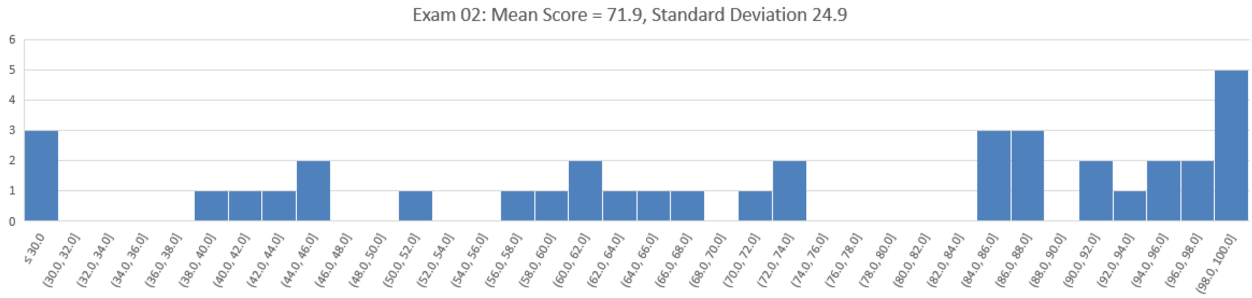
CE407 Separations **Solution Exam 02** **November 20, 2025**



1. **(25 points)** A multi-component fractionating column equipped with a total condenser is devised to separate the following feed stream:

Component	Mole fraction, $x_{i,F}$	Relative Volatility, α_{ij}
1) Ethanol (LK)	0.40	3.95
2) Propanol	0.05	2.05
3) Butanol (HK)	0.45	1.00
4) Pentanol	0.10	0.50

If the column is designed to give a **95%** recovery of Ethanol in the distillate and a **99%** recovery of Butanol in the bottoms, what will be the mole fraction of each component in the distillate and bottoms streams?

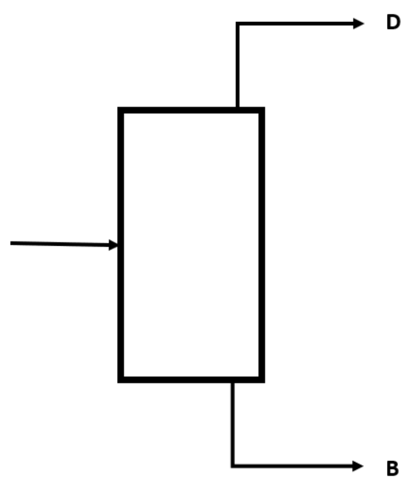
Solution

Applying the stated recoveries leads to the following information:

100 mole basis

100 mole total
1: 40 moles (LK)
2: 5 moles

3: 45 moles (HK)
4: 10 moles



1: 0.95 * 40 = 38.0 moles (LK)
2: δ moles
3: 45 – 44.55 = 0.45 moles (HK)
4: 0 moles

Legend
1: Ethanol
2: Propanol
3: Butanol
4: Pentanol

1: 40 – 38.0 = 2.00 moles (LK)
2: 5 - δ moles
3: 0.99 * 45 = 44.55 moles (HK)
4: 10 moles

Next, we use the Fenske equation to solve for N_{min} for this system

- $i = 1, j = 3$

$$N_{min} + 1 = \frac{ln\left[\frac{Dx_{1D}/Bx_{1B}}{Dx_{3D}/Bx_{3B}}\right]}{ln\,\overline{\alpha_{1,3}}} = \frac{ln\left[\frac{38.0/2.00}{0.45/44.55}\right]}{ln\,3.95} = 5.4884$$

Now, we apply the Fenske equation and the known value for N_{min} to the distributed component, Butanol, and the Heavy Key, Pentanol

- $i = 2, j = 3$

$$N_{min} + 1 = \frac{ln\left[\frac{Dx_{2D}/Bx_{2B}}{Dx_{3D}/Bx_{3B}}\right]}{ln\,\overline{\alpha_{2,3}}} = \frac{ln\left[\frac{\delta/(5-\delta)}{0.45/44.55}\right]}{ln\,2.05} = 5.4884$$

$$5.4884 * ln(2.05) = ln\left[\frac{\delta/(5-\delta)}{0.45/44.55}\right] = 3.9398$$

