CE407 Exam 02 July 03, 2020

1. **(40 pts)** 500 kg/hr of a feed solution containing a solute (C) mass fraction of 0.25 is to be extracted using 125 kg/hr of a recycled solvent stream (solute C mass fraction = 0.04, solvent mass fraction B = 0.95. The exiting raffinate shall be 0.10 mass fraction solute on a solvent free basis. Use the following equilibria data and phase diagrams on next pages to determine how many extraction stages are required. ***Use the McCabe-Thiele method to determine the required number of stages.***

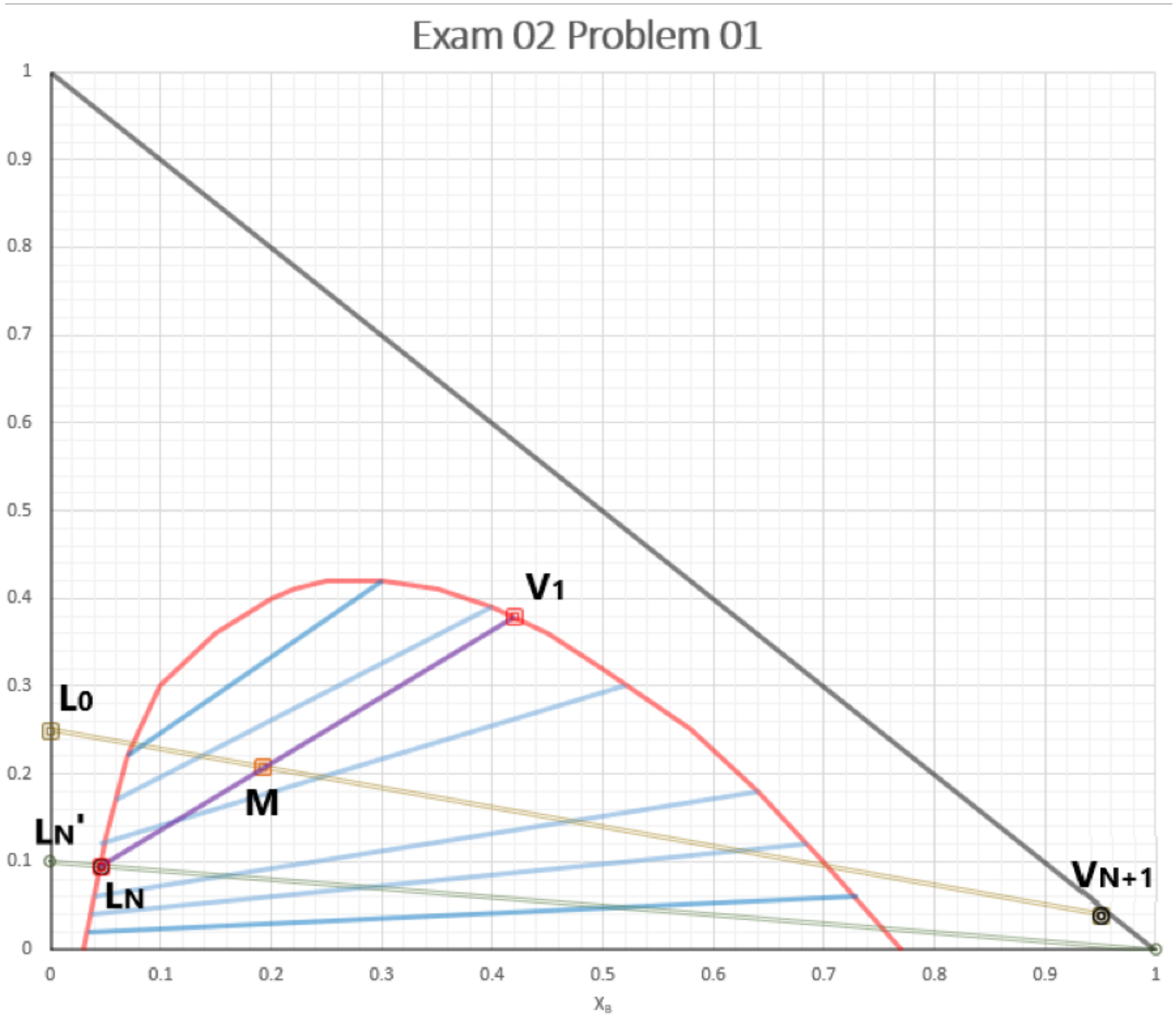
Diluent Rich (Raffinate)		Solvent Rich (Extract)	
x (x_B)	**y (x_C)**	**x (x_B)**	**y (x_C)**
0.07	0.22	0.30	0.42
0.06	0.17	0.40	0.39
0.045	0.12	0.52	0.30
0.04	0.06	0.64	0.18
0.038	0.04	0.685	0.12
0.035	0.02	0.73	0.06

Solution

- Step 1
  - Mark $\mathbf{L_0}$ at (0, 0.25)
  - Mark $\mathbf{L_N'}$ at (0, 0.10)
  - Mark $\mathbf{V_{N+1}}$ at (0.95, 0.04)
- Step 2
  - Draw line from pure solvent (1, 0) to $\mathbf{L_N'}$**.** The point where this line intersects the diluent rich side of the phase boundary is $\mathbf{L_N}$ and is roughly equal to (0.045,0.095). $\mathbf{L_N}$ is approximately 0.095.
- Step 3
  - Draw line from $\mathbf{L_0}$ to $\mathbf{V_{N+1}}$ and add Mixture point

