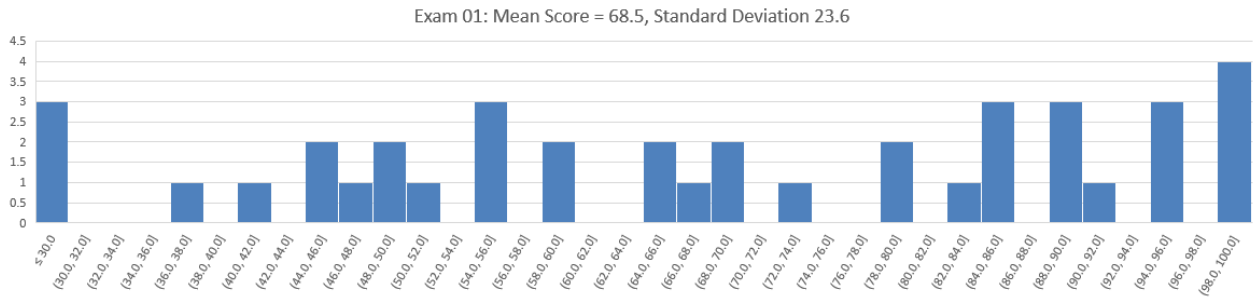
CE407 Exam 01 **Solution** **Thursday, 10/09/2025**



1. **(40 points)** A 1000 mol/min feed of saturated liquid with a composition of 0.56 mole fraction toluene and 0.44 mole fraction ethylbenzene is fed to a fractionating column. The mole fraction of toluene in the distillate is $x_D = 0.95$ and the mole fraction $x_B = 0.1$ in the bottom product. The column is equipped with a total condenser. The column is to operate with a reflux ratio R = 1.71.
   a) How many ideal stages are required for this separation?
   b) What is optimal feed stage?
   c) What is the required rate of cooling at the condenser (q_c) and the required rate of heating (q_r) to the reboiler? **Answer in units of** $\frac{kJ}{hour}$

- Equilibrium data provided via the attached T_{xy} diagram and Equilibrium Chart
  - Read temperatures on the T_{xy} diagram to the nearest degree
- Define reference states such that the liquid enthalpy is zero for each pure component at 383.9 K.
- Heat capacity of Water 4.186 kJ/kg C. Allow for a 10 C temperature rise in the cooling water.
- Steam: use 159 psig steam, which has a latent heat of evaporation of 1986 kJ/kg
- **$H_x(T,x) = (185 – 28x) * (T – 383.9)$ J/mol, T in K**
- **$H_y(T,y) = 43307 – 5307\,y + (170 – 30\,y) * (T – 383.9)$ J/mol, T in K**

Problem 1 **(40 points)**

a) and b) See McCabe-Thiele diagram.