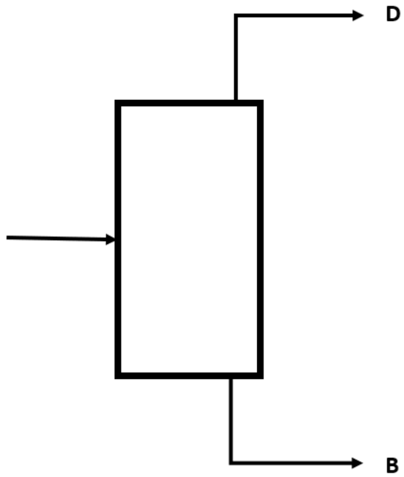
$$e^{3.9398} = \frac{\delta/(5-\delta)}{0.45/44.55} = 51.4083$$

$$\delta/(5-\delta) = 51.4083 * 0.45/44.55 = 0.5193$$

$$\delta = 1.71$$

$$5 - \delta = 3.29$$

100 mole basis



1: 38.0 moles (LK)
2: 1.71 moles
3: 0.45 moles (HK)
4: 0 moles
40.16 Total Moles

100 mole total
1: 40 moles (LK)
2: 5 moles

3: 45 moles (HK)
4: 10 moles

Legend
1: Ethanol
2: Propanol
3: Butanol
4: Pentanol

1: 2.00 moles (LK)
2: 3.29 moles
3: 44.55 moles (HK)
4: 10 moles
59.84 Total Moles

Component	x_{di}	x_{bi}
Ethanol	0.946	0.033
Propanol	0.043	0.055
Butanol	0.011	0.744
Pentanol	0.000	0.167

2. **(40 points)** A source of ore containing a valuable salt has been located. The mined material contains $\frac{0.17\ tons\ salt}{ton\ of\ total\ material}$. The flow rate of pure water entering stage N is 7.0 tons/hour. We need to recover 90% of the incoming salt.

If 4.0 tons of total material are to be processed per hour what is:

a) The concentration of the strong extract exiting stage 1
b) The required number of ideal stages

Data for solution retention on the ore is provided on the following page.

The mass fraction of the exiting raffinate is given as $x_N = 0.0453$
THEREFORE YOU DO NOT NEED TO ITERATIVELY SOLVE FOR x_N

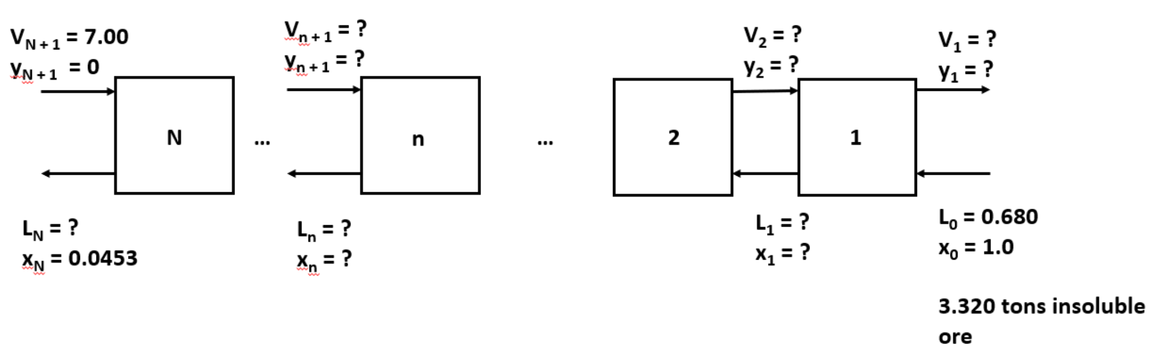
Solution

One hour basis:

$$4.0\ tons\ total\ material * \frac{0.17\ tons\ salt}{ton\ of\ total\ material} = 0.680\ tons\ salt$$

$$4.0\ tons\ total\ material - 0.510\ tons\ salt = 3.320\ tons\ ore$$

The battery of leaching vessels has the following information:



Problem states that we will recover 90% of the salt in the extract:

$$y_1\,V_1 = 0.9 * (x_0\,L_0) = 0.9 * 0.680 = 0.612\ tons\ salt$$

First we use a **salt balance across the entire battery:**

$$y_{N+1}\,V_{N+1} + x_0\,L_0 = x_N\,L_N + y_1\,V_1$$

$$0 * 7.00 + 0.680 = x_N\,L_N + 0.612$$

$$x_N\,L_N = 0.680 - 0.612 = 0.068\ ton\ salt$$

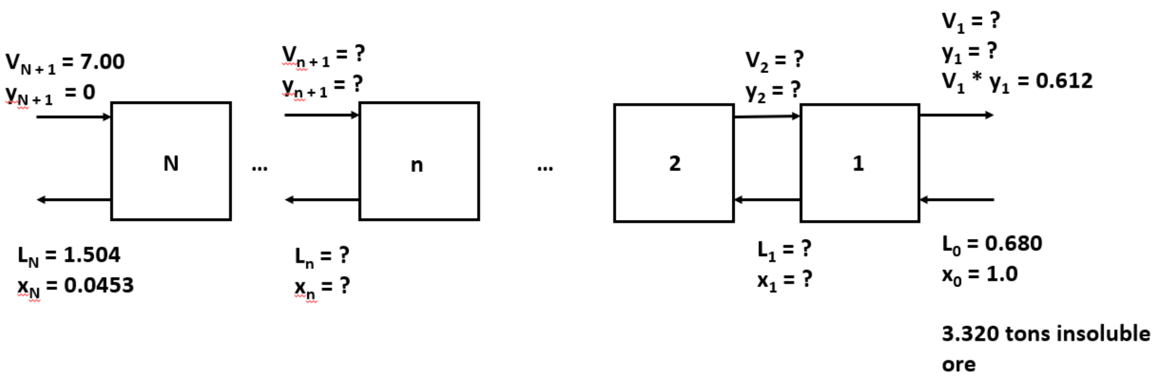
Solve for $\boldsymbol{L_N}$

$\boldsymbol{x_N = 0.0453}$

We interpolate the retention data to obtain $\mathbf{0.4 + \frac{0.0453 - 0.04}{0.06 - 0.04} * (0.6 - 0.4) = 0.453\ \frac{\textit{tons solution}}{\textit{ton raw ore}}}$

$$\therefore\ \boldsymbol{L_N = 0.453\ \frac{\textit{tons solution}}{\textit{ton raw ore}} * 3.320\ \textit{ton ore} =\ 1.504\ \textit{ton solution}}$$

Solution Balance across the entire battery:



$$\boldsymbol{L_0 + V_{N+1} = L_N + V_1}$$

$$\mathbf{0.680 + 7.00 = 1.504 +} \boldsymbol{V_1}$$

$$\boldsymbol{V_1 = 6.176}$$

$$\boldsymbol{y_1 * V_1 = 0.612}$$

$$\boldsymbol{y_1 = \frac{0.612}{6.176} = 0.099}$$

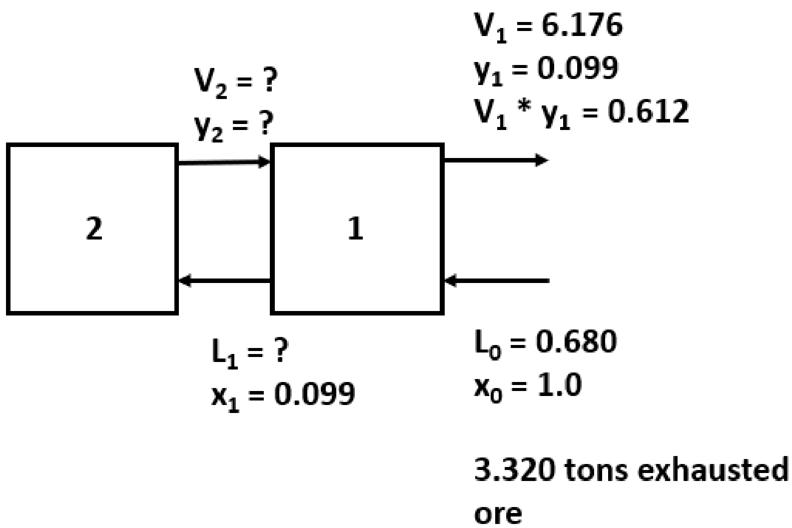
a) $\boldsymbol{y_1 = 0.099}$

In order to determine the number of stages we need to construct an operating line. We already have two points:

$(x_N, y_{N+1}) = (\mathbf{0.0453, 0})$ and $(x_0, y_1) = (\mathbf{1.0, 0.099})$

The next step is to do balances around stage 1:

We know that for ideal stages $x_n = y_n$, therefore $x_1 = y_1 = \mathbf{0.099}$



We interpolate the retention data to obtain $\mathbf{0.9} + \frac{0.099-0.10}{0.10-0.08} * (\mathbf{0.9} - \mathbf{0.8}) = \mathbf{0.895}\ \frac{\textit{tons solution}}{\textit{ton raw ore}}$