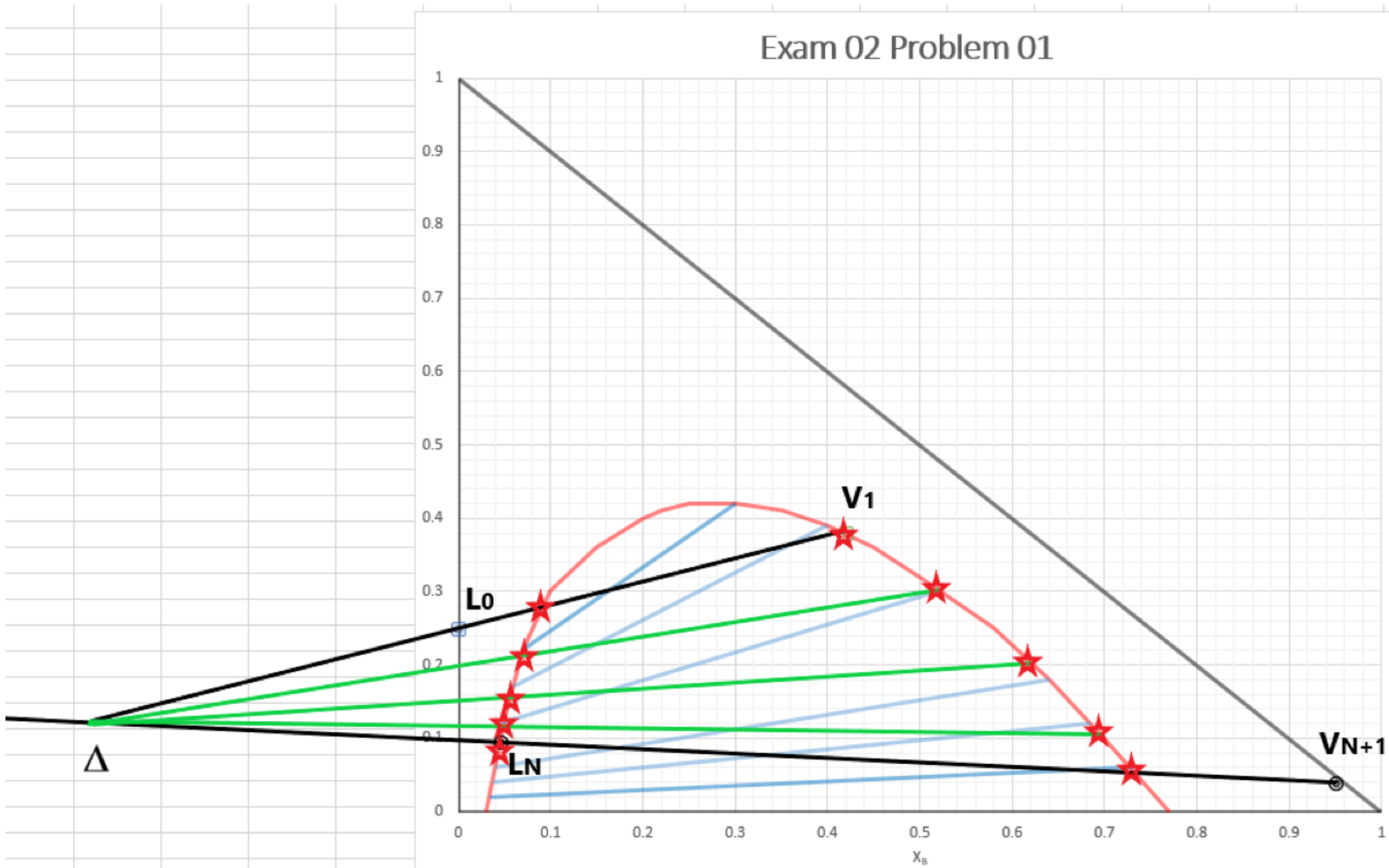
$$x_M = \frac{F x_F + V_{N+1} x_S}{F + V_{N+1}} = \frac{500 * 0.25 + 125 * 0.04}{500 + 125} = 0.208$$

- Step 4
    - Draw line from L_N through **M** until it extends to the solvent rich side of the phase boundary. **V_1 = 0.38 = x_c.**



- Step 5
    - (Switch to a new graph with just $\mathbf{L_N}$, $\mathbf{L_0}$, $\mathbf{V_1}$, and $\mathbf{V_{N+1}}$ shown)
    - Draw lines $\overline{\boldsymbol{V_1 L_0}}$ and $\overline{\boldsymbol{V_{N+1} L_N}}$, their intersection is the Δ point.
- Step 6
    - Anchor your rule on the delta point
    - Mark off pairs of points for the operating line. Where the ruler crosses the Raffinate (left-hand) boundary of the phase boundary you will record $\mathbf{x_c}$ as **x**. Where the ruler crosses the right-hand (Extract) boundary record $\mathbf{x_c}$ as **y**.
    - Choose enough points to generate a reasonably smooth operating curve.

○



○ **Operating Line**

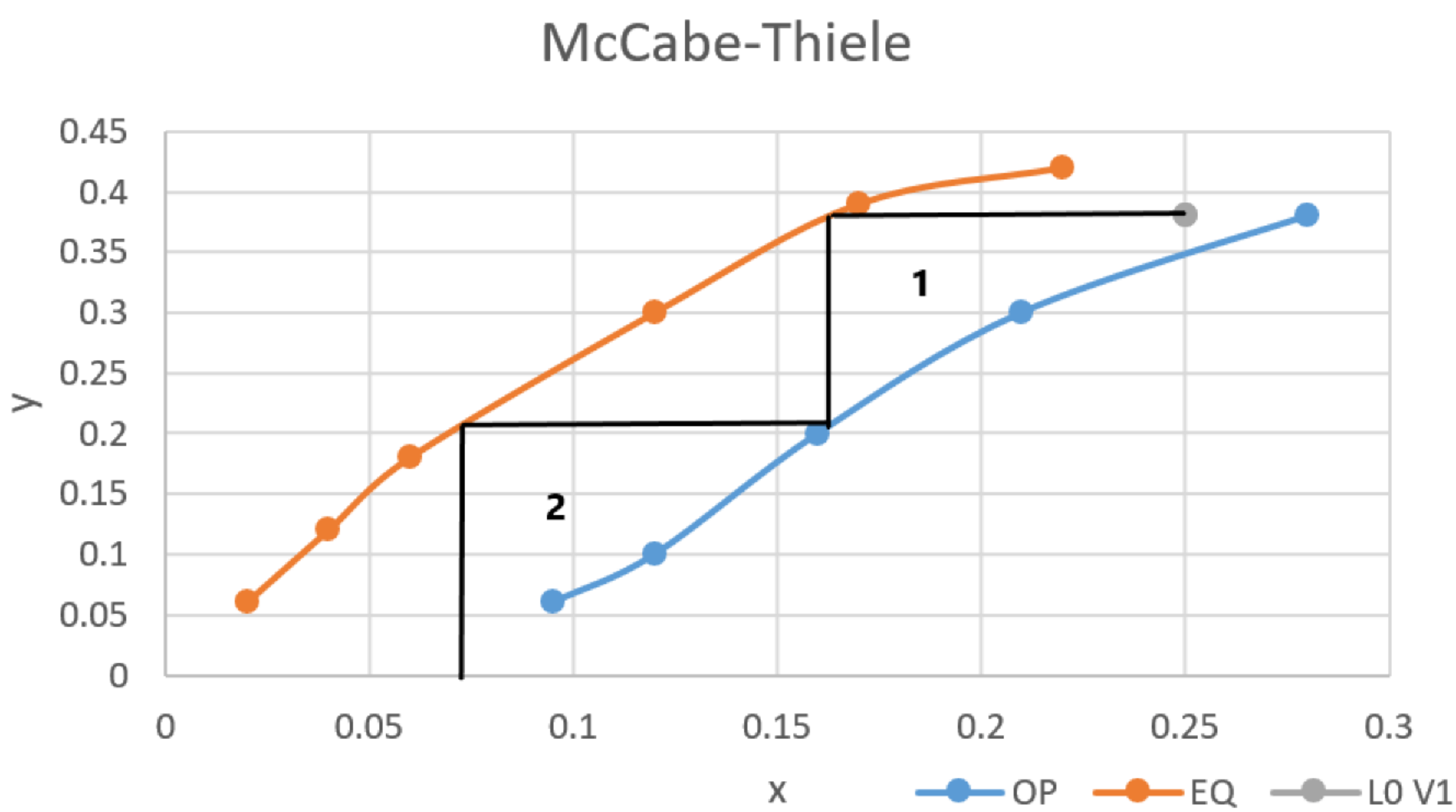
x	y
0.28	0.38
0.21	0.30
0.16	0.20
0.12	0.10
0.095	0.06

