CE 400 / CE 500

Process Safety Management

Lecture 23 Probability and Statistics II

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Binomial Distribution

- Much like it sounds, this is a distribution where the outcome is an a/b or yes/no situation
- A Conforming Item is one that meets quality specifications (or works as designed)
- The count of non-conforming items is a discrete, random variable
- For a Binomial Distribution, the outcome of nonconformity, D, has a constant probability of p
- The outcome of conformity, S, has a constant probability of $q = 1 - p$ (Rule 1)
- By Rule 3, We know that the probability of a series of outcomes is obtained by multiplying the probability of the individual outcomes.
  - (The constant probability indicates that each event is independent of the previous events)
  - That assumption may not always be true – one failure may cause another in some cases…
- This terminology is for quality systems, next lecture we will switch to reliability terminology

Binomial Distribution, continued

- Uppercase denotes the random variable X
- X = number of non-conforming units
- Lowercase represents a specific value it can take, x
- For a number of n trials, X is the number of non-conforming results
- The probability of a given x is P(X=x) = $\boldsymbol{f(x)}$
- S = Satisfactory
- D = Defective

# of non-conforming in a sample of n = 2	Outcomes Yielding x	*f*(x)
0	SS	$qq = q^2$
1	Either SD or DS	$qp + pq = 2qp$
2	DD	$pp = p^2$

- Note that sum of $\boldsymbol{f}(\boldsymbol{x})$ must be 1
  - $q^2 + 2qp + p^2 = 1$
  - This is $(q + p)^2$. Note that $(q + p) = 1$ (Rule 1)

Binomial Distribution, continued

- General Case:
  - p = probability of individual trial being successful
  - n = number of trials
  - X = total number of successful trials
  - P(x) = probability of having exactly X = x successful trials out of n total trials

$$P(x) = \frac{n!}{(n-x)!\,x!} * p^x * (1-p)^{n-x}$$

  - Where n! is n factorial: $n! = n * (n-1) * (n-2) * \cdots * 1$

Poisson Distribution

- Used to model the number of times an event will occur in an interval of time or space
- Assumptions
  - k is the number of times the event occurs in the interval and is 0,1,2,3,etc
  - Events occur independently
  - The ***average*** rate that the event occurs is constant over the intervals
    - One could predict that if equipment is not maintained the failure rate will increase with time
  - Two events cannot occur instantaneously
  - The actual probability distribution is a Binomial Distribution where the number of trials is significantly larger than the number of successes (or failures, depending on your point of view) considered (i.e. the events are somewhat rare)
- P(k events in the interval) = $e^{-\lambda}\frac{\lambda^k}{k!}$
- λ = average number of events in the interval
- k = 0, 1, 2, 3, etc
- k! = k * (k-1) *(k-2) * … * 2 * 1   (k factorial)

Poisson Example

On a particular river, overflow floods occur once every 100 years on average. Calculate the probability of k = 0, 1, 2, 3, 4, 5, or 6 overflow floods in a 100-year interval, assuming the Poisson model is appropriate.

Because the average event rate is one overflow flood per 100 years, λ = 1

$$P(k \text{ overflow floods in 100 years}) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{1^k e^{-1}}{k!}$$

$$P(k = 0 \text{ overflow floods in 100 years}) = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} \approx 0.368$$

$$P(k = 1 \text{ overflow flood in 100 years}) = \frac{1^1 e^{-1}}{1!} = \frac{e^{-1}}{1} \approx 0.368$$

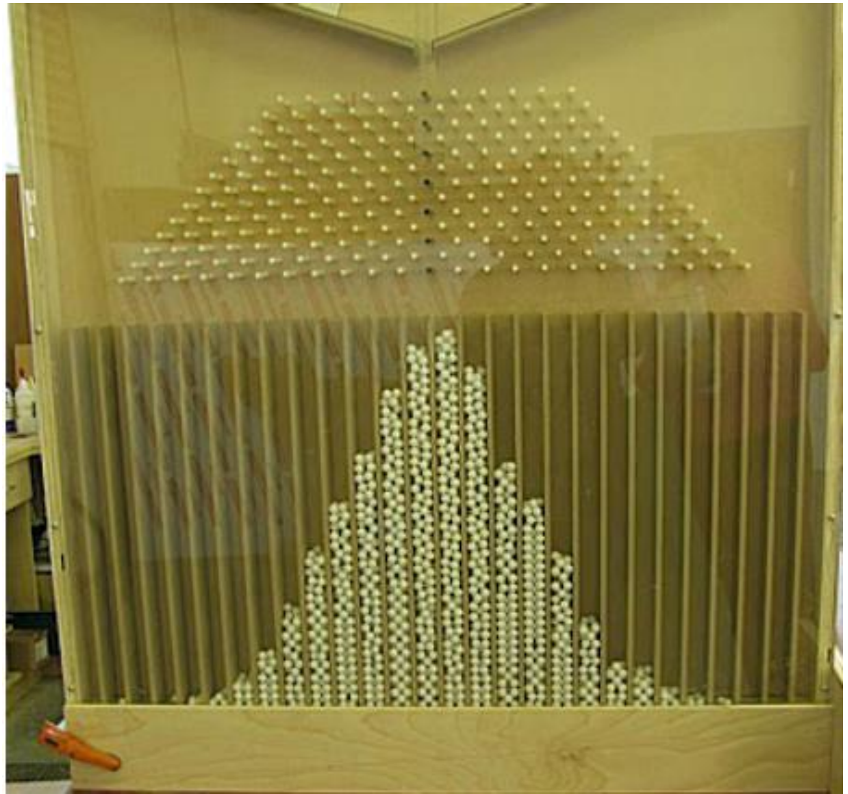
$$P(k = 2 \text{ overflow floods in 100 years}) = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} \approx 0.184$$