$\boldsymbol{L_1 = 0.895 * 3.320 = 2.971\ ton\ solution}$

Solution Balance

$$\boldsymbol{L_0 + V_2 = L_1 + V_1}$$

$$\mathbf{0.680} + \boldsymbol{V_2} = \mathbf{2.971} + \mathbf{6.176}$$

$$\boldsymbol{V_2 = 8.467\ tons}$$

Salt Balance

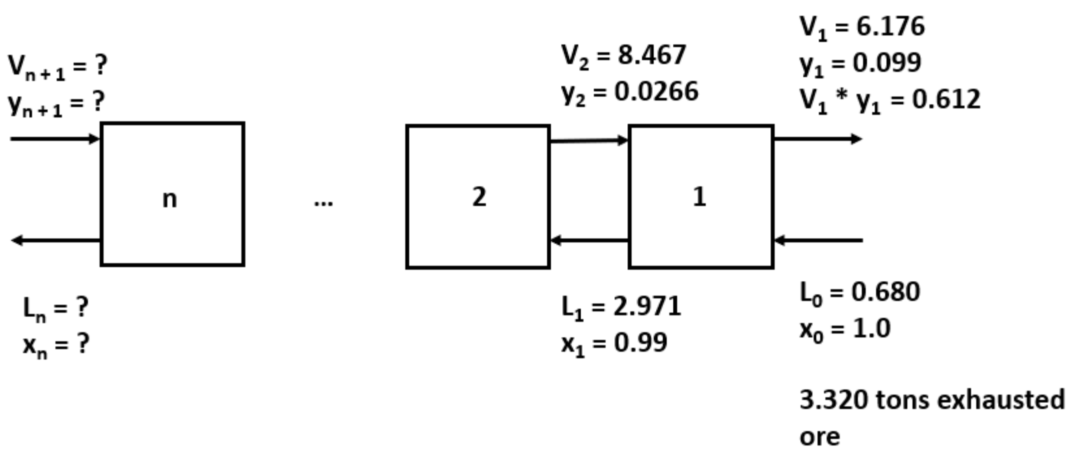
$$\boldsymbol{x_0\, L_0 + y_2\, V_2 = x_1\, L_1 + y_1\, V_1}$$

$$\mathbf{0.680} + \boldsymbol{y_2} * \mathbf{8.467} = \mathbf{0.099} * \mathbf{2.971} + \mathbf{0.099} * \mathbf{6.176}$$

$$y_2 = 0.0266$$

Now we have the point $(x_1, y_2) = (0.099, 0.0266)$

Now we will do balances containing stages 1 to n:



We can choose an arbitrary value for x_n due to the fact that the stage n is an arbitrary location in the battery.

Choose $x_n = 0.08$ this leads to $0.80 \frac{tons\ solution}{ton\ raw\ ore}$ from the retention data.

Therefore $L_n = 0.80 * 3.320 = 2.656\ ton\ solution$

Solution Balance

$$L_0 + V_{n+1} = L_n + V_1$$

$$0.680 + V_{n+1} = 2.656 + 6.176$$

$$V_{n+1} = 8.152\ ton\ solution$$

Salt Balance

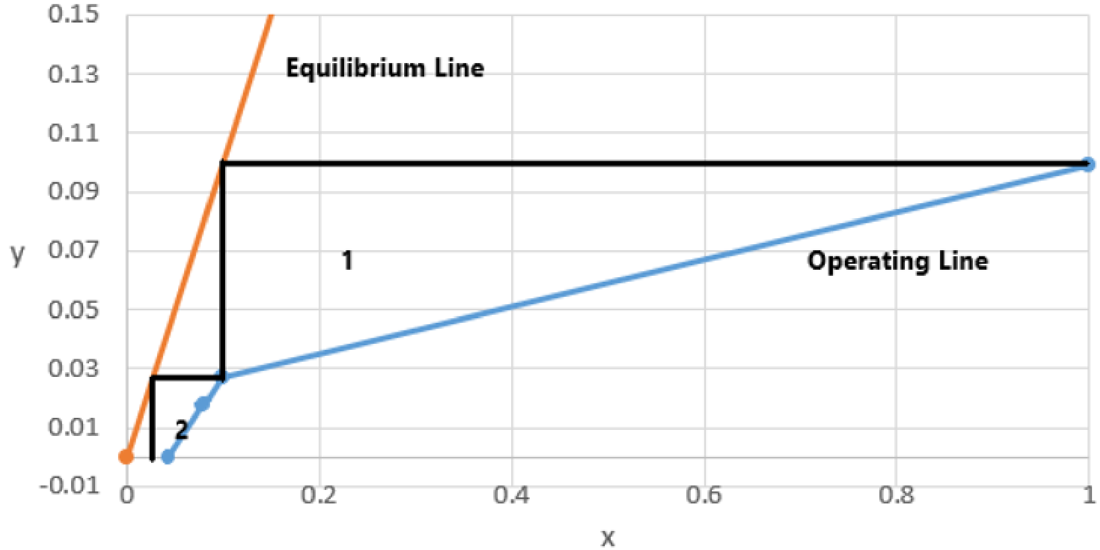
$$x_0 L_0 + y_{n+1} V_{n+1} = x_n L_n + y_1 V_1$$