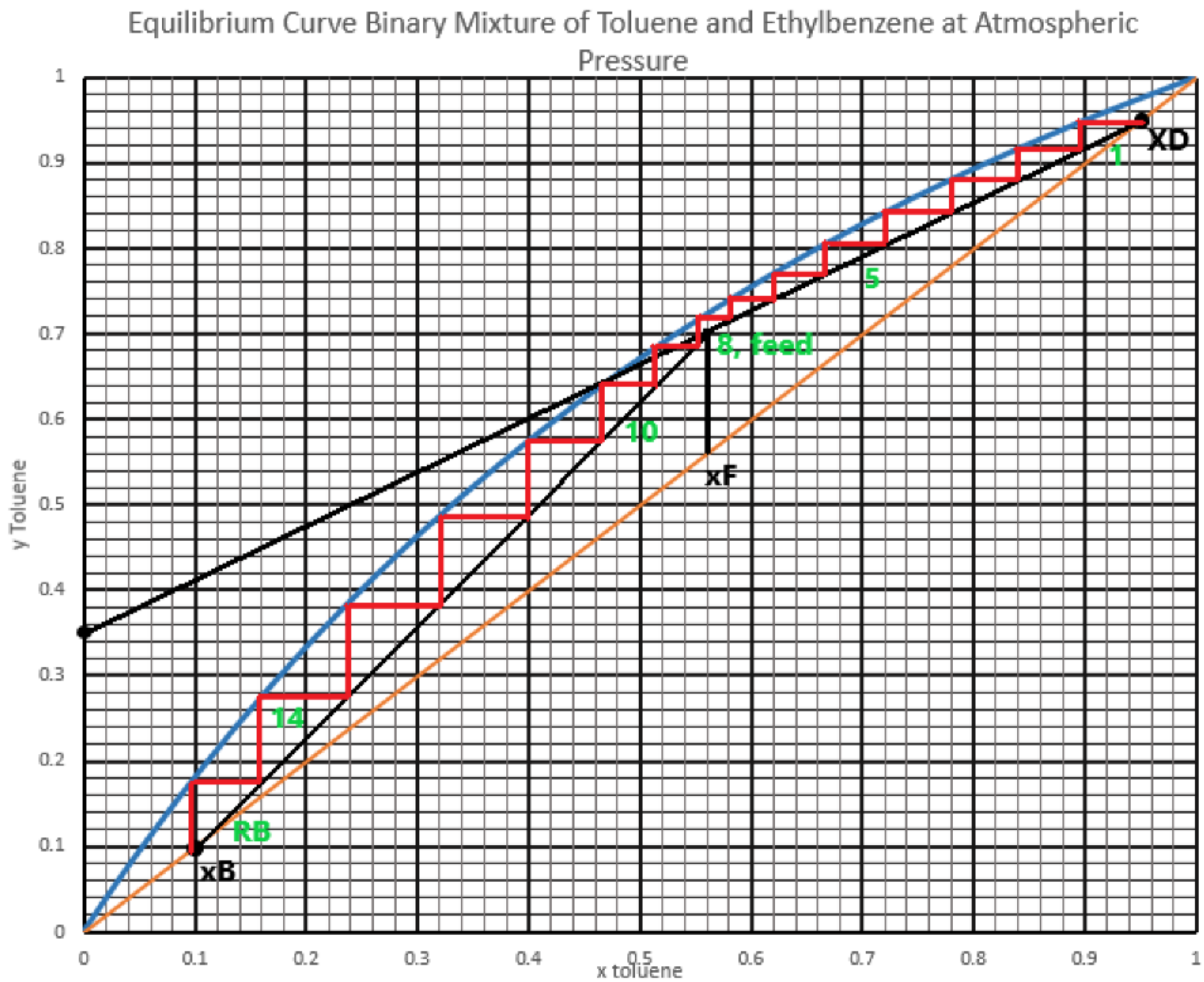
For R = 1.71 and $x_D = 0.9$ The Rectifying operating line will pass through $(x_D, x_D) = (0.95, 0.95)$ and the intercept $\left(0, \frac{x_D}{R+1}\right) = \left(0, \frac{0.95}{1.71+1}\right) = (0, 0.35)$

Because we have a saturated liquid feed, the feed line will be a vertical line located at $x = x_F = 0.56$

The Stripping line will pass from the intersection of the feed and Rectifying lines to the point $(x_B, x_B) = (0.1, 0.1)$

Step off the number of stages from $x = x_D = 0.95$ to $x = x_B = 0.10$

Stages required: 14 plus RB. Feed at stage 8.



