A gas stream with composition 5 mole percent benzene, 95 mole percent air on an oil-free basis is to be scrubbed with an absorption oil to reduce the benzene content to 0.1 mole percent on an oil-free basis (in order that Elroy may breath it safely). The absorber will operate at atmospheric pressure with inlet temperatures of 20 and 30 °C for the gas and liquid, respectively. The gas enters saturated with oil at the inlet temperature and can be assumed to leave as a saturated gas at 30 °C. Entering oil is pure. Calculate the required number of transfer units N_{Oy} if the liquid rate is 1.3 times the minimum.

Data: heat capacities:

air	(v)	29 J / mol °C
benzene	(v)	96 J / mol °C
benzene	(l)	138 J / mol °C
oil	(v)	140 J / mol °C
oil	(l)	170 J / mol °C

Vapor pressures in mm Hg:

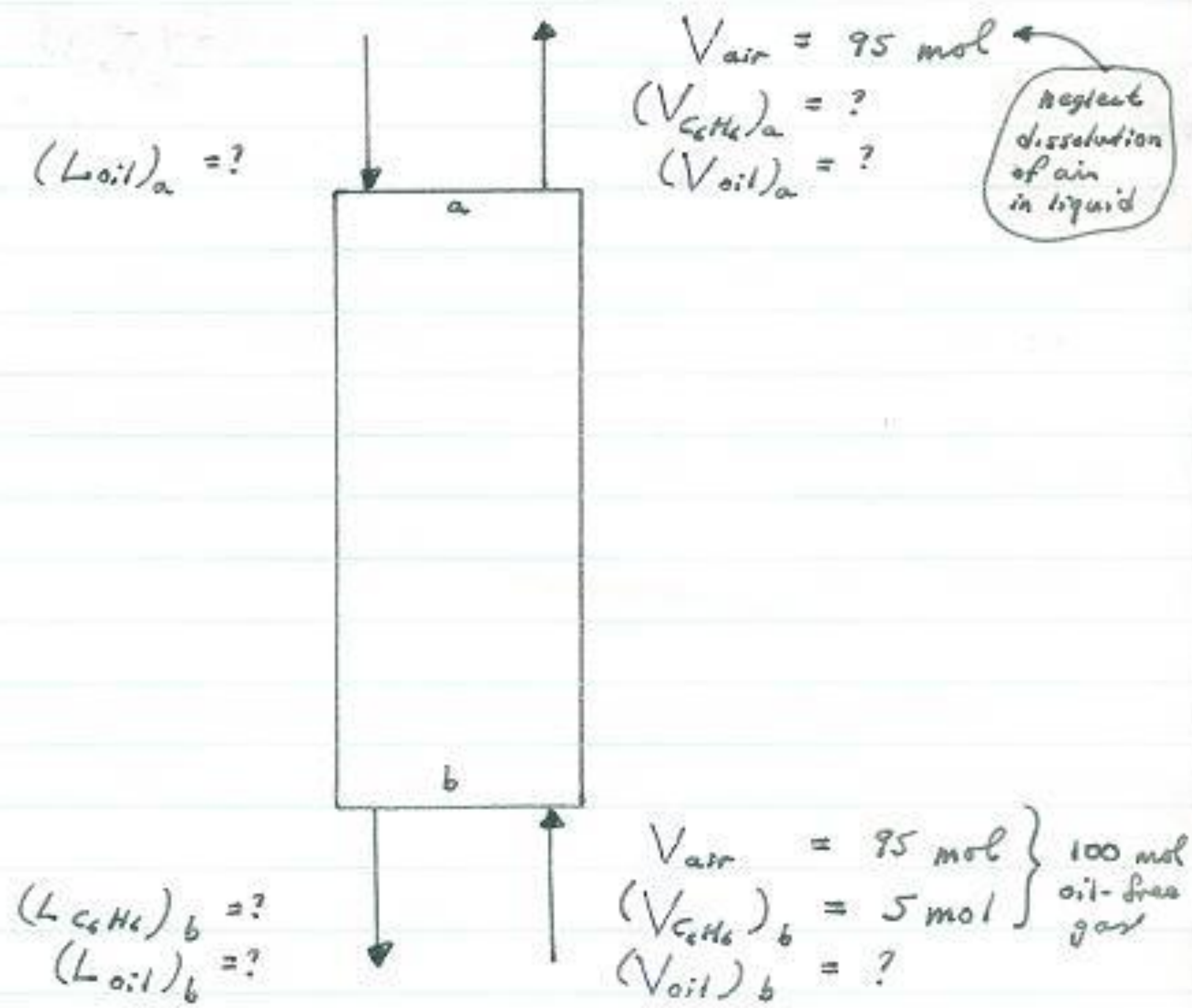
T (°C)	benzene	oil
10	46	
15	59	
20	75	22
25	95	
30	119	37
35	148	
40	183	
45	223	
50	271	
55	327	
60	391	
65	466	
70	551	
75	648	
80	760	

