

CE 407 Notes

Binary distillation theory of Murphree tray efficiency

Consider a binary distillation process.

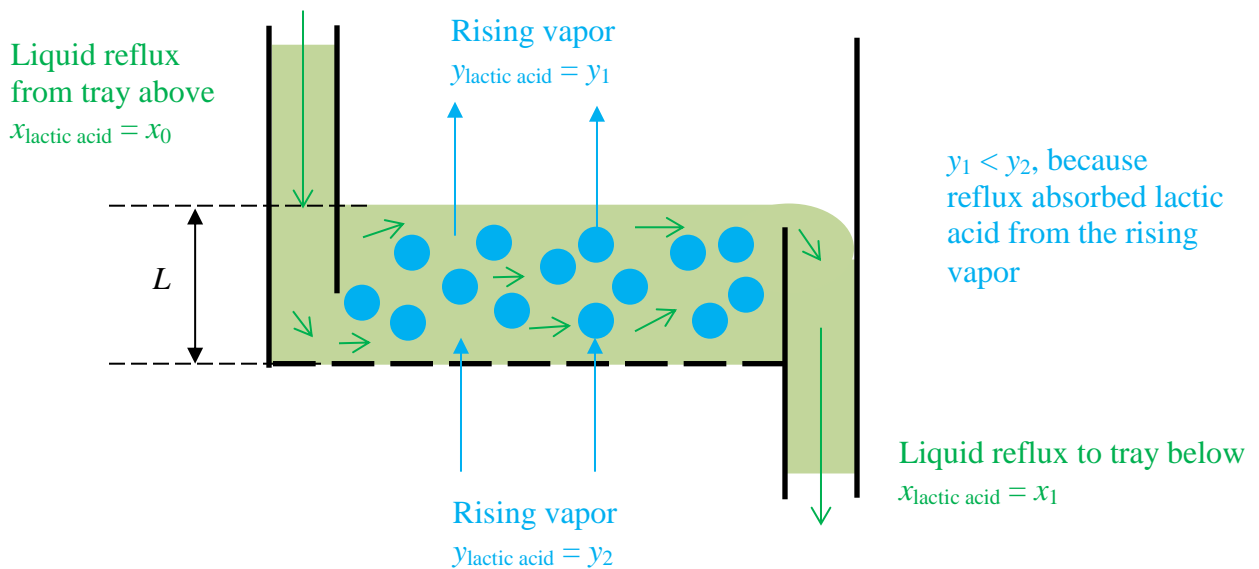
Octanol (“OCT”) is the light component (more volatile, lower boiling); its boiling point at e.g. 14 mm Hg is 94 °C.

Lactic acid (“LA”) is the heavy component (less volatile, higher boiling); its boiling point at e.g. 14 mm Hg is 132 °C.