$$0.680 + y_{n+1} * 8.152 = 0.08 * 2.656 + 0.099 * 6.176$$

$$y_{n+1} = 0.0177$$

Now we have the point $(x_n, y_{n+1}) = (0.08, 0.0177)$

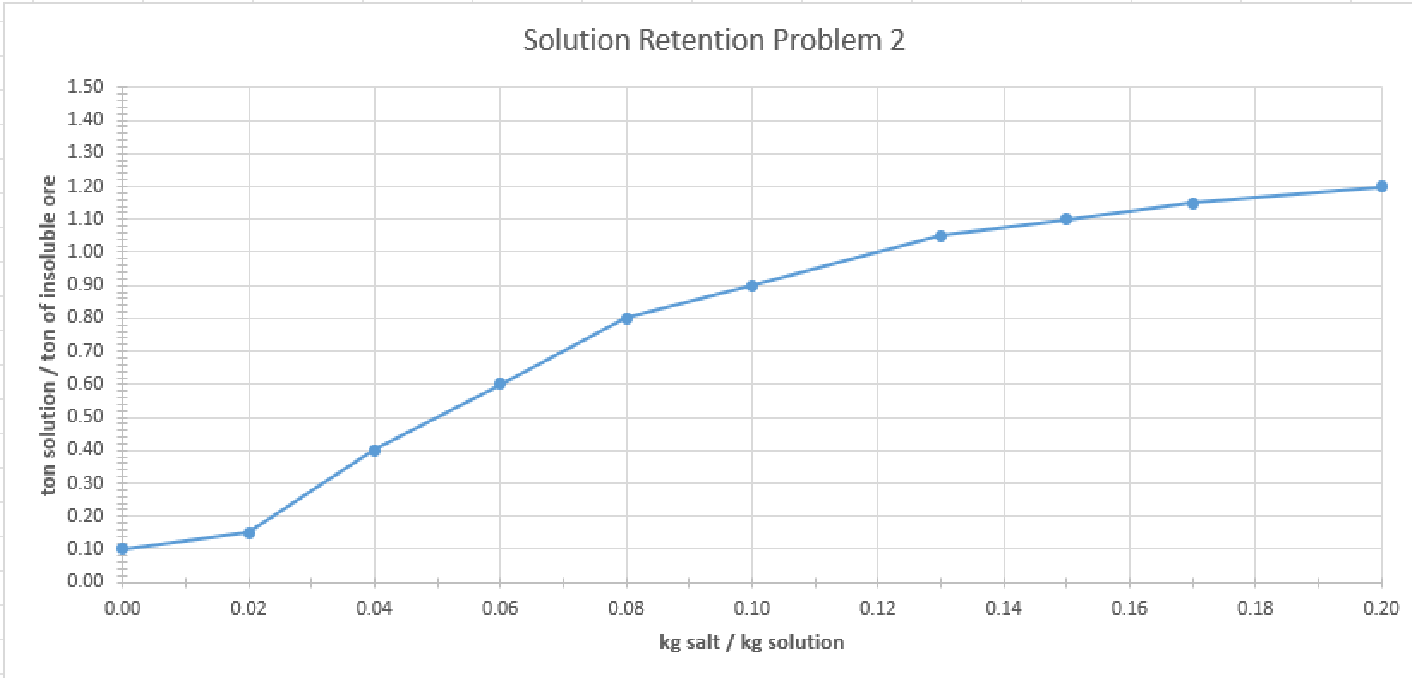
We can now plot the operating and equilibrium lines. (Remember that equilibrium line is $x_n = y_n$)