- Step 7
  - **Generate Equilibrium Curve**
  - From data given in problem statement, take the $\mathbf{x_c}$ value for Raffinate as **x** and the $\mathbf{x_c}$ value for the Extract as **y**

x (xC)	y (xC)
0.22	0.42
0.17	0.39
0.12	0.30
0.06	0.18
0.04	0.12
0.02	0.06

- Step 8
  - Plot Equilibrium and Operating Curves
  - Mark the point $(\boldsymbol{L_0}, \boldsymbol{V_1}) = (\mathbf{0.25}, \mathbf{0.38})$
  - Step off stages from $(\boldsymbol{L_0}, \boldsymbol{V_1})$ until you reach $\boldsymbol{L_N}$ which is when $\boldsymbol{x} = \mathbf{0.095}$

This separation requires 2 stages


McCabe-Thiele
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
y
1
2
0
0.05
0.1
0.15
0.2
0.25
0.3
x
OP
EQ
L0 V1

2. **(40 pts)** Benzene will be stripped from a valuable oil by countercurrent contact with air in a tower packed with 2.0" ceramic Raschig rings. The contaminated oil (composition 98 mole % oil and 2 mole % benzene) will enter the tower at 2500 mol/hr and 95% of the entering benzene is to be removed. The flow rate of the incoming air will be 37,500 mol/hr. The density and viscosity of the dilute oil/benzene solution are well approximated by the properties of pure oil. The vapor phase behaves ideally. The tower will operate isothermally at 25 C and at a total pressure of 1 atm. The tower diameter shall be determined to give ΔP/ft of packing equal to 50% that of ΔP_{flood}/ft. The equilibrium curve at these conditions is **y = 0.125 x**. You may regard the operating line as being straight. Calculation of the flooding velocity should be based on flow rates at the **top** of the tower, where they are largest. The density of the vapor at the top of the column can be approximated as that of air.

Data:
Air:
MW = 28.9

Benzene:
MW = 78.11
Sc = 1.76 in air
Sc = 3500 in oil

Oil:
MW = 106
ρ = 0.83 g/cm^3
ν = 2.84 cSt
μ = 2.36 cP

Packing:
Table included in attachments
Gas Constant = 0.73024 ft^3 atm / (R lbmol) R is degrees Rankine
Gas Constant = 82.05745 cm^3 atm / (K mol)

a. What is the required diameter for the tower? Use the following graph and the correlation $\Delta P_{flood}/ft = 0.115\ F_P^{0.7}$ inches water column per foot of packing

b. What height of packing is required? Base your solution on y – y*. Use the correlations from lecture to determine H_x and H_y.

$$H_{Oy} = H_y + m\,\frac{V}{L}\,H_x$$

Solution: Problem states that 95% of the Benzene will be removed from the liquid stream:

Moles Benzene in the Liquid stream at the top, a: = 0.02 * 2500 = 50 moles benzene

Moles Benzene in the liquid stream at the bottom, b: = (1 - 0.95) * 50 = 2.5 moles benezene

Mole fraction of Benzene in the liquid stream at the bottom:

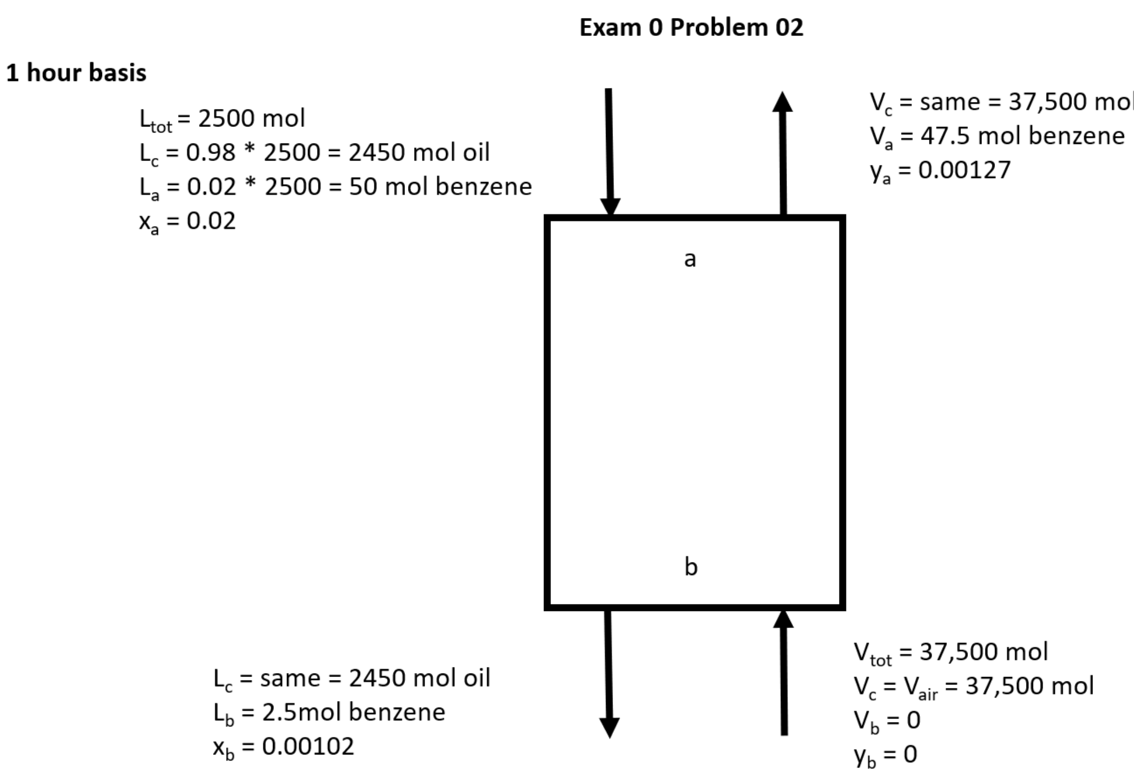
$$x_b = \frac{2.5}{2.5 + 2450} = 0.00102$$

Moles of benzene in the vapor stream at top:

50 moles entering with liquid at the top + 0 moles entering with vapor at the bottom – 2.5 moles exiting with liquid at the bottom = **47.5 moles benzene exiting with vapor at the top**

Mole fraction benzene in exiting vapor:

$$y_a = \frac{47.5}{47.5 + 37500} = 0.00127$$



a. Now we use the flooding correlation given:

$$\Delta P_{flood} = 0.115 * F_P^{0.7}$$

(Note that for F_p = 65 this correlation actually should be ignored and we can just go with $\Delta P_{flood} = 2.0\text{ " } WC$). But as you were directed to use the above correlation, I will show that calculation.