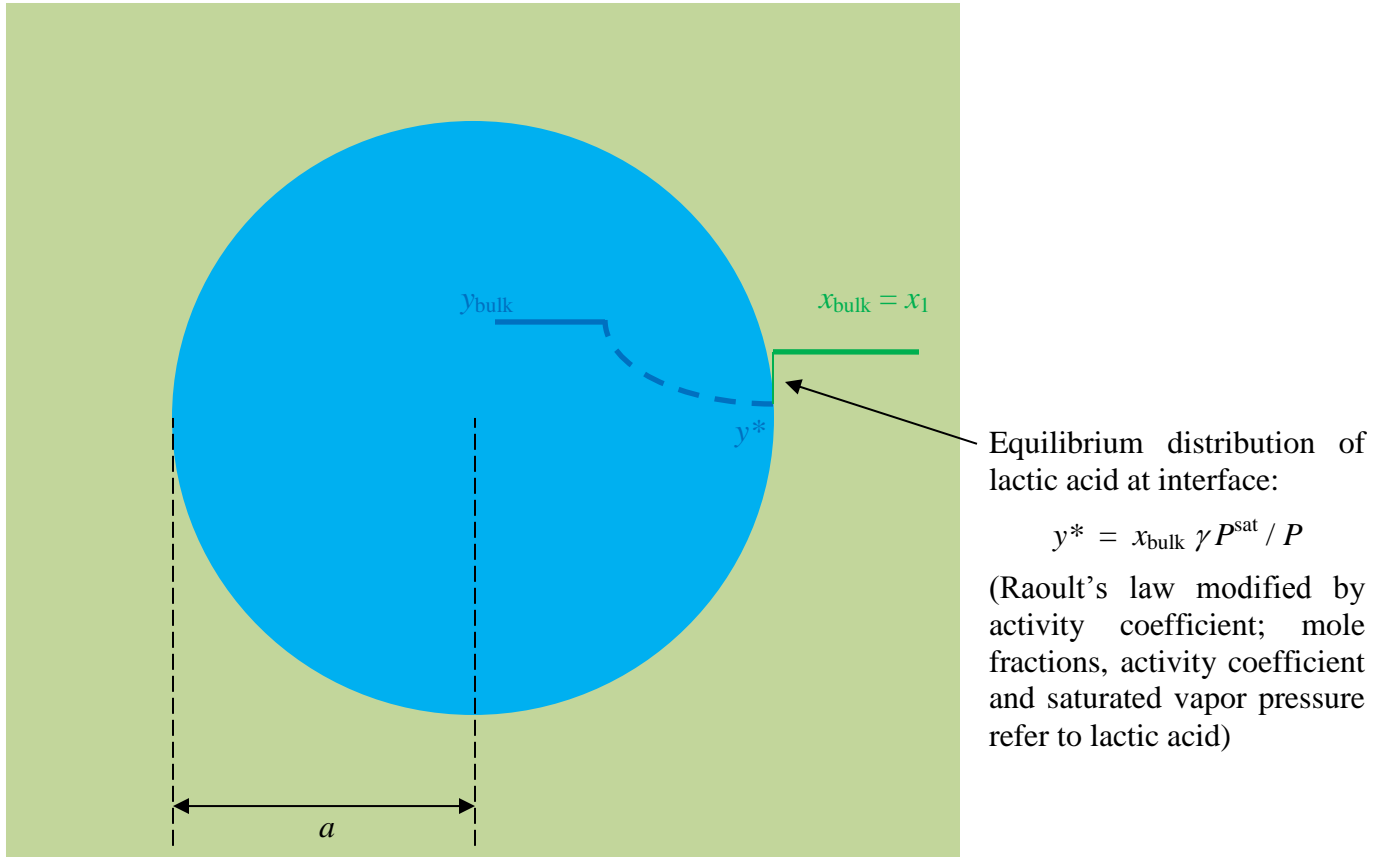
Comment: In this set of notes we focus on the heavy component, lactic acid, and mole fractions x and y refer to it. This departs from the usual convention according to which x and y refer to the light component. In general, please go by the usual convention.

Q: How does the falling liquid reflux absorb lactic acid from the rising vapor?

A: Lactic acid transfers from the interior of each vapor bubble to the liquid-vapor interface, and from there to the bulk liquid.



Let's focus on one bubble of vapor surrounded by liquid.



What equation describes the physics?

$$d(\text{moles of lactic acid in bubble}) / dt = -(4\pi R^2) \times k_y (y_{\text{bulk}} - y^*)$$

where

- a = radius of bubble,
- k_y = mass transfer coefficient for transfer of lactic acid through octanol-rich gas,
- y_{bulk} = gas-phase mole fraction of lactic acid in bulk of bubble,
- y^* = $y^*(x_{\text{bulk}})$ = gas-phase mole fraction of lactic acid at liquid-vapor interface
 $= x_{\text{bulk}} \gamma P^{\text{sat}} / P$ (Raoult's law modified by activity coefficient), and
- x_{bulk} = mole fraction of lactic acid in liquid (assumed well-mixed).