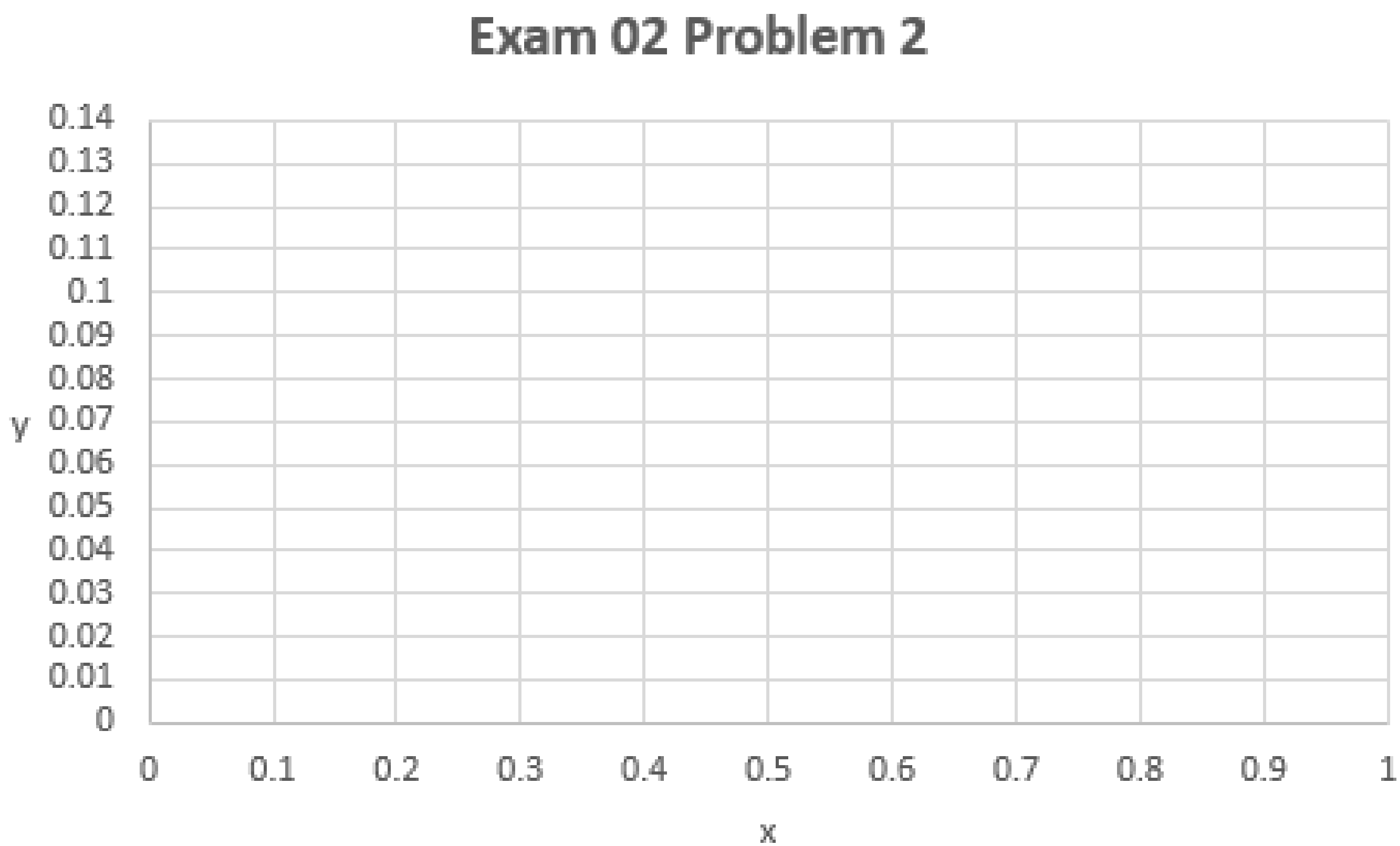


b) There are 2 ideal stages required.