c) Required Heating and Cooling Loads

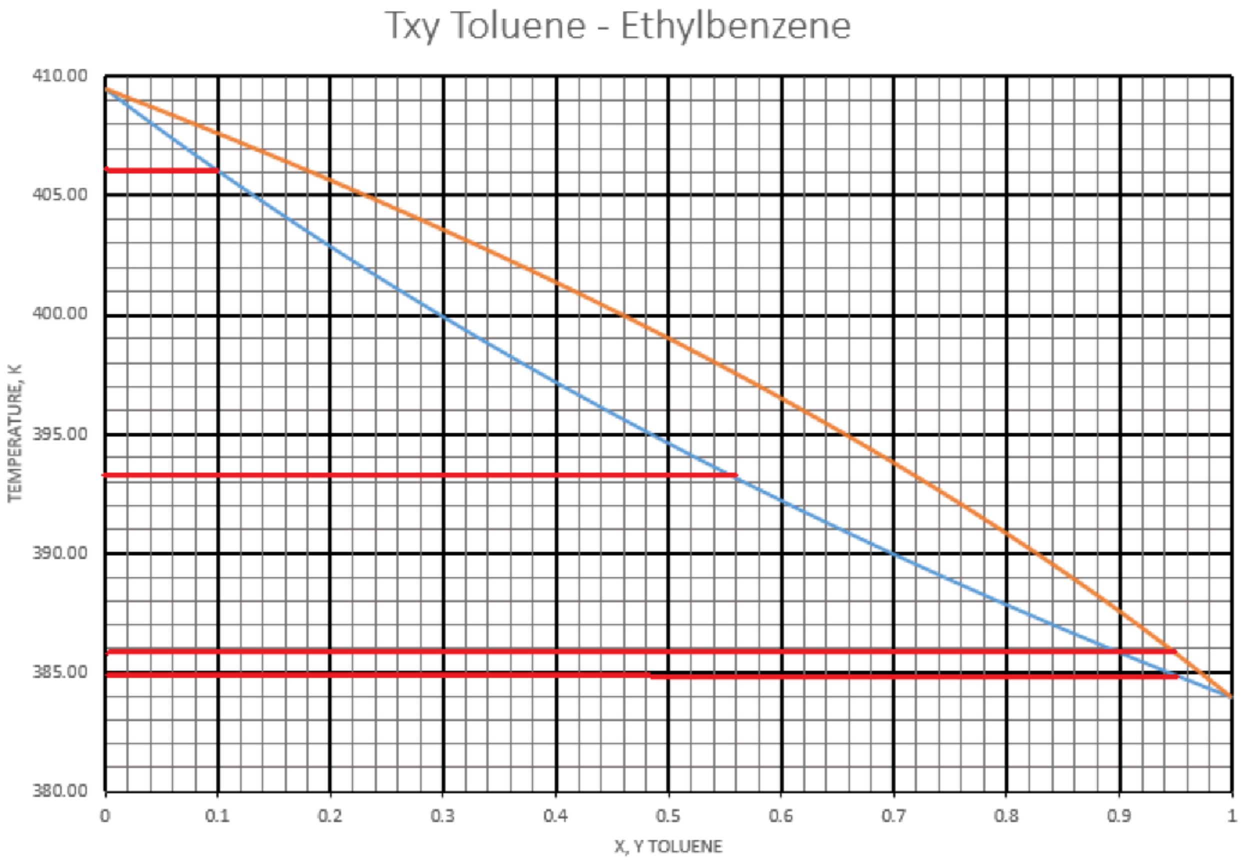
Temperatures can be read off of T_{xy} diagram

Temperature of Distillate/x_0 is that of saturated liquid at x = 0.95 and is 385 K

Temperature of Vapor entering condenser is that of saturated vapor at y = 0.95 and is 386 K

Temperature of Feed is that of saturated liquid at x = 0.56 and is 393 K

Temperature of Bottoms is that of saturated liquid at x = 0.1 and is 406 K



From Lecture 08:

$$-q_c = D(1 + R)(H_{x,0} - H_{y,1})$$

$$q_r - q_c = DH_D + BH_B - FH_F$$

Total Mole Balance: F = D + B

Toluene Mole Balance: $x_F F = x_D D + x_B B$

$x_F = 0.56$, F = 1000 mol/min, $x_D = 0.95$, $x_B = 0.10$

Solves for: **D = 541.2 and B = 458.8 mol/minute**

D = 32,472 and B = 27,528 mol/hr

F = 60,000 mol/hr (= 1000 mol/min * 60)

Can also use formulas depending on mole fractions of feed, distillate, and bottoms.

From problem statement:

- $H_x(T,x) = (185 – 28x) * (T – 383.9)$ J/mol
- $H_y(T,y) = 43307 – 5307 y + (170 – 30 y) * (T – 383.9)$ J/mol

$H_{x\,0} = H_D = H_x(385,\ 0.95) = (185 - 28 * 0.95) * (385 - 383.9) = 174.2$ $\quad H_x$ in $\frac{J}{mol}$ T in K

$H_{y\,1} = H_y(386,\ 0.95) = 43307 - 5307 * 0.95 + (170 - 30 * 0.95) * (386 - 383.9) = 38{,}563$

H_y in $\frac{J}{mol}$ $\quad T$ in K

$H_F = H_x(393,\ 0.56) = (185 - 28 * 0.56) * (393 - 383.9) = 1540.8$ $\quad H_x$ in $\frac{J}{mol}$ $\quad T$ in K