Other data:

compound	boiling point (°C)	H of vaporization at boiling point (J / mol)
benzene	80	30790
oil	111	33300

Assume ideal gas and ideal liquid solution (==> Raoult's law valid & no ΔH of mixing). Neglect dissolution of air in the liquid. Choose reference states such that H = 0 for pure components in gaseous state at 30 °C.

Basis: 100 mol oil-free entering gas



(i) Preliminaries.

Entering gas: Saturated with oil at 20 °C.

Equil. condition: $y_{oil} P = \underbrace{x_{oil}}_{1\,\dagger} P_{oil}^{sat}$

so

$$y_{oil} = \frac{P_{oil}^{sat}}{P} = \frac{22\ \text{mmHg}}{760\ \text{mmHg}} = 0.02895.$$

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† If one had air + C_6H_6 gas in equil. with oil liquid, some C_6H_6 would be in liquid, but so little that take $x_{oil} \approx 1$.

Then by def. of mole fraction

$$y_{oil} = 0.02895 = \frac{(V_{oil})_b}{\underbrace{100\text{ mol} + (V_{oil})_b}_{\text{total moles}}}$$

$$\Rightarrow (V_{oil})_b = \left(\frac{0.02895}{1-0.02895}\right)(100\text{ mol}) = 2.981\text{ mol}$$

<u>Exiting gas:</u> 0.1 mole % C_6H_6 on oil-free basis:

$$0.001 = \frac{(V_{C_6H_6})_a}{\underbrace{95\text{ mol} + (V_{C_6H_6})_a}_{\text{total oil-free moles}}}$$

$\Rightarrow (V_{C_6H_6})_a = 0.095$ mol. Next, saturated with oil at 30°C $\Rightarrow$ $y_{oil} = 1 \cdot \frac{P_{oil}^{sat}}{P} = \frac{37}{760} = 0.04868$

so $$0.04868 = \frac{(V_{oil})_a}{95\text{ mol} + 0.095\text{ mol} + (V_{oil})_a}$$

or $(V_{oil})_a = 4.866$ mol

<u>Entering & exiting oil:</u> Not now, but can relate $(L_{oil})_a$ and $(L_{oil})_b$ by material balance:

$$\overbrace{(L_{oil})_a + \underbrace{(V_{oil})_b}_{2.981\text{ mol}}}^{\text{in}} = \overbrace{(L_{oil})_b + \underbrace{(V_{oil})_a}_{4.866\text{ mol}}}^{\text{out}}$$

or $$(L_{oil})_a = (L_{oil})_b + 1.885\text{ mol} \qquad (1)$$

<u>Benzene balance:</u>

entering oil is pure

$$\overbrace{\underbrace{\cancel{(L_{C_6H_6})_a}}_{0} + (V_{C_6H_6})_b}^{\text{in}} = \overbrace{(L_{C_6H_6})_b + (V_{C_6H_6})_a}^{\text{out}}$$

or $(L_{C_6H_6})_b = 5\text{ mol} - 0.095\text{ mol} = 4.905\text{ mol}$

(ii) <u>Enthalpies</u>

(ref. state)

<u>Pure air (g)</u>: $H_y = \cancelto{0}{H(30°C)} + C_{py}(T - 30°C)$

$= 29(T-30)$

<u>Pure benzene (g)</u>: $H_y = \cancelto{0}{H_y(30°C)} + C_{py}(T - 30°C)$

$= 96(T-30)$

<u>Pure oil (g)</u>: $H_y = \cancelto{0}{H_y(30°C)} + C_{py}(T - 30°C)$

$= 140(T-30)$

<u>Pure benzene (l)</u>:

heat gas to b.p.