From the data table given we see that for 2 “ ceramic Raschig Rings **F_p = 65** and **f_p = 0.92**

$$\Delta P_{flood} = 0.115 * 65^{0.7} = 2.14 \frac{"H20}{ft}$$

We are instructed to work at 50% of the flooding pressure drop, therefore we will use 1.07 “ water per foot of packing.

To use the attached chart we must calculate $\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}}$

As indicated in the problem statement, we will evaluate this at the top of the tower where flooding is the most likely to happen.

Although we do not know the cross-sectional are required to calculate **G_x** or **G_y**, the ratio of them is the same as the ratio of the mass flows of the liquid and vapor.

Mass flow of vapor:

At top = 37500 mol air/hr * 28.9 g / mol + 47.5 mol benzene/hr * 78.11 g /mol = 1087.5 kg/hr = $S * G_y$

Mass flow of liquid:

At top = 2450 mol oil/hr * 106 g/mol + 50 mol benzene/hr * 78.11 g/mol = 263.6 kg/hr = $S * G_x$

$$\frac{G_x}{G_y} = \frac{S * G_x}{S * G_y} = \frac{263.6}{1087.5} = 0.243$$

The density of the liquid can be approximated as the density of oil,

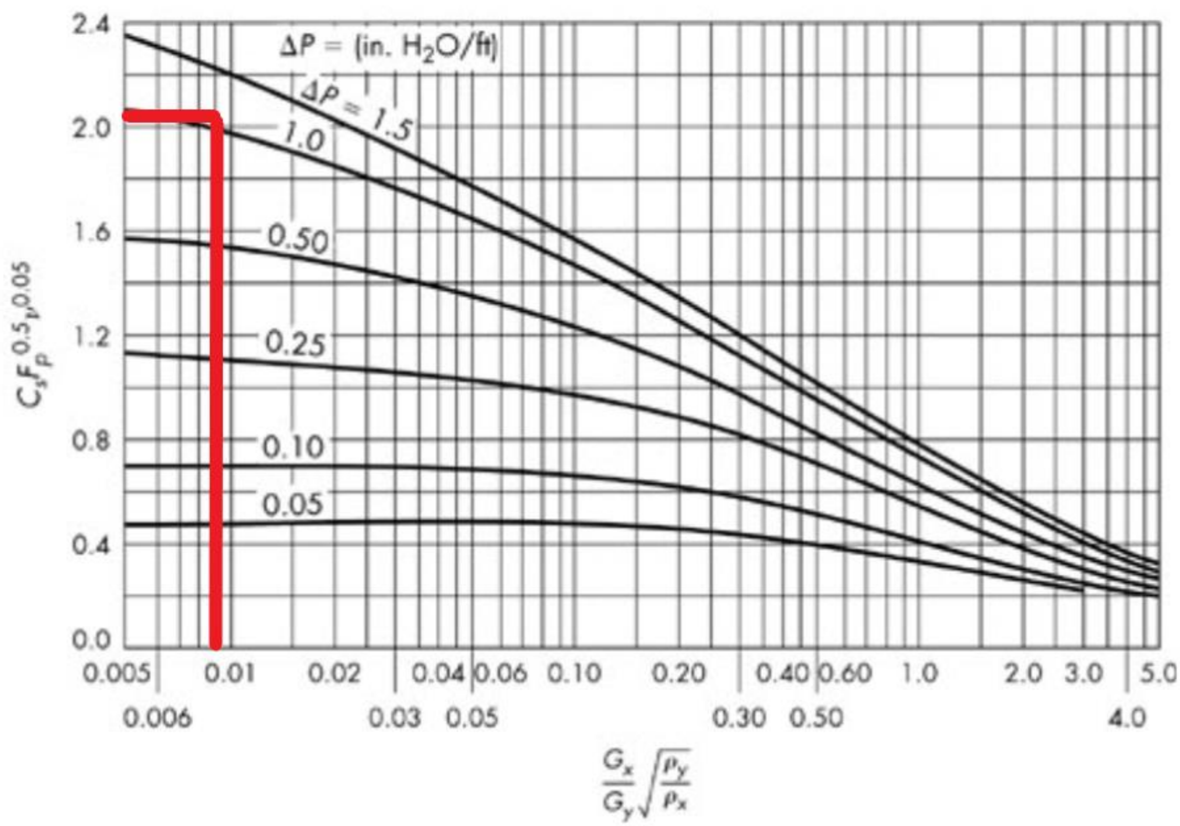
$$\rho_x = 0.83\ g/cm^3$$

The density of the vapor can be obtained using the ideal gas law:

$$\rho_y = \frac{MW\ P}{RT} = \frac{28.9\ \frac{g}{mol}\ 1\ atm}{82.05745\ \frac{cm^3 atm}{K\ mol}\ 298.15\ K} = 0.00118\ \frac{g}{cm^3}$$

Now:

$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}} = 0.243\sqrt{\frac{0.00118}{0.83}} = 0.0092$$



From graph we can read (by interpolating between the lines from 1.0 and 1.5 "water/ft) that

$$C_s\, F_p^{0.5}\, \nu^{0.05} = 2.0$$

$$C_s\, 65^{\,0.5}\, 2.84^{\,0.05} = 2.0$$

$$C_s = 0.235$$

By definition:

$$C_s = u_0 \sqrt{\frac{\rho_y}{\rho_x - \rho_y}}$$

$$0.235 = u_0 \sqrt{\frac{0.00118}{0.83 - 0.00118}} = 0.0377\, u_0$$

$$u_0 = 6.23\, ft/s$$

Calculate volumetric flow:

$$1087.5\, \frac{kg}{hr} * \frac{1000\, g}{kg} * \frac{cm^3}{0.00118\, g} * \frac{hr}{3600\, s} * \left(\frac{in}{2.54\, cm}\right)^3 * \left(\frac{ft}{12\, in}\right)^3 = 9.04\, \frac{ft^3}{s}$$

The required area is:

$$Area = \frac{volumetric\ flow}{linear\ velocity} = \frac{9.04\ \frac{ft^3}{s}}{6.23\ ft/s} = 1.45\ ft^2 = \frac{\pi D^2}{4}$$

$$D = 1.36\ ft$$

b) Now that we have the cross-sectional area we can calculate **G$_x$** and **G$_y$** independently:

Mass flow of vapor:

1087.5 kg/hr * (2.2 lb/kg) = 2397.5 lb/hr = $S * G_y$

Mass flow of liquid:

263.6 kg/hr * (2.2 lb/kg) = 581.1 lb/hr = $S * G_x$

$$G_x = \frac{581.1\ \frac{lb}{hr}}{1.45\ ft^2} = 400.8\ \frac{lb}{ft^2\ hr}$$

$$G_y = \frac{2397.5\ \frac{lb}{hr}}{1.45\ ft^2} = 1653.4\ \frac{lb}{ft^2\ hr}$$

We can now calculate the height of the transfer unit:

$$H_x = 0.9\ ft \left(\frac{G_x/\mu}{1500\ \frac{lb}{ft^2\ hr} \Big/ 0.891\ cP} \right)^{0.3} \left(\frac{S_c}{381} \right)^{0.5} \frac{1}{f_p}$$

$$H_x = 0.9\ ft \left(\frac{400.8\ \frac{lb}{ft^2\ hr} \Big/ 2.36\ cP}{1500\ \frac{lb}{ft^2\ hr} \Big/ 0.891\ cP} \right)^{0.3} \left(\frac{3500}{381} \right)^{0.5} \frac{1}{0.92} = 1.48\ ft$$

$$H_y = 1.4\ ft \left(\frac{G_y}{500\ \frac{lb}{ft^2\ hr}} \right)^{0.3} \left(\frac{1500\ \frac{lb}{ft^2\ hr}}{G_x} \right)^{0.4} \left(\frac{S_c}{0.66} \right)^{0.5} \frac{1}{f_p}$$

$$H_y = 1.4\ ft \left(\frac{1653.4}{500\ \frac{lb}{ft^2\ hr}} \right)^{0.3} \left(\frac{1500\ \frac{lb}{ft^2\ hr}}{400.8} \right)^{0.4} \left(\frac{1.76}{0.66} \right)^{0.5} \frac{1}{0.92} = 6.03\ ft$$

Now we can calculate height of overall transfer unit:

$$\boldsymbol{H_{Oy} = H_y + m\,\frac{V}{L} * \; H_x}$$

From problem statement we know that y = 0.125 x and therefore m = 0.125

$$V/_L = \frac{37500 + 47.5}{2500} = 15.0$$

Now:

$$\boldsymbol{H_{Oy} = 6.03\,ft + 0.125 * 15.0 * \; 1.48\,ft = 8.81\,ft}$$

Now we will calculate the number of transfer units:

$$\boldsymbol{N_{Oy} = \frac{y_b - y_a}{\overline{\overline{(y - \; y^*)_{lm}}}}}$$

$y_a = 0.00127$

$y_b = 0$

$y_a^* = 0.125 * \; x_a = 0.125 * 0.02 = 0.0025$

$y_b^* = 0.125 * \; x_b = 0.125 * 0.00102 = 0.00013$

$y_b - y_a = 0 - 0.00127 = -0.00127$

$y_a - y_a^* = 0.00127 - 0.0025 = -0.00123$

$y_b - y_b^* = 0 - 0.00013 = -0.00013$

$$\overline{\boldsymbol{(y - \; y^*)_{lm}}} = \frac{(y_a - y_a^*) - \; (y_b - y_b^*)}{\ln\frac{y_a - y_a^*}{y_b - y_b^*}} = \frac{-0.00123 - (-0.00013)}{\ln\frac{-0.00123}{-0.00013}} = \frac{-0.0011}{2.24723} = -0.0004895$$

$$\boldsymbol{N_{Oy} = \frac{y_b - y_a}{\overline{\overline{(y - \; y^*)_{lm}}}}} = \frac{-0.00127}{-0.0004895} = \boldsymbol{2.595}$$

The required height of the packing can be calculated as:

$$\boldsymbol{Z_t = H_{Oy} * \; N_{Oy} = 8.81\,ft * \; 2.595 = 22.9\,f}$$

3. (20 pts) *n*-Heptane undergoes mass transfer from a bulk gas (air + n-heptane) phase, where its mole fraction $y = 0.03$,to a bulk liquid (mineral oil + n-heptane) phase, where its mole fraction $x = 0.005$, through a gas-liquid interface. Temperature and pressure are 35 C and 1.0 atmosphere. The vapor pressure of n-Heptane at this temperature is 74.02 mm Hg. Mass transfer coefficients are as follows:

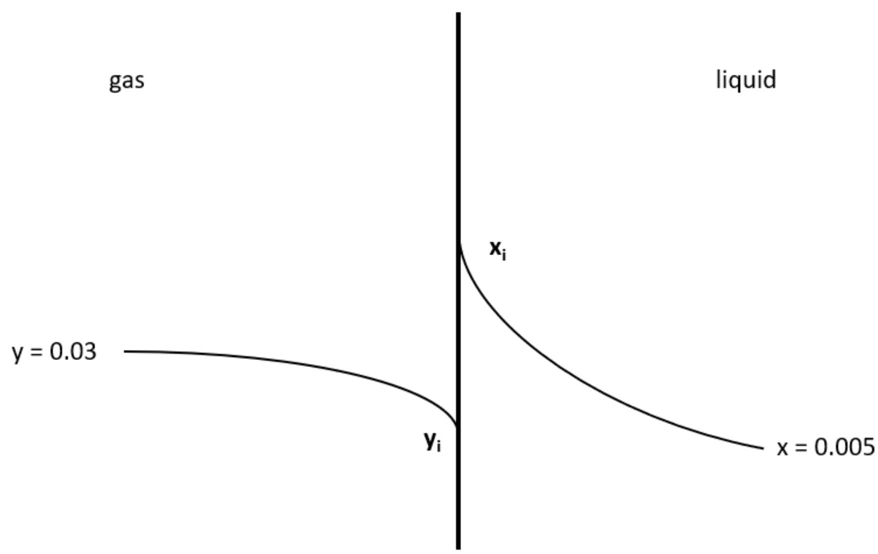
$$k_y = 7.0 * \ 10^{-6} \ \frac{mol}{cm^2 s}$$
$$k_x = 3.5 * \ 10^{-6} \ \frac{mol}{cm^2 s}$$

Assuming validity of Raoult's Law for the equilibrium relation,

a. What are the overall mass transfer coefficients K_y and K_x?
b. What is the molar flux of n-heptane from gas to liquid?