$H_B = H_x(406,\ 0.1) = (185 - 28 * 0.1) * (406 - 383.9) = 4026.6$ $\quad H_x$ in $\frac{J}{mol}$ $\quad T$ in K

$$-q_c = D(1 + R)\left(H_{x,0} - H_{y,1}\right)$$

$$-q_c = 32{,}472\ \frac{mol}{hr} * (1 + 1.71) * (174.2 - 38563)\ \frac{J}{mol}$$

$$q_c = 3.38 * 10^9\ \frac{J}{hr} = 3.38 * 10^6\ \frac{kJ}{hr}$$

$$q_r - q_c = DH_D + BH_B - FH_F$$

$$q_r - 3.38 * 10^9 \frac{J}{hr} = 32{,}472\ \frac{mol}{hr} * 174.2\ \frac{J}{mol} + 27{,}528 * 4026.6 - 60{,}000 * 1540.8$$

$$q_r = 3.40 * 10^9\ \frac{J}{min} = 3.40 * 10^6\ \frac{kJ}{min}$$

2. **(35 pts)** 500 mol/min of contaminated mineral oil (composition 85 mole percent oil, 15 mole percent benzene) is to be purified by contact with air in a stripping tower operating isothermally and isbarically at 25 C and 1.0 atm. Ninety-five percent of the benzene in the oil must be removed. Air enters pure.

   a) What is the minimum flow rate of air required to achieve the desired cleanup, corresponding to an infinite number of stages?
   Note that operating line will curve AWAY from equilibrium line

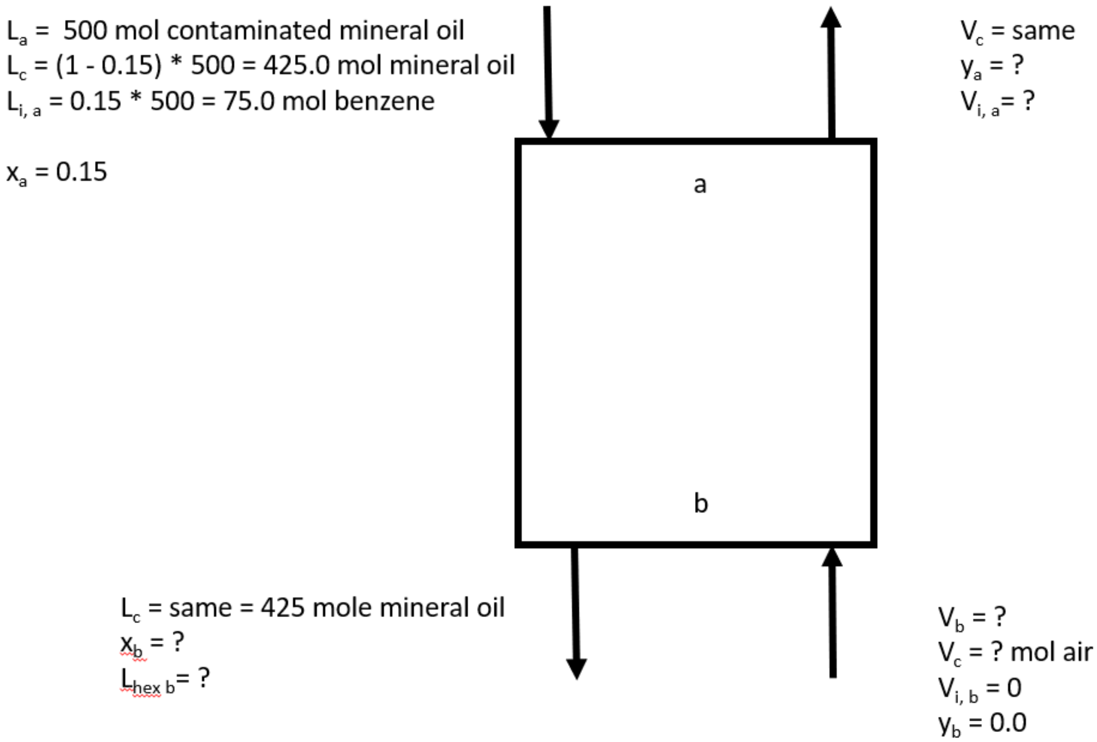
$$\boldsymbol{P_{Benzene}^{sat} = 95.17\ mm\ Hg}$$

   b) If 4500 mol/min of air is used how many ideal stages will be required. Note: Calculate four values of the operating line so that you can document it's curvature. Equilibrium line is included on the provided graph

As usual neglect any evaporation of mineral oil or dissolution of air in the liquid. Assume validity of Raoult's Law for benzene.

