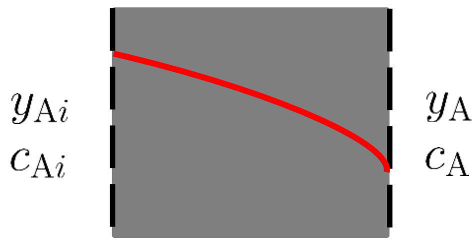
All about mass transfer coefficients

Solute = A; all mass transfer coefficients (k_y, etc.) refer to transport of this species

Presentation in context of one-component mass transfer (A diffusing/transferring through non-diffusing/transferring B)



Presentation in context of gas phase; everything carries over to liquid phase ($k_y \rightarrow k_x$, $y_{\mathrm{A}i} \rightarrow x_{\mathrm{A}i}$, $y_{\mathrm{A}} \rightarrow x_{\mathrm{A}}$)

Solute flux

$$N_{\mathrm{A}} = k_{\mathrm{y}} \ln\left(\frac{1-y_{\mathrm{A}}}{1-y_{\mathrm{A}i}}\right) = k_y \frac{y_{\mathrm{A}i}-y_{\mathrm{A}}}{\overline{(1-y_{\mathrm{A}})}_L} \cong k_y(y_{\mathrm{A}i}-y_{\mathrm{A}})$$

k'_y

where

$$\begin{aligned}\overline{(1-y_{\mathrm{A}})}_L &= \frac{(1-y_{\mathrm{A}})-(1-y_{\mathrm{A}i})}{\ln\left[(1-y_{\mathrm{A}})/(1-y_{\mathrm{A}i})\right]} \\ &= \frac{y_{\mathrm{A}i}-y_{\mathrm{A}}}{\ln\left[(1-y_{\mathrm{A}})/(1-y_{\mathrm{A}i})\right]}\end{aligned}$$

If transport is by pure molecular diffusion then

$$k_y = \frac{Dc}{L} \quad = \frac{D_v \rho_M}{B_T}$$ in book notation

With flow $k_y > \dfrac{Dc}{L}$

1. Mass transfer coefficients for various solute concentration units

$$k_y(y_{Ai} - y_A) = k_y(y_{Ai} - y_A) \times \frac{c}{c} = \boxed{\frac{k_y}{c}}(c_{Ai} - c_A)$$

$$= k_c(c_{Ai} - c_A)$$

$$\begin{aligned} k_y(y_{Ai} - y_A) &= k_y(y_{Ai} - y_A) \times \frac{P/RT}{c} \\ &= \frac{k_y}{cRT}(Py_{Ai} - Py_A) \\ &= \boxed{\frac{k_c}{RT}}(p_{Ai} - p_A) \\ &= k_G(p_{Ai} - p_A) \end{aligned}$$

Correlations generally given for k_c, which has the dimensions of a velocity (units e.g. m/s)

2. Dimensionless groups

Definitions

L = characteristic length (= diameter for round objects — cylinder and sphere — called D_p in book and “diam” by Elroy, who reserves the symbol D for diffusion coefficients)

U = characteristic velocity

ρ = fluid density

μ = fluid viscosity

ν = fluid kinematic viscosity = μ/ρ

Fluid is mixture of solute A and solvent B. The concentration of A varies with position (decreasing with increasing distance from a surface that A is being transported away from). Fluid properties are well approximated by pure solvent B properties for dilute solutions.

Dimensionless groups that are inputs to correlations

Reynolds number

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{(\rho U) L}{\mu} = \frac{G L}{\mu} \quad \text{where} \quad G = \rho U \text{ = mass velocity}$$

$$= \frac{U L}{\mu / \rho} = \frac{U L}{\nu} \quad \text{where} \quad \nu = \mu / \rho \text{ = kinematic viscosity}$$

Schmidt number

$$\mathrm{Sc} = \frac{\nu}{D} = \frac{\mu}{\rho D}$$

Dimensionless groups that are outputs of correlations

Sherwood number

$$\mathrm{Sh} = \frac{k_c L}{D} \quad \text{[so that } k_c = (D/L) \times \mathrm{Sh}\text{]}$$

Alternate non-dimensionalization of mass transfer coefficient is the Stanton number for mass transfer

$$\mathrm{St}_M = \frac{k_c}{U} = \frac{k_c L}{D} \times \frac{\nu}{UL} \times \frac{D}{\nu} = \frac{k_c L}{D} \times \left(\frac{UL}{\nu} \times \frac{\nu}{D}\right)^{-1} = \frac{\mathrm{Sh}}{\mathrm{Re\ Sc}}$$

Colburn j factor for mass transfer

$$j_M = \frac{k_c}{U}\left(\frac{\mu}{\rho D}\right)^{2/3} = \mathrm{St}_M\ \mathrm{Sc}^{2/3} = \frac{\mathrm{Sh}}{\mathrm{Re\ Sc}^{1/3}}$$

Why introduce the Colburn j factor? Because it was found to be approximately equal to the ½ times the Fanning friction factor for f_{Fanning} for turbulent flow in smooth pipes

$$j_M = \mathrm{St}_M\ \mathrm{Sc}^{2/3} = \frac{\mathrm{Sh}}{\mathrm{Re}\ \mathrm{Sc}^{1/3}} \quad = \quad \frac{f_{\mathrm{Fanning}}}{2}$$

This so-called Colburn or Colburn–Chilton analogy allows fluid mechanical correlations to inform mass transfer

"In general, j_M is a function of Re." In other words, j_M does a pretty good job of encapsulating the Schmidt number dependence of mass transfer coefficients.

3. Commonly used correlations

Be **sure** to see textbook for ranges of validity and caveats

Mass transfer to walls of pipe with turbulent flow

$$\mathrm{Sh} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Sc}^{1/3}$$

Equivalent to

$$j_M = 0.023\ \mathrm{Re}^{-0.2}$$

Mass transfer in gas phase within wetted wall tower

Here the “wall” for the gas is the wavy surface of the liquid running down the tower

$$\mathrm{Sh} = 0.023\ \mathrm{Re}^{0.81}\ \mathrm{Sc}^{0.44}$$

(not very different from the preceding correlation)

Flow perpendicularly past a single cylinder

$$\mathrm{Sh} = 0.61\ \mathrm{Re}^{1/2}\ \mathrm{Sc}^{1/3}$$

Flow past a single sphere

$$\mathrm{Sh} = 2.0 + 0.6\ \mathrm{Re}^{1/2}\ \mathrm{Sc}^{1/3}$$

Flow through a packed bed of spherical particles

$$\mathrm{Sh} = 1.17\ \mathrm{Re}^{0.585}\ \mathrm{Sc}^{1/3}$$

Equivalent to

$$j_M = 1.17\ \mathrm{Re}^{-0.415}$$

Note that the G in the Reynolds number is based on the **superficial** fluid velocity, i.e., volumetric flow rate divided by **total** cross-sectional area of the bed (even though the part of the cross-sectional area is blocked by the spheres)