## CE407 Exam 01 Solution: Mean = 81 10/24/2023

1. ( 40 points) $A 1000 \mathrm{~mol} / \mathrm{min}$ feed of saturated liquid with a composition of 0.5 mole fraction toluene and 0.5 mole fraction ethylbenzene is fed to a fractionating column. The mole fraction of toluene in the distillate is $x_{D}=0.9$ and the mole fraction $x_{B}=0.1$ in the bottoms product. The column is equipped with a total condenser. The column is to operate with a reflux ratio $R=2.5$.
a) How many ideal stages are required for this separation?
b) What is optimal feed stage?
c) What is the required rate of cooling at the condenser $\left(q_{c}\right)$ and the required rate of heating $\left(q_{r}\right)$ to the reboiler? Give answer in $k J / h o u r$
d) What are the required rates of cooling water and steam? Give answer in kg/hour

- Equilibrium data provided via the attached $\mathrm{T}_{\mathrm{xy}}$ diagram and Equilibrium Chart
- Define reference states such that the liquid enthalpy is zero for each pure component at 383.9 K.
- Heat capacity of Water $4.186 \mathrm{~kJ} / \mathrm{kg}$ C. Allow for a 10 C temperature rise in the cooling water.
- Steam: use 159 psig steam, which has a latent heat of evaporation of $1986 \mathrm{~kJ} / \mathrm{kg}$
- $H_{x}(T, x)=(185-28 x) *(T-383.9) \mathrm{J} / \mathrm{mol}$
- $\mathrm{H}_{\mathrm{y}}(\mathrm{T}, \mathrm{y})=43307-5307 \mathrm{y}+(170-30 \mathrm{y})$ * $(\mathrm{T}-383.9) \mathrm{J} / \mathrm{mol}$


## Solution

a) and b) See Excel Spreadsheet and McCabe-Thiele diagram.

For $\mathrm{R}=2.5$ and $\mathrm{x}_{\mathrm{D}}=0.9$ The R operating line has the equation $y=\frac{2.5}{2.5+1} x+\frac{0.9}{2.5+1}=0.714 x+0.257$
The intersection of the feed line and the $R$ Op line occurs at ( $0.5,0.61$ ). (Feed line is a vertical line at $\mathrm{x}=$ 0.5)

Draw the Rectifying line from $(0.9,0.9)$ to $(0.5,0.61)$ or to $(0,0.257)$
The Stripping line will pass from the intersection point to the point $(x B, x B)=(0.1,0.1)$
Stages required: 9 plus RB. Feed at stage 5.

Problem 1 VLE Toluene and Ethylbenzene

c) Required Heating and Cooling Loads

Temperatures can be read off of $T_{x y}$ diagram
Temperature of Distillate $/ x_{0}$ is that of saturated liquid at $x=0.9$ and is 386 K
Temperature of Vapor entering condenser is that of saturated vapor at $\mathrm{y}=0.9$ and is 388 K
Temperature of Feed is that of saturated liquid at $x=0.5$ and is 394.5 K
Temperature of Bottoms is that of saturated liquid at $x=0.1$ and is 406 K

## Txy Toluene - Ethylbenzene



From Lecture 08:

$$
\begin{gathered}
-q_{c}=D(1+R)\left(H_{x, 0}-H_{y, 1}\right) \\
q_{r}-q_{c}=D H_{D}+B H_{B}-F H_{F}
\end{gathered}
$$

Total Mole Balance: F = D + B
Toluene Mole Balance: $x_{F} F=x_{D} D+x_{B} B$
$\mathrm{x}_{\mathrm{F}}=0.5, \mathrm{~F}=1000 \mathrm{~mol} / \mathrm{min}, \mathrm{x}_{\mathrm{D}}=0.90, \mathrm{x}_{\mathrm{B}}=0.10$
Solves for: $\quad D=500$ and $B=500$ mol $/$ minute
$D=30,000$ and $B=30,000 \mathrm{~mol} / \mathrm{hr}$
$F=60,000 \mathrm{~mol} / \mathrm{hr} \quad\left(=1000 \mathrm{~mol} / \mathrm{min}^{*} \mathbf{6 0}\right)$
Can also use formulas depending on mole fractions of feed, distillate, and bottoms.
From problem statement:

- $H_{x}(T, x)=(185-28 x) *(T-383.9) \mathrm{J} / \mathrm{mol}$
- $\mathrm{H}_{\mathrm{y}}(\mathrm{T}, \mathrm{y})=43307-5307 \mathrm{y}+(170-30 \mathrm{y}) *(\mathrm{~T}-383.9) \mathrm{J} / \mathrm{mol}$
$H_{x 0}=H_{D}=H_{x}(386,0.9)=(185-28 * 0.9) *(386-383.9)=335.6 \quad H_{x}$ in $\frac{J}{m o l} T$ in $K$
$H_{y 1}=H_{y}(388,0.9)=43307-5307 * 0.9+(170-30 * 0.9) *(388-383.9)=39117$
$H_{y}$ in $\frac{J}{m o l} \quad T$ in $K$
$H_{F}=H_{x}(394.5,0.5)=(185-28 * 0.5) *(394.5-383.9)=1812.6 \quad H_{x}$ in $\frac{J}{m o l} \quad T$ in $K$
$H_{B}=H_{x}(406,0.1)=(185-28 * 0.1) *(406-383.9)=4026.6 \quad H_{x}$ in $\frac{J}{m o l} \quad T$ in K

$$
\begin{gathered}
-q_{c}=D(1+R)\left(H_{x, 0}-H_{y, 1}\right) \\
-q_{c}=30,000 *(1+2.5) *(335.6-39117) \\
q_{c}=4.07 * 10^{9} \frac{\mathrm{~J}}{\mathrm{hr}}=4.07 * 10^{6} \frac{\mathrm{~kJ}}{\mathrm{hr}} \\
q_{r}-q_{c}=D H_{D}+B H_{B}-F H_{F} \\
q_{r}-4.07 * 10^{9} \frac{\mathrm{~J}}{\mathrm{hr}}=30,000 \frac{\mathrm{~mol}}{\mathrm{hr}} * 335.6 \frac{\mathrm{~J}}{\mathrm{~mol}}+30,000 * 4026.6-60,000 * 1812.6 \\
q_{r}=4.09 * 10^{9} \frac{\mathrm{~J}}{\mathrm{hr}}=4.09 * 10^{6} \frac{\mathrm{~kJ}}{\mathrm{hr}}
\end{gathered}
$$

d) Cooling water requirement

$$
\begin{aligned}
& \mathrm{C}_{\mathrm{p}}=4.186 \mathrm{~kJ} /(\mathrm{kg} \mathrm{C}) \\
& \mathrm{q}_{\mathrm{c}}=(\text { mass flow cooling water }) * \mathrm{C}_{\mathrm{p}, \text { water }} *\left(\mathrm{~T}_{\text {out }}-\mathrm{T}_{\text {in }}\right) \\
& 4.07 * 10^{6} \mathrm{~kJ} / \mathrm{hr}=\mathrm{m} * 4.186 \mathrm{~kJ} /(\mathrm{kg} \mathrm{C}) * 10 \mathrm{C} \\
& \mathrm{~m}_{\text {cooling water }}=97,229 \mathrm{~kg} / \mathrm{hr}
\end{aligned}
$$

Steam Requirement
$q_{r}=($ mass flow steam $) * \Delta H^{\text {vap }}{ }_{\text {steam }}$
$4.09 * 10^{6} \mathrm{~kJ} / \mathrm{hr}=\mathrm{m} * 1986 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{m}_{\text {steam }}=2059 \mathrm{~kg} / \mathrm{hr}$
2. ( 30 points) A $200 \mathrm{~mol} /$ hour stream of contaminated air (composition 96 mole percent air, 4 mole percent toxin) must be cleaned up by countercurrent contact with clean water in an absorption tower operating isothermally at 25 C and atmospheric pressure. The exiting air should have toxin mole fraction equal to 0.001. Entering water is pure.

If the entering water flow rate is 25.0 mole/hour, how many ideal stages are required? Calculate at least four points on the operating curve in order to capture its shape.

As usual, you may neglect evaporation of water as well as dissolution of air in the liquid. Points on the ( $\mathrm{x}, \mathrm{y}$ ) equilibrium curve can be determined from the following table of equilibrium solubility data at 25 C :

| Mole Fraction in Liquid Phase, $\mathbf{x}$ | Mole Fraction in Vapor Phase, $\mathbf{y}$ |
| :---: | :---: |
| $\mathbf{0 . 0 0}$ | $\mathbf{0 . 0 0 0}$ |
| 0.10 | 0.006 |
| 0.20 | 0.015 |
| 0.30 | 0.030 |
| 0.40 | 0.050 |

Solution
Preliminary Calculations
1 hour basis

$$
\begin{aligned}
& \begin{array}{l}
\mathrm{L}_{\mathrm{c}}=25 \text { mole } \\
\mathrm{x}_{\mathrm{a}}=0
\end{array} \\
&
\end{aligned}
$$

Calculate the moles of toxin at a end of tower, $V_{t a}$ :

$$
\begin{gathered}
y_{a}=0.001=\frac{V_{t a}}{V_{t a}+V_{c}}=\frac{V_{t a}}{V_{t a}+192} \\
V_{t a}=0.1922
\end{gathered}
$$