Solution

Ex01, Problem 02
1 minute basis



Benzene transferred from liquid to vapor:

$$0.95 * 75\ mol = 71.25\ mol\ benzene$$

Entering air was pure, so

$$V_{i,a} = 0 + 71.25 = 71.25\ mol\ benzene$$

(0 moles from incoming air and 71.25 moles transferred from liquid)

Benzene remaining in Exiting Liquid:

$$L_{i,b} = 75 - 71.25\ mol = 3.75\ mol\ benzene$$

The mole fraction of benzene in the exiting liquid is:

$$x_b = \frac{L_{i,b}}{L_{i,b} + L_c} = \frac{3.75}{3.75 + 425} = 0.00875$$

a) Minimum Flow of Air – for this stripping operation the operating line curves away from the equilibrium line, so the first contact **WILL** occur at $x = x_a$

For minimum air:

$$y_a = y^*(x_a) \qquad \text{(i.e. y is in equilibrium with } x_a\text{)}$$

$$y_a = \frac{P^{sat}}{P} = \frac{95.17}{760}\, x_a = 0.1252 * 0.15 = 0.01878$$

By definition of mole fraction:

$$y_a = 0.01878 = \frac{V_{i,a}}{V_{i,a} + V_{c,min}} = \frac{71.25}{71.25 + V_{c,min}}$$

$$\boldsymbol{V_{c,min} = 71.25\ mol * \frac{1 - 0.01878}{0.01878} = 3722.7\ mol\ air}$$

b)

Graphical Method, which accounts for curvature of the Operating Line

$$\boldsymbol{y_a = \frac{V_{i,a}}{V_{i,a} + V_c} = \frac{71.25}{71.25 + 4500} = 0.01559}$$

Operating Line:

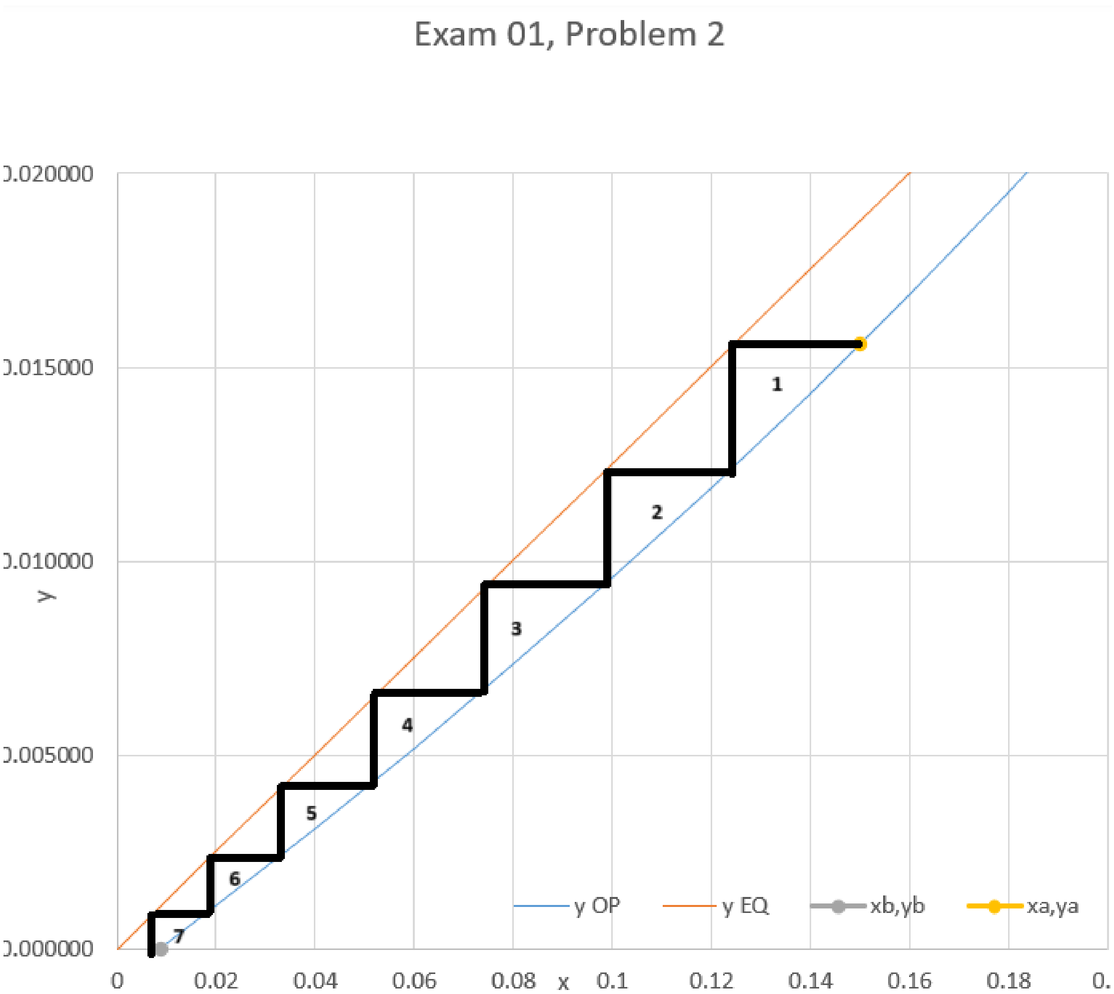
$$\boldsymbol{y = 1 - \left[\frac{1}{1 - y_a} + \frac{L_c}{V_c}\left(\frac{1}{1 - x} - \frac{1}{1 - x_a}\right)\right]^{-1}}$$

