated to a scale that is smaller than the desired uncertainty. With this method the operator needs to make only two readings, and make sure no fluid splashes out of the container. Furthermore, several blocks can be tested before the container needs to be emptied and recharged with fluid. Thus the measurements are the least time-consuming.

Conclusions

None of the three methods tested were satisfactory, at least as practiced in the laboratory. It is likely that the imprecision or inaccuracy of each method could be eliminated by better design of the apparatus and by strictly following the experimental procedure. Despite the limitations of the experimental data, the results showed the superiority of one method. This method involves adding the block to a known volume of fluid and then subtracting the resultant final volume from the initial volume. In the experimental studies this method suffered from low accuracy, but alternative designs can be conceived that will improve the accuracy.

Recommendations

It is recommended that an apparatus be designed for measuring volume based upon fluid displacement. As pointed out in the discussion section, care must be taken to ensure that the new equipment will have sufficient accuracy. This equipment should then be added to the production line and operators should be trained in its proper use. These steps should meet the requirements imposed by our customer.

References


Appendix 1: Sample Calculations

1) Calculation of Volume using Density
   a) \( \rho = 2.70 \text{ g/cm}^3 \)
b) \( m = 10.34 \text{ g (measured)} \)

c) \( V = \frac{m}{\rho} = \frac{10.34 \text{ g}}{2.70 \text{ g/cm}^3} = 3.83 \text{ cm}^3 \), from equation (4)

2) Calculation of Volume from Dimensions

a) \( \lambda_w = \lambda_l = \lambda_d = 0.025 \text{ cm (estimated uncertainty in the ruler reading)} \)
b) \( w = 3.34 \text{ cm (measured)} \)
c) \( l = 1.51 \text{ cm (measured)} \)
d) \( d = 0.080 \text{ cm (measured)} \)
e) \( V = lwd = 4.03 \text{ cm}^3 \), from equation (1)

f) \( \lambda_r = \sqrt{\left(\frac{\partial V}{\partial w}\right)^2 \lambda_w^2 + \left(\frac{\partial V}{\partial l}\right)^2 \lambda_l^2 + \left(\frac{\partial V}{\partial d}\right)^2 \lambda_d^2}
\)

\( = \sqrt{(ld)^2 \lambda_w^2 + (wd)^2 \lambda_l^2 + (lw)^2 \lambda_d^2} \)

\( = 0.145 \text{ cm}^3 \)

3) Calculation of Volume by Displacing Fluid in a Partially Filled Container

a) \( V_i = 27 \text{ cm}^3 \) (measured)
b) \( V_f = 30 \text{ cm}^3 \) (measured)
c) \( \lambda_{V_i} = \lambda_{V_f} = 1 \text{ cm}^3 \) (estimated uncertainty in beaker scale reading)
d) \( V_{tot} = V_2 - V_1 = 3 \text{ cm}^3 \), from equation (3)

e) \( \lambda_{V_{tot}} = \sqrt{\left(\frac{\partial V_{tot}}{\partial V_i}\right)^2 \lambda_{V_i}^2 + \left(\frac{\partial V_{tot}}{\partial V_f}\right)^2 \lambda_{V_f}^2}
\)

\( = \sqrt{(-1)^2 \lambda_{V_i}^2 + (1)^2 \lambda_{V_f}^2} \)

\( = 1.41 \text{ cm}^3 \)

4) Calculation of Volume by Collecting Displaced Fluid Volume

a) \( 3.6 \pm 0.05 \text{ cm}^3 \)
Figure 1. Schematic representation of the method wherein the displaced fluid was collected.