For Problem 2

x	retention
0.00	0.10
0.02	0.15
0.04	0.40
0.06	0.60
0.08	0.80
0.10	0.90
0.13	1.05
0.15	1.10
0.17	1.15
0.20	1.20




Exam 02 Problem 2
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

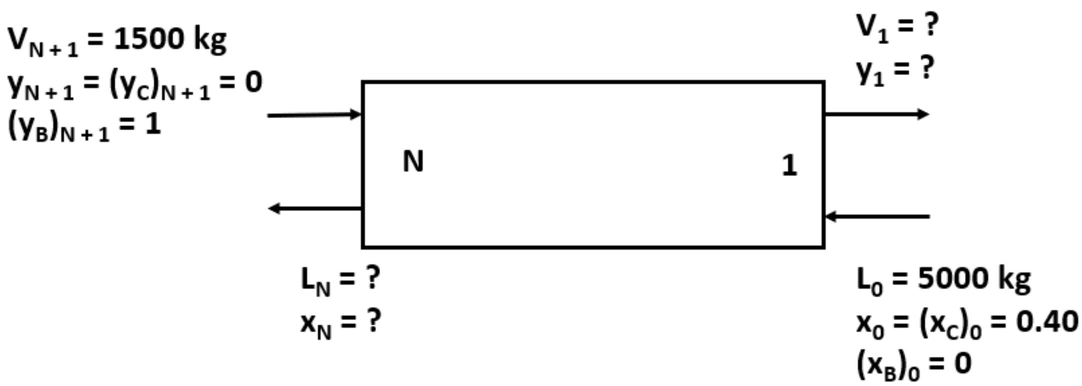
3. **(35 points)** Consider a countercurrent liquid extraction process. The 5000 kg/hr feed $(\boldsymbol{L_0})$ stream comprises 40 mass % ethanol (solute $\boldsymbol{C}$) and 60 mass % water (diluent $\boldsymbol{A}$). The 1500 kg/hr entering solvent $(\boldsymbol{V_{N+1}})$ stream is pure benzene (solvent $\boldsymbol{B}$). The exiting raffinate should have 10.0 mass % ethanol on a benzene-free basis.
   a) What will be the flow rate of the exiting extract?
   b) What will be the composition of the exiting extract?

   The phase diagram for mixtures of water, benzene, and ethanol is provided as Figure 3.

   Solution:

1 hour basis

A (diluent) = water
B (solvent) = benzene
C (solute) = ethanol



Start by calculating the fictitious mixture point
$\boldsymbol{M}$ = the rate at which liquid enters the system = $\boldsymbol{L_0 + V_{N+1} = 6500\ kg}$

$$\boldsymbol{x_M = (x_c)_M = \frac{x_0\, L_0 + y_{N+1}\, V_{N+1}}{L_0 + V_{N+1}}}$$

$$\boldsymbol{x_M = (x_c)_M = \frac{0.40 * 5000 + 0 * 1500}{5000 + 1500} = 0.308}$$

$$\boldsymbol{(x_B)_M = \frac{(x_B)_0\, L_0 + (y_B)_{N+1}\, V_{N+1}}{L_0 + V_{N+1}} = \frac{0 * 5000 + 1 * 1500}{5000 + 1500} = 0.231}$$