$$y = 1 - \left[\frac{1}{1-0.01559} + \frac{425}{4500}\left(\frac{1}{1-x} - \frac{1}{1-0.15}\right)\right]^{-1}$$

I used Excel to generate operating and equilibrium lines

B7 =1-(1/(1-B$2)+ E$2/F$2*(1/(1-A7)-1/(1-A$2)))^(-1)

	A	B	C	D	E	F
1	**xa**	**ya**	**xb**	**yb**	**Lc**	**Vc**
2	0.15	0.015586546	0.008746	0	425	4500
3						
4						
5	x	y OP	y EQ			
6	0	-0.000834	0			
7	0.02	0.001092909	0.002504			
8	0.04	0.00309226	0.005008			
9	0.06	0.005168187	0.007512			
10	0.08	0.007325173	0.010016			
11	0.1	0.009568062	0.01252			
12	0.12	0.011902088	0.015024			
13	0.14	0.014332924	0.017528			
14	0.16	0.016866718	0.020032			
15	0.18	0.019510154	0.022536			
16	0.2	0.022270505	0.02504			



Round up to 7 ideal stages

3. **(25 pts)** A 100 mole/min feed stream comprising 30 mole percent benzene and 70 mole percent toluene is separated by continuous distillation at atmospheric pressure in a column fitted with a total condenser. Process specifications require a mole fraction of 0.997 benzene in the distillate. The feed enters as a saturated liquid. The reflux ratio R_D = 2.0. Use the analytical procedure (Kremser equation) for x > 0.9. Note that one point on the equilibrium curve is **(x,y) = (0.9000000, 0.9587266)**.

Advice: Carry calculations out to at least 5 decimal places!

How many stages are required to go from x = 0.9 to x_d = 0.997?

Note that you do not need a VLE curve because you are not determining the stages between x_b and x = 0.9

Solution:

Problem 3 **25 points**

Now we will use the Kremser equation to calculate the number of stages required to get from cutoff value of x = 0.9 to the desired value of x_D = 0.993.

$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{\ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$

$\boldsymbol{y_a}$ = (the point on the R OP line at x = x_D) = x_D **= 0.99700**
$\boldsymbol{y_b}$ = (the point on the R OP line at x = 0.90)

$$\frac{R}{R+1} * 0.9 + \frac{x_D}{R+1} = y_b$$

$$\frac{2.0}{2.0+1} * 0.9 + \frac{0.997}{2.0+1} = \mathbf{0.93233}$$

y_b^* = the point on the "equilibrium line" at x_{cutoff} = 0.9. It was given in the problem statement that **(0.9000000, 0.9587266)** is a point on the equilibrium curve, therefore:
$\boldsymbol{y_b^*}$ **= 0.9587266**

In order to determine y_a^* we will need to calculate an equation for the "equilibrium line". This line will pass through the point **(0.9000000, 0.9587266)** and the point **(1, 1)** because equilibrium ***ALWAYS*** passes through **(1, 1)**. Solving for the line that passes through these two points leads to $\boldsymbol{y^* = 0.412734375\, x + 0.587265625}$

$\boldsymbol{y_a^*}$ **=** (the point on the equilibrium line where x = x_D) **= 0.99876**

Placing these values into the Kremser equation above leads to a value of **N= 5.6**

Solution:

Problem 3 using x_D **= 0.99300** **25 points**

Now we will use the Kremser equation to calculate the number of stages required to get from cutoff value of x = 0.9 to the desired value of x_D = 0.993.