Solution:

Mole fractions refer to n-Heptane



a. **Overall Mass Transfer Coefficients**

$$\boldsymbol{K_y} = \left(\frac{\boldsymbol{1}}{\boldsymbol{k_y}} + \frac{\boldsymbol{m}}{\boldsymbol{k_x}}\right)^{-1} \text{ and } \boldsymbol{K_x} = \left(\frac{\boldsymbol{1}}{\boldsymbol{k_x}} + \frac{\boldsymbol{1}}{\boldsymbol{m\,k_y}}\right)^{-1}$$

Assuming Raoult's Law leads to equilibrium being expressed as:

$$\boldsymbol{y_i} = \frac{\boldsymbol{P^{sat}_{n-heptane}}}{\boldsymbol{P}} * \ \boldsymbol{x_i}$$

$$P_{n-heptane}^{sat} = 74.02\ mm\ Hg$$

$$y_i = \frac{74.02\ mm\ Hg}{760} * x_i = 0.09739\ x_i$$

$y_i = m * x_i = 0.09739\ x_i$, so $m = 0.09739$

$$K_y = \left(\frac{1}{k_y} + \frac{m}{k_x}\right)^{-1}$$

$$K_y = \left(\frac{1}{7.0 * 10^{-6} \frac{mol}{cm^2 s}} + \frac{0.09739}{3.5 * 10^{-6} \frac{mol}{cm^2 s}}\right)^{-1} = 5.859 * 10^{-6} \frac{mol}{cm^2 s}$$

$$K_x = \left(\frac{1}{k_x} + \frac{1}{m k_y}\right)^{-1}$$

$$K_x = \left(\frac{1}{3.5 * 10^{-6} \frac{mol}{cm^2 s}} + \frac{1}{0.09739 * 7.0 * 10^{-6} \frac{mol}{cm^2 s}}\right)^{-1} = 5.706 * 10^{-7} \frac{mol}{cm^2 s}$$

b. Molar Flux of *n*-Heptane

Calculate flux using overall coefficients (only need to do one method…)

Overall Gas Phase Calculation:

$N_{n-heptane} = K_y * (y - y^*)$ where $y^* = mx = 0.09739 * 0.005 = 4.870 * 10^{-4}$

$$N_{n-heptane} = 5.859 * 10^{-6} \frac{mol}{cm^2 s} * (0.03 - 4.870 * 10^{-4}) = 1.729 * 10^{-7} \frac{mol}{cm^2 s}$$

Overall Liquid Phase Calculation:

$N_{n-heptane} = K_x * (x^* - x)$ where $x^* = \frac{y}{m} = \frac{0.03}{0.09739} = 0.3080$

$$N_{n-heptane} = 5.706 * 10^{-7} \frac{mol}{cm^2 s} * (0.3080 - 0.005) = 1.729 * 10^{-7} \frac{mol}{cm^2 s}$$

To do the interfacial methods you must solve for the intefacial mole fractions:

Equal Molar Flux requirement -

$$k_y * (y - y_i) = k_x * (x_i - x)$$

Equilibrium requirement –

$$y_i = 0.09739\ x_i$$

Substitute equilibrium into molar flux… (drop the 10^{-6}, as it appears on both sides…)

$$7.0 * (0.03 - y_i) = 3.5 * \left(\frac{y_i}{0.09739} - 0.005\right)$$

$$y_i = \frac{7.0 * 0.03 + 3.5 * 0.005}{7 + {}^{3.5}/_{0.09739}}$$

$$y_i = 0.0053$$

$$x_i = \frac{y_i}{0.09739} = \frac{0.0053}{0.09739} = 0.0544$$

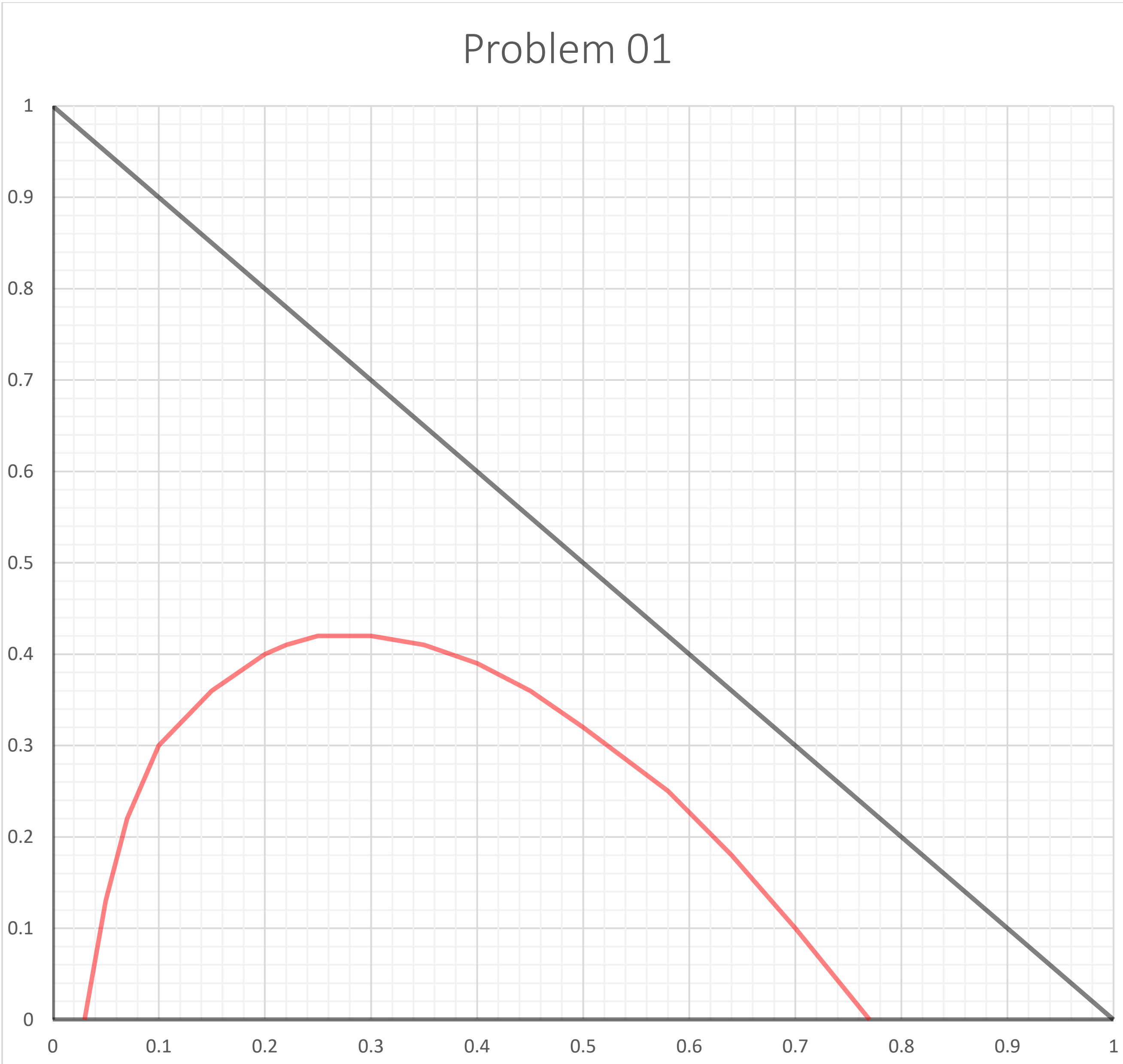
Gas Phase Calculation:

$$N_{n-heptane} = k_y * (y - y_i) = 7.0 * 10^{-6} \frac{mol}{cm^2 s} * (0.03 - 0.0053)$$
$$= 1.729 * 10^{-7} \frac{mol}{cm^2 s}$$

Liquid Phase Calculation

$$N_{n-heptane} = k_x * (x_i - x) = 3.5 * 10^{-6} \frac{mol}{cm^2 s} * (0.0544 - 0.005)$$
$$= 1.729 * 10^{-7} \frac{mol}{cm^2 s}$$

All methods to calculate the flux lead to the same answer!