$$H_x = \cancelto{0}{H_y(30°C)} + C_{py}(80°C - 30°C) + \Delta H_{v\to l}(80°C) + C_{px}(T - 80°C)$$

condense

change temp of liquid

$$= 0 + 96(80-30) + (-30790) + 138(T-80)$$

$$\Delta H_{v \to l} = -\Delta H_{l \to v}$$

$$H_x = -25990 + 138(T-80)$$

Can also write this as:

$$H_x = -25990 + 138(T - 30 + 30 - 80)$$

$$= -32890 + 138(T-30)$$

<u>Pure oil (l)</u>:

$$H_x = H_y(30°C) + C_{py}(111°C - 30°C) + \Delta H_{v\to l}(111°C) + C_{px}(T - 111°C)$$

$$= 0 + 140(111-30) + (-33300) + 170(T - 111°C)$$

$$= -21960 + 170(T-111)$$

or

[all: H in J/mol and T in °C]

$$H_x = -35730 + 170(T-30)$$

Entering gas: 20°C

$$(VH_y)_b = \left(\sum_i V_i\, H_{y_i}\right)_b + 0$$

ideal gas mixture is ideal solution a fortiori ∴ no ΔH of mixing

$$= (95)\cdot 29(20-30) + (5)\cdot 96(20-30) + (2.981)\cdot 140(20-30)$$

$$= -0.365\times10^{5}\ \text{J}$$

Exiting gas: 30°C By choice of ref. state, $H_{y_i} = 0$

so $(VH_y)_a = 0$ J

Entering liquid:

$$(LH_x)_a = (L_{oil})_a\ [-35730 + 170(30-30)]\ \text{J/mol}$$
$$= (L_{oil})_a\ (-35730\ \text{J/mol})$$

Exiting liquid:

$$(LH_x)_b = (L_{oil})_b\ [-35730 + 170(T_{x_b}-30)] + \underbrace{(L_{C_6H_6})_b}_{4.905\ \text{mol}}\ [-32890 + 138(T_{x_b}-30)]$$

temp. of exiting liquid unknown

Statement of enthalpy balance:

$$\underbrace{(LH_x)_a + (VH_y)_b}_{\text{in}} = \underbrace{(LH_x)_b + \overbrace{(VH_y)_a}^{0\ \text{J/mol}}}_{\text{out}}$$

or

$$\left.\begin{aligned} &(L_{oil})_a(-35730) + (-0.365\times10^5)J \\ &\qquad = (L_{oil})_b\left[-35730 + 170(T_{x_b}-30)\right] \\ &\qquad\quad + (4.905)\left[-32890 + 138(T_{x_b}-30)\right] \end{aligned}\right\} \quad (2)$$

(iii) Equilibrium relation

At given liquid temperature, Raoult's law:

$$y_{C_6H_6}\,P = x_{C_6H_6}\,P^{sat}_{C_6H_6}$$

or

$$x_{C_6H_6} = \left(\frac{P}{P^{sat}_{C_6H_6}}\right) y_{C_6H_6}$$

(iv) Minimum liquid

Op. line touches equil. curve at "b" end.

$$\therefore \quad x_b = (x_{C_6H_6})_b = \frac{P}{P^{sat}_{C_6H_6}}(y_{C_6H_6})_b$$

$$\frac{5 \text{ mol}}{(95+5+2.981)\text{ mol}} = 0.04855$$

Given: $T_{x_b} = 40°C$. From table, $P^{sat}_{C_6H_6} = 183$ mm Hg.

Then $x_b = \left(\frac{760}{183}\right)(0.04855) = 0.2016$

By def. of mole fraction

$$x_b = 0.2016 = \frac{(L_{C_6H_6})_b}{(L_{oil})_b + (L_{C_6H_6})_b}$$

4.905 mol

$\Rightarrow (L_{oil})_b = 19.425$ mol.

Then $(L_{oil})_a = 21.310$ mol by (1).