$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{\ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$

$\boldsymbol{y_a}$ = (the point on the R OP line at x = x_D) = x_D **= 0.99300**
$\boldsymbol{y_b}$ = (the point on the R OP line at x = 0.90)

$$\frac{R}{R+1} * 0.9 + \frac{x_D}{R+1} = y_b$$

$$\frac{2.0}{2.0+1} * 0.9 + \frac{0.993}{2.0+1} = \mathbf{0.93100}$$

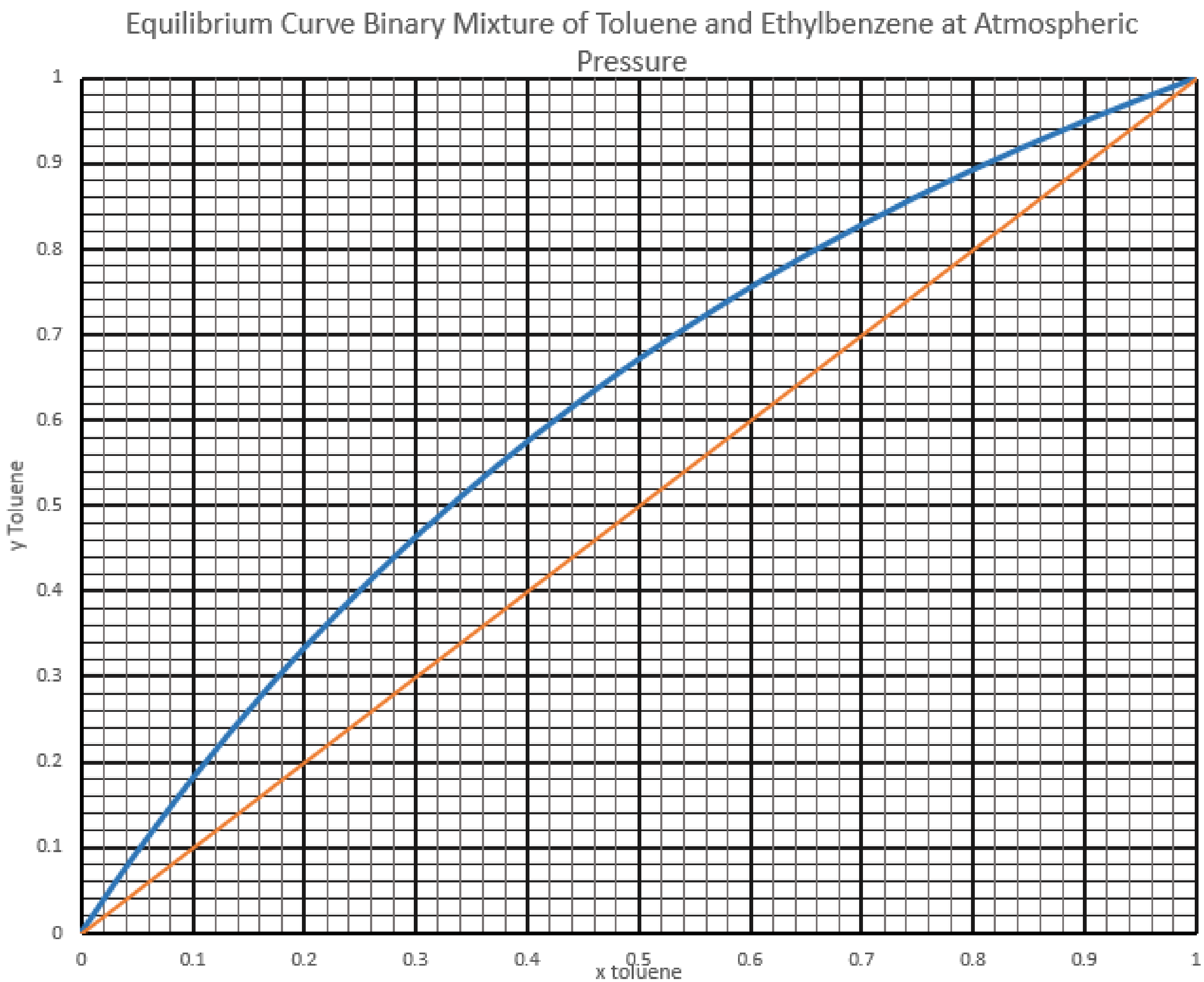
y_b^* = the point on the "equilibrium line" at x_{cutoff} = 0.9. It was given in the problem statement that **(0.9000000, 0.9587266)** is a point on the equilibrium curve, therefore:
$\boldsymbol{y_b^*}$ **= 0.9587266**

In order to determine y_a^* we will need to calculate an equation for the "equilibrium line". This line will pass through the point **(0.9000000, 0.9587266)** and the point **(1, 1)** because equilibrium ***ALWAYS*** passes through **(1, 1)**. Solving for the line that passes through these two points leads to $\boldsymbol{y^* = 0.412734375\, x + 0.587265625}$