Problem 01
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
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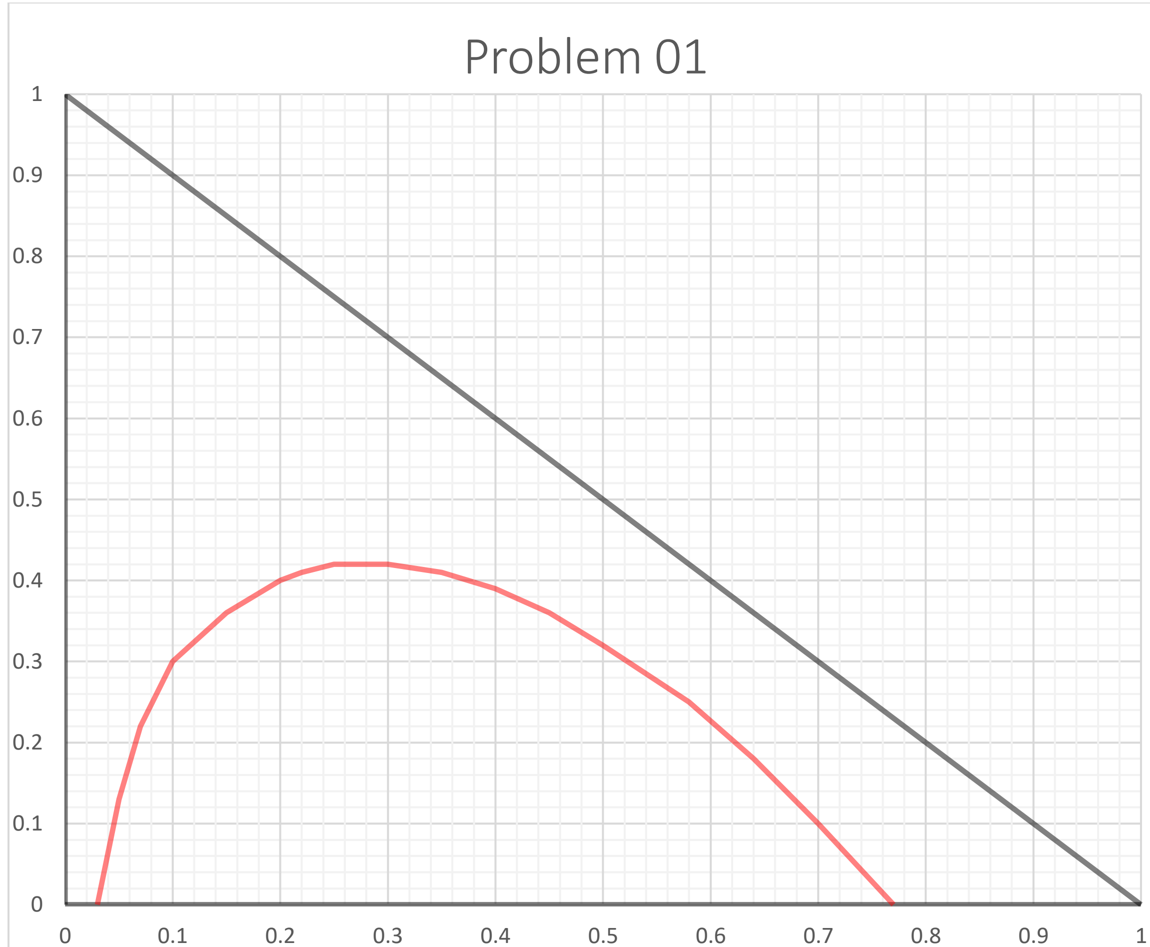
0.6
0.7
0.8
0.9
1