Substitute T_{x_b}, $(L_{oil})_b$, $(L_{oil})_a$ into enthalpy balance (2) and check if eq. satisfied:

$$LHS = -7.979 \times 10^5 \text{ J}$$
$$RHS = -8.156 \times 10^5 \text{ J} \quad \text{(pretty close)}$$

By trial,[†] conclude $T_{x_b} = 42.8\ °C$, $\boxed{(L_{oil})_a = 24.176 \text{ mol}}$
$(L_{oil})_b = 22.291$ mol for min. liquid.

(v) N_{Oy} for actual operating conditions

$$(L_{oil})_a = (1.3)\underbrace{(24.176 \text{ mol})}_{\text{min. entering oil}} = 31.429 \text{ mol}$$

Then $(L_{oil})_b = 29.544$ mol by (1).
Now solve enthalpy balance (2) for temperature T_{x_b}. Get $T_{x_b} = 40.08°C \approx 40°C$. Also,

$$x_b = \frac{4.905}{29.544 + 4.905} = 0.1424$$

Assume that T_x is linear with x.
Make table:

x	T_x (°C)	$P^{sat}_{C_6H_6}$ (mm Hg)[†]	y^*
$x_a =$ 0	30	119	0
0.0356	32.5	133	0.0062
0.0712	35	148	0.0139
0.1068	37.5	165	0.0232
$x_b =$ 0.1424	40	183	0.0343

† Remember: the correct way to interpolate in a table of p^{sat} vs. T(°C) is $\ln p^{sat}$ linear with $\underbrace{1/(T + 273.15)}_{\text{abs. temp.}}$.

$$\frac{0.095}{95 + 0.095 + 4.866} \approx 0.00095$$ $\boxed{7}$

Since solutions dilute, approx. op. line as linear:

$$y = y_a + \left(\frac{y_b - y_a}{x_b - x_a}\right)(x - x_a)$$

$$= \quad 0.04855 \quad 0.1424 \quad 0$$

or

$$y = 0.00095 + 0.3343\,x$$

Make t

Make table:

y_a

i	x_i	y_i on op line	y_i^* in equil. with x_i	$(\Delta N_{OY})_i$
0	0	0.00095	0	
1	0.0356	0.0128	0.0062	4.06
2	0.0712	0.0248	0.0139	1.40
3	0.1068	0.0366	0.0232	0.97
4	0.1424	0.0486	0.0343	0.87
				7.3

y_b (rounded)

[For composition change from y_{i-1} to y_i, assume equil. curve ≈ linear. Then can use eq. (22-18).

(op. line already assumed straight)

Sample calc. for $i = 1$:

$$y_1 - y_0 = 0.0128 - 0.00095 = 0.01185$$

$$(y - y^*)_L = \frac{(y-y^*)_i - (y-y^*)_{i-1}}{\ln\left[\frac{(y-y^*)_i}{(y-y^*)_{i-1}}\right]}$$

$$= \frac{(y_1 - y_1^*) - (y_0 - y_0^*)}{\ln\left(\frac{y_1 - y_1^*}{y_0 - y_0^*}\right)}$$

$$= \frac{(0.0128 - 0.0062) - (0.00095 - 0)}{\ln\left(\frac{0.0128 - 0.0062}{0.00095 - 0}\right)}$$

$$= 0.002915$$

$$(\Delta N_{Oy})_i = \frac{y_i - y_{i-1}}{(y - y^*)_L} = \frac{0.01185}{0.002915} = 4.06 \Big]$$

$$\boxed{\text{If the liquid rate is 1.3 times the minimum, the required number of transfer units is } N_{Oy} = 7.3.}$$

Note: if one had just used eq. $N_{Oy} = \frac{y_b - y_a}{(y - y^*)_L}$

(which assumes equil. curve to be perfectly straight) <u>without</u> subdividing composition interval from y_a to y_b, one would have gotten

$$N_{Oy} = \frac{0.0486 - 0.00095}{\left\{\frac{(0.0486 - 0.0343) - (0.00095 - 0)}{\ln\left[\frac{0.0486 - 0.0343}{0.00095 - 0}\right]}\right\}} = 9.7$$

≈ 30 % error !