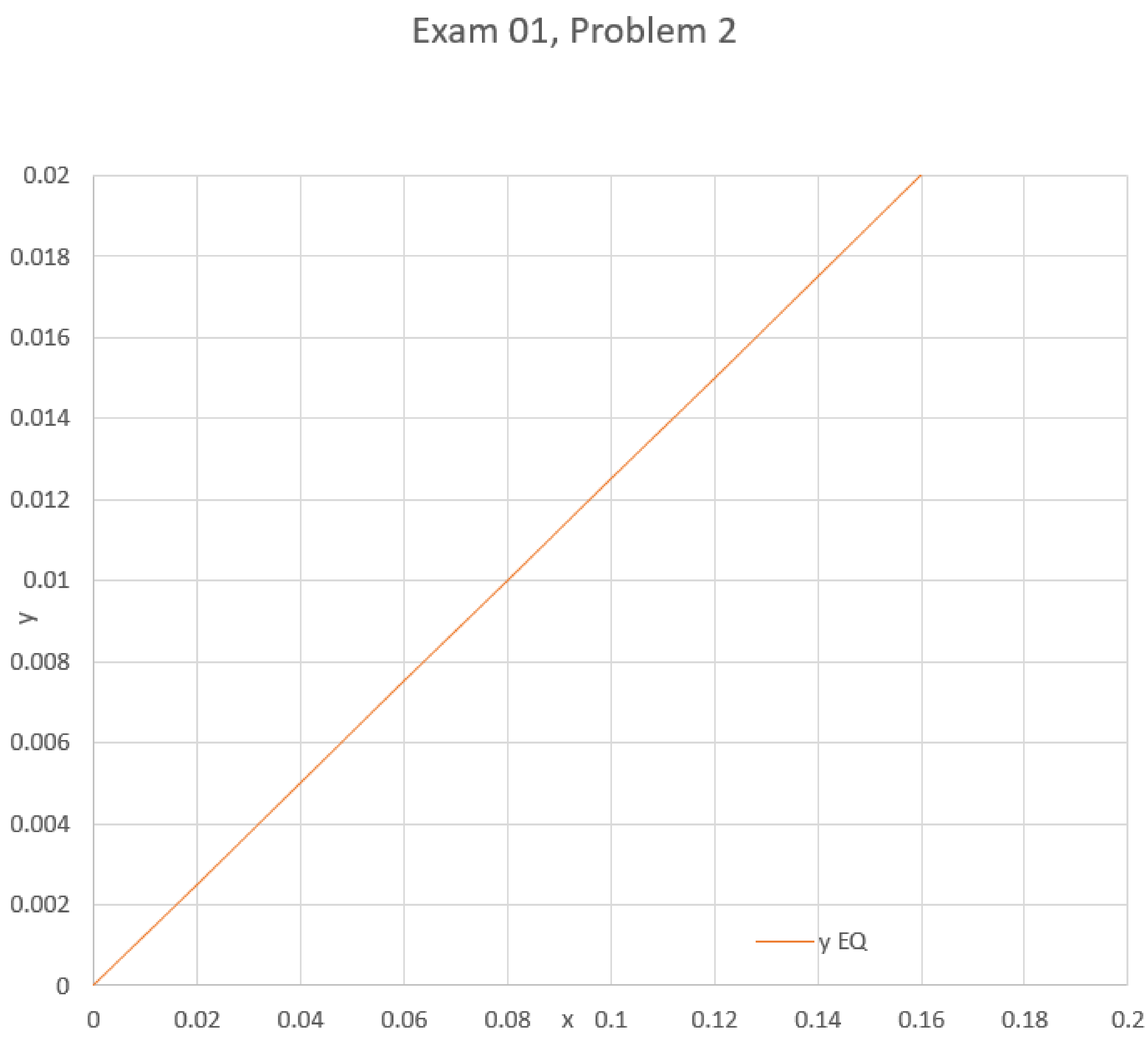
$\boldsymbol{y_a^*}$ **=** (the point on the equilibrium line where x = x_D) **= 0.9971109**

Placing these values into the Kremser equation above leads to a value of **N = 4.0**

For Problem 01



For Problem 01


Exam 01, Problem 2
0.02
0.018
0.016
0.014
0.012
0.01
y
0.008
0.006
0.004
0.002
0
0
0.02
0.04
0.06
0.08
x
0.1
0.12
0.14
0.16
0.18
0.2
y EQ