Problem 01
1
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1

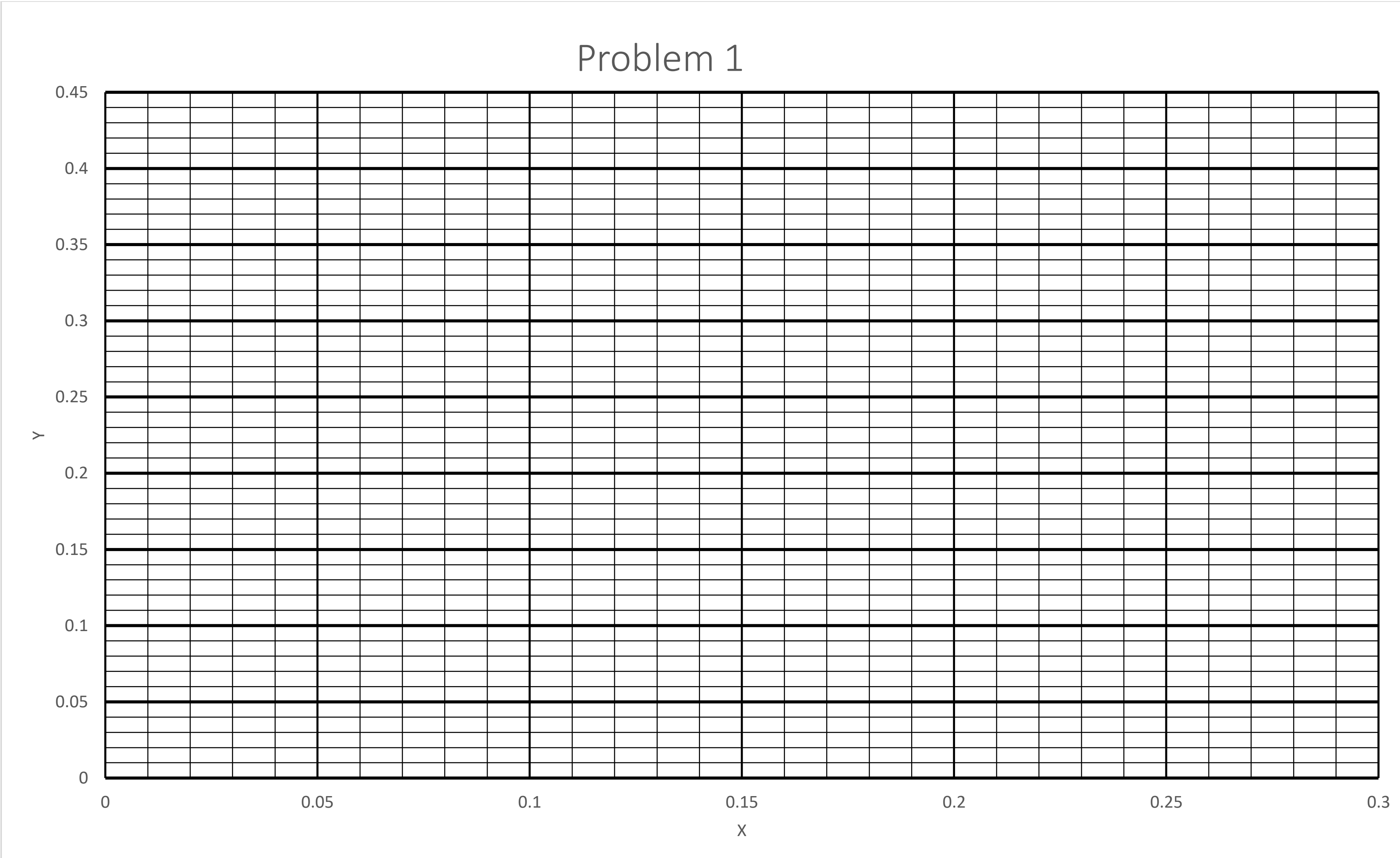
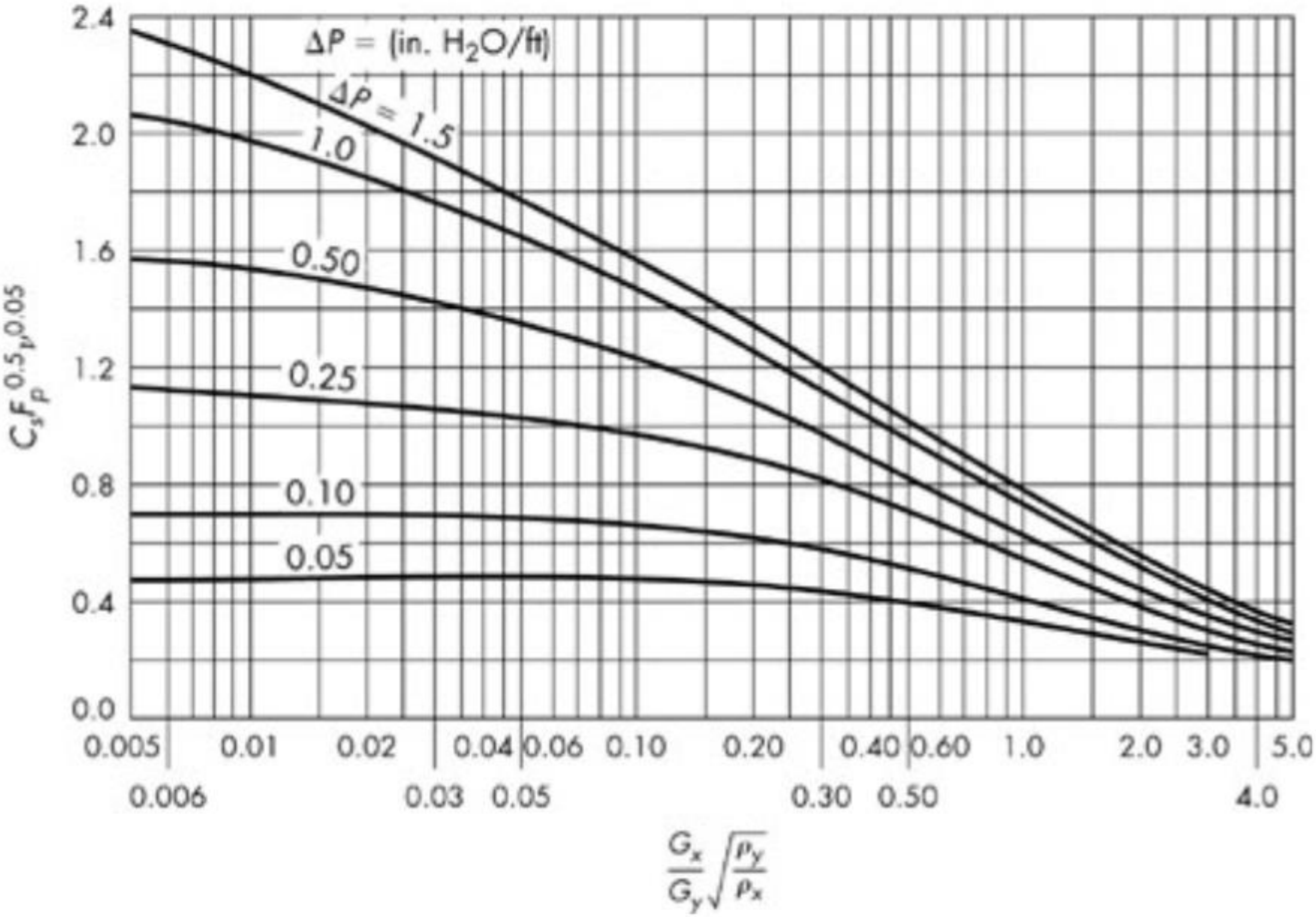

Problem 1
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Y
0
0.05
0.1
0.15
0.2
0.25
0.3
X

TABLE 18.1
Characteristics of dumped tower packings[12,15b,27]

Type	Material	Nominal size, in.	Bulk density,† lb/ft^3	Total area,† ft^2/ft^3	Porosity ε	Packing factors† F_p	f_p
Raschig rings	Ceramic	$\frac{1}{2}$	55	112	0.64	580	1.52§
		1	42	58	0.74	155	1.36§
		$1\frac{1}{2}$	43	37	0.73	95	1.0
		2	41	28	0.74	65	0.92§
Pall rings	Metal	1	30	63	0.94	56	1.54
		$1\frac{1}{2}$	24	39	0.95	40	1.36
		2	22	31	0.96	27	1.09
	Plastic	1	5.5	63	0.90	55	1.36
		$1\frac{1}{2}$	4.8	39	0.91	40	1.18
Berl saddles	Ceramic	$\frac{1}{2}$	54	142	0.62	240	1.58§
		1	45	76	0.68	110	1.36§
		$1\frac{1}{2}$	40	46	0.71	65	1.07§
Intalox saddles	Ceramic	$\frac{1}{2}$	46	190	0.71	200	2.27
		1	42	78	0.73	92	1.54
		$1\frac{1}{2}$	39	59	0.76	52	1.18
		2	38	36	0.76	40	1.0
		3	36	28	0.79	22	0.64



ΔP = (in. H2O/ft)
ΔP = 1.5
1.0
0.50
0.25
0.10
0.05
2.4
2.0
1.6
1.2
0.8
0.4
0.0
$C_s F_p^{0.5} \nu^{0.05}$
0.005
0.006
0.01
0.02
0.03
0.04
0.05
0.06
0.10
0.20
0.30
0.40
0.50
0.60
1.0
2.0
3.0
4.0
5.0
$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}}$