The bubble forms at one of the holes at the bottom of the tray at time $t = 0$. It rises through the liquid, and pops through the surface at time L / u_{bubble} , where L is the depth of liquid on the tray, and u_{bubble} is the speed at which the bubble rises due to buoyancy. In other words, the residence time of the bubble in the liquid is L / u_{bubble} .

Q: What's the upshot of all this?

A: E.V. Murphree showed in 1925 that

$$\frac{y_1 - y_2}{y^* - y_2} = 1 - \exp\left(-\frac{k_y AL}{u_{\text{bubble}}}\right) = \eta_M$$

(with slight differences in notation and formulation; cf. Eq. (6) in Murphree (1925)), where A is the interfacial area per mole of vapor on the tray. The quantity η_M is called the Murphree efficiency or tray efficiency. The larger the mass transfer coefficient (k_y), the larger the interfacial area (A) or the longer the residence time of the bubble spends on the tray (L / u_{bubble}), the closer η_M is to unity, and the more nearly equal y_1 is to y^* . A tray for which the vapor leaving the tray has a lactic acid mole fraction perfectly in equilibrium with that of the liquid leaving the tray (i.e., $y_1 = y^*(x_1)$) is called an ideal or theoretical tray ($\eta_M = 1 = 100\%$).

Q: What is the take-home message we want to remember here?

A: In distillation (and other staged operations), we are usually given a tray efficiency η_M . When we work in terms of η_M , mass transfer seems not to enter distillation calculations. However, the joy of mass transfer is actually embedded in η_M .

Problem: Suppose the volume fraction of bubbles in the froth (bubbly liquid on tray) is $\phi = 0.4$. What is A , the interfacial area per mole of vapor on the tray, for a bubble diameter of (a) 2 mm and (b) 2 cm? (The important parameter A appears in the equation for η_M at the top of the previous page.) Assume atmospheric pressure.

Solution: One bubble has volume $4\pi a^3/3$ and surface (interfacial) area $4\pi a^2$. For a given total volume of froth (bubbly liquid on the tray) V ,

$$\text{Vapor volume in total volume } V = \phi V$$

$$\begin{aligned} \text{Moles of vapor in total volume } V &= (\text{vapor volume}) \times (\text{molar density from ideal gas law}) \\ &= (\phi V) \times (P / RT). \end{aligned}$$

Also,

$$\begin{aligned} \text{Number of bubbles } N_{\text{bubble}} \text{ in total volume } V &= (\text{vapor volume}) / (\text{volume of one bubble}) \\ &= (\phi V) / (4\pi a^3/3) \end{aligned}$$

$$\begin{aligned} \text{Surface area of bubbles in total volume } V &= N_{\text{bubble}} \times (4\pi a^2) = [\phi V / (4\pi a^3/3)] \times (4\pi a^2) \\ &= 3\phi V / a \end{aligned}$$

With these intermediate results,

$$\begin{aligned} A &= (\text{surface area of bubbles in total volume } V) / (\text{moles of vapor in total volume } V) \\ &= (3\phi V / a) / [(\phi V) \times (P / RT)] = 3RT / (aP) \end{aligned}$$

Note that the volume fraction of bubbles in the froth actually cancels out of the calculation, and is therefore not needed. The desired numerical answers are

$$\begin{aligned} \text{(a) } A &= 3 \times [8.314 \text{ J/(mol K)}] \times (298.15 \text{ K}) / [(0.001 \text{ m}) \times (101325 \text{ Pa})] \\ &= 73.4 \text{ m}^2/\text{mol} \quad \leftarrow \text{ look at that high surface area, Batman!} \end{aligned}$$

$$\begin{aligned} \text{(b) } A &= 3 \times [8.314 \text{ J/(mol K)}] \times (298.15 \text{ K}) / [(0.01 \text{ m}) \times (101325 \text{ Pa})] \\ &= 7.34 \text{ m}^2/\text{mol} \quad \leftarrow \text{ look at that low surface area, Batman!} \end{aligned}$$