Toxin Mass Balance:

$$
\begin{gathered}
x_{a} L_{a}+y_{b} V=V_{t a}+L_{t b} \\
0 * L_{a}+0.04 * 200=0.1922+L_{t b} \\
L_{t b}=7.808 \quad \text { equals moles of toxin in liquid at } \mathrm{b} \text { end }
\end{gathered}
$$

Now:

$$
x_{b}=\frac{L_{t b}}{L_{t b}+L_{c}}=\frac{7.808}{7.808+25}=0.238
$$

When $L_{c}=25 \mathrm{~mol} / \mathrm{hr}$ we can construct the operating line with the following values:

$$
\begin{gathered}
x_{a}=0 \\
y_{a}=0.001 \\
V_{c}=192 \\
L_{c}=25
\end{gathered}
$$

Calculate several points for the operating line using the equation

$$
y=1-\left[\frac{1}{1-y_{a}}+\frac{L_{c}}{V_{c}}\left(\frac{1}{1-x}-\frac{1}{1-x_{a}}\right)\right]^{-1}
$$

| Mole Fraction in Liquid Phase, $\mathbf{x}$ | $\mathbf{y}$ on Operating Line |
| :---: | :---: |
| 0.00 | 0.001 |
| 0.10 | 0.015 |
| 0.20 | 0.032 |
| 0.30 | 0.054 |
| 0.40 | 0.081 |

We will step off from $y_{b}=0.04$ to $y_{a}=0.001$
Or from $x_{b}=0.238$ to $x_{a}=0$
Plot on next page shows we need 4 stages

Problem 2

3. (30 points) A multi-component fractionating column equipped with a total condenser is devised to separate the following feed stream:

| Component | Mole fraction, $\mathrm{X}_{\mathrm{i}, \mathrm{F}}$ | Relative Volatility, $\mathrm{\alpha}_{\mathrm{ij}}$ |
| :--- | :--- | :--- |
| 1) Ethanol | 0.03 | 7.9 |
| 2) Propanol (LK) | 0.43 | 4.1 |
| 3) Butanol | 0.05 | 2.0 |
| 4) Pentanol (HK) | 0.44 | 1.0 |
| 5) Octanol | 0.05 | 0.13 |

If the column is designed to give a $95 \%$ recovery of Propanol in the distillate and a $98 \%$ recovery of Pentanol in the bottoms, what will be the composition of the distillate and bottoms streams?

## Solution

Applying the stated recoveries leads to the following information:


Next, we use the Fenske equation to solve for $\mathrm{N}_{\text {min }}$ for this system

- $i=2, j=4$

$$
N_{\min }+1=\frac{\ln \left[\frac{D x_{2 D} / B x_{2 B}}{D x_{4 D} / B x_{4 B}}\right]}{\ln \overline{\alpha_{2,4}}}=\frac{\ln \left[\frac{40.85 / 2.15}{0.88 / 43.12}\right]}{\ln 4.1}=4.8450
$$

Now, we apply the Fenske equation and the known value for $\mathrm{N}_{\text {min }}$ to the distributed component, Butanol, and the Heavy Key, Pentanol

- $\quad i=3, j=4$

$$
N_{\min }+1=\frac{\ln \left[\frac{D x_{3 D} / B x_{3 B}}{D x_{4 D} / B x_{4 B}}\right]}{\ln \overline{\alpha_{3,4}}}=\frac{\ln \left[\frac{\delta /(5-\delta)}{0.88 / 43.12}\right]}{\ln 2.0}=4.8450
$$

$$
4.8450 * \ln (2.0)=\ln \left[\frac{\delta /(5-\delta)}{0.88 / 43.12}\right]=3.3583
$$

$$
e^{3.3583}=\frac{\delta /(5-\delta)}{0.88 / 43.12}=28.7406
$$

$$
\delta /(5-\delta)=28.7406 * 0.88 / 43.12=0.5865
$$

$$
\delta=1.85
$$

$$
5-\delta=3.15
$$

Problem 3
100 mole basis


| Component | $\mathbf{x}_{\mathrm{di}}$ | $\mathbf{x}_{\mathbf{b i}}$ |
| :---: | :---: | :---: |
| Ethanol | 0.064 | 0.000 |
| Propanol | 0.877 | 0.040 |
| Butanol | 0.040 | 0.059 |
| Pentanol | 0.019 | 0.807 |
| Octanol | 0.000 | 0.094 |

Problem 1
Problem 1 VLE Toluene and Ethylbenzene


Problem 1
Txy Toluene - Ethylbenzene


Problem 2