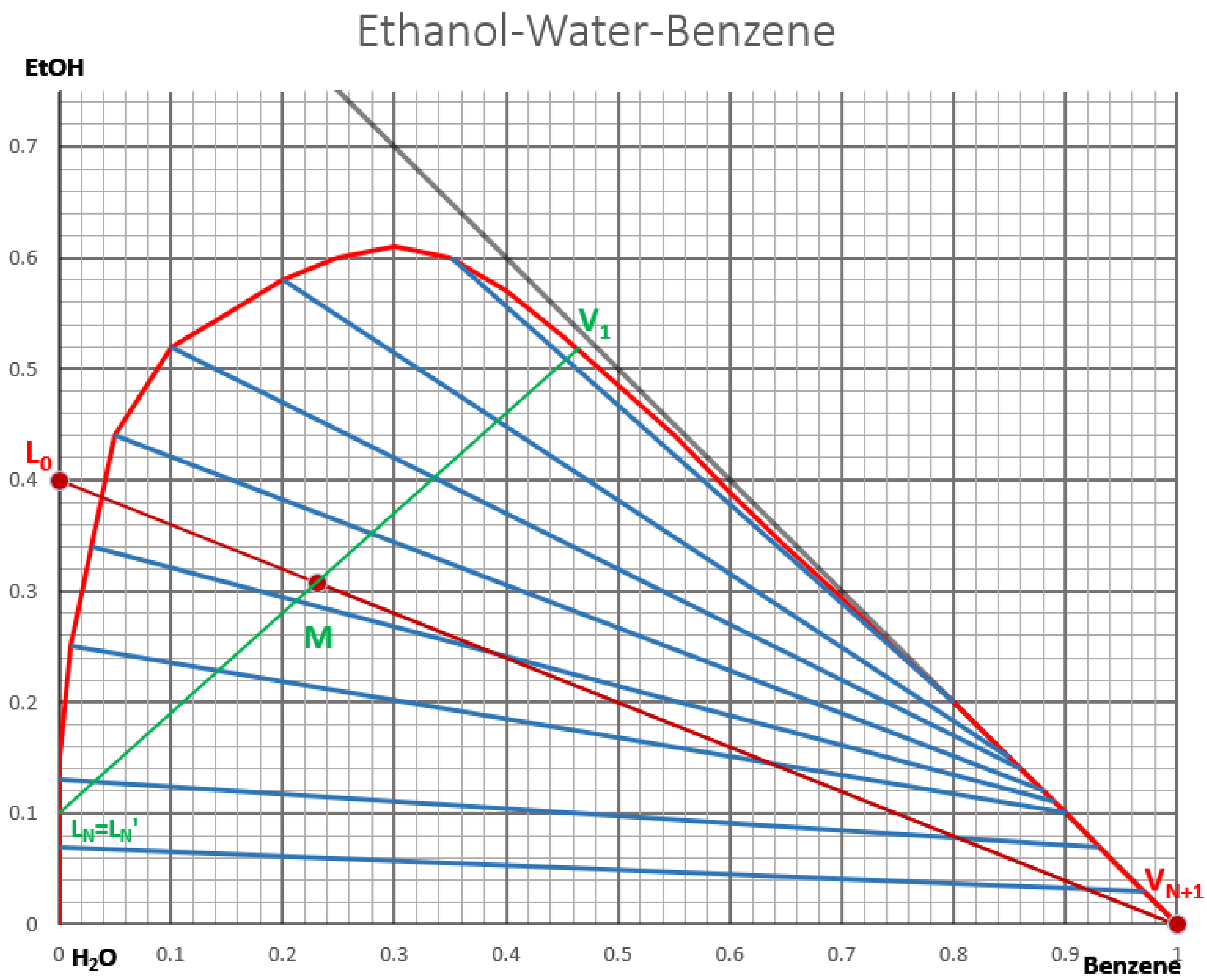
Locate $\boldsymbol{M}$ as a point on the line $\overline{\boldsymbol{L_0\, V_{N+1}}}$ where $\boldsymbol{x_M = (x_c)_M = 0.308}$ or $\boldsymbol{(x_B)_M = 0.231}$.

Could also just plot point $\boldsymbol{((x_B)_M, x_M) = (0.231, 0.308)}$.

We know the exiting raffinate on a nonanol-free basis, $\boldsymbol{L'_N = 0.10}$, because the left-hand side of the two-phase boundary is essential at $\boldsymbol{(x_B)_N = 0}$, we can state that $\boldsymbol{L_N \approx L'_N = 0.10}$

We also know that M is also equal to the rate at which liquid exits the system and therefore lies on the line $\overline{L_N V_1}$. We can extend the line $\overline{L_N V_M}$ to reach the two phase boundary in order to locate V_1. Reading off the graph:

$$(y_B)_1 = 0.465 \text{ and } (y_c)_1 = 0.519$$



The lack of subscripts in the following equation implies that the following mass fractions are all for solute

$$\frac{V_1}{L_N} = \frac{x_M - x_N}{y_1 - x_M} = \frac{0.308 - 0.100}{0.519 - 0.308} = 0.986$$

$$L_N = \frac{M}{1+V_1/L_N} = \frac{6500}{1+0.986} = 3273\ kg \quad \text{and} \quad V_1 = M - L_N = 6500 - 3273 = 3227\ kg$$

The same equations apply to the solvent, B, concentrations:

$$\frac{V_1}{L_N} = \frac{(x_B)_M - (x_B)_N}{(y_B)_1 - (x_B)_M} = \frac{0.2308 - 0}{0.465 - 0.2308} = 0.985$$

$L_N = \frac{M}{1+V_1/L_N} = \frac{6500}{1+0.985} = 3275\ kg$ and $V_1 = M - L_N = 6500 - 3275 = 3225\ kg$

Both calculations yield essentially the same answer in this case.

a) $V_1 = 3227\ kg$

b) $(y_B)_1 = 0.465$ and $(y_c)_1 = 0.519$

Figure 3 for Problem 3