The table below gives the probability for 0 to 6 overflow floods in a 100-year period.

k	P(k overflow floods in 100 years)
0	0.368
1	0.368
2	0.184
3	0.061
4	0.015
5	0.003
6	0.0005

Normal Distribution

- This is a special case of the Binomial Distribution
- A continuous function that occurs when the number of possible outcomes is sufficiently large that it approaches becoming a continuous variable



Normal Distribution

- The probability of seeing a value of x (where x is considered to be a continuous variable)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

- μ is the mean
- σ^2 is the variance

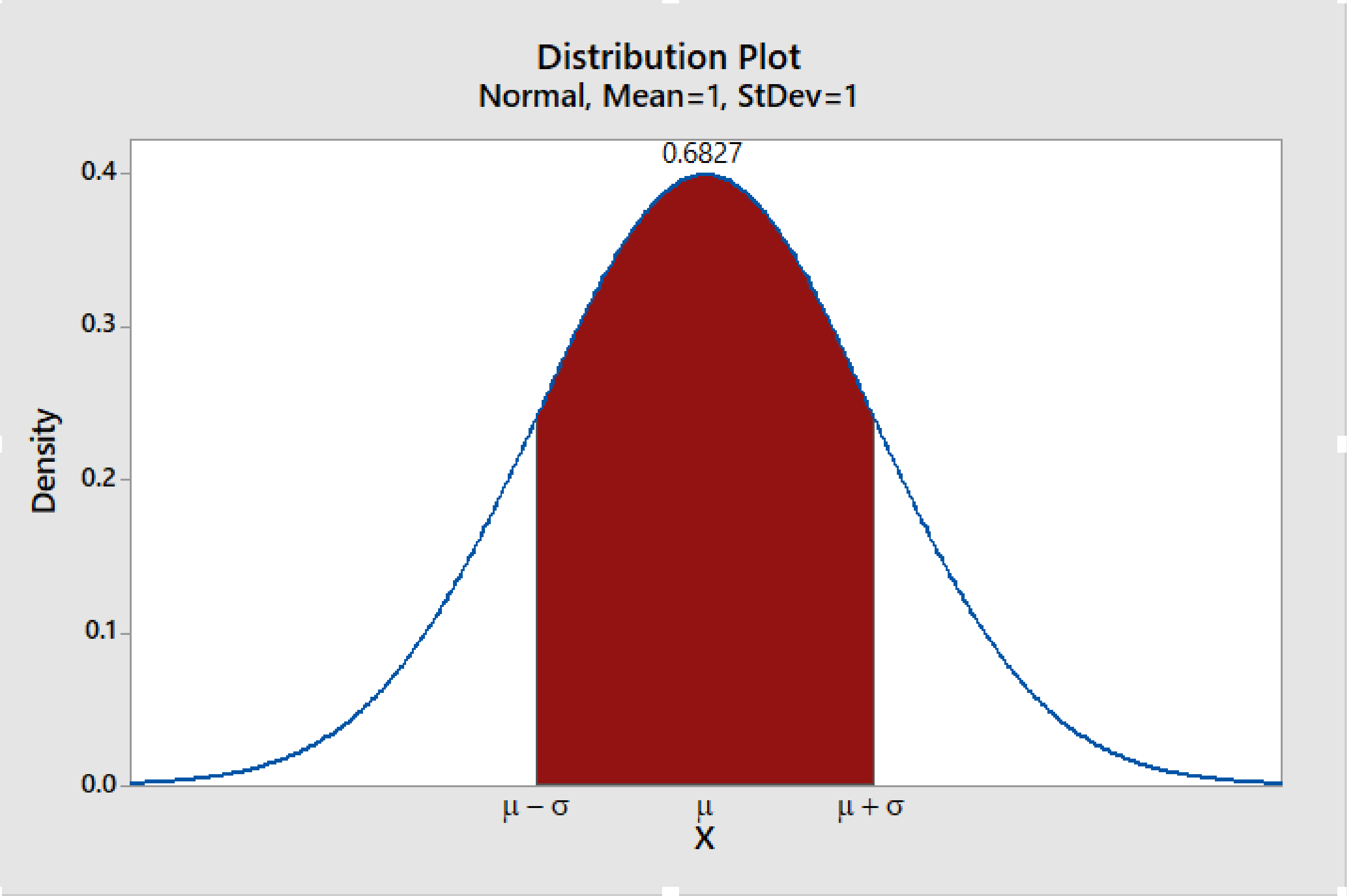
Normal Distribution

$$N(\mu,\sigma^2)$$

- N specifies that this is a Normal Distribution
- μ is the mean
- σ^2 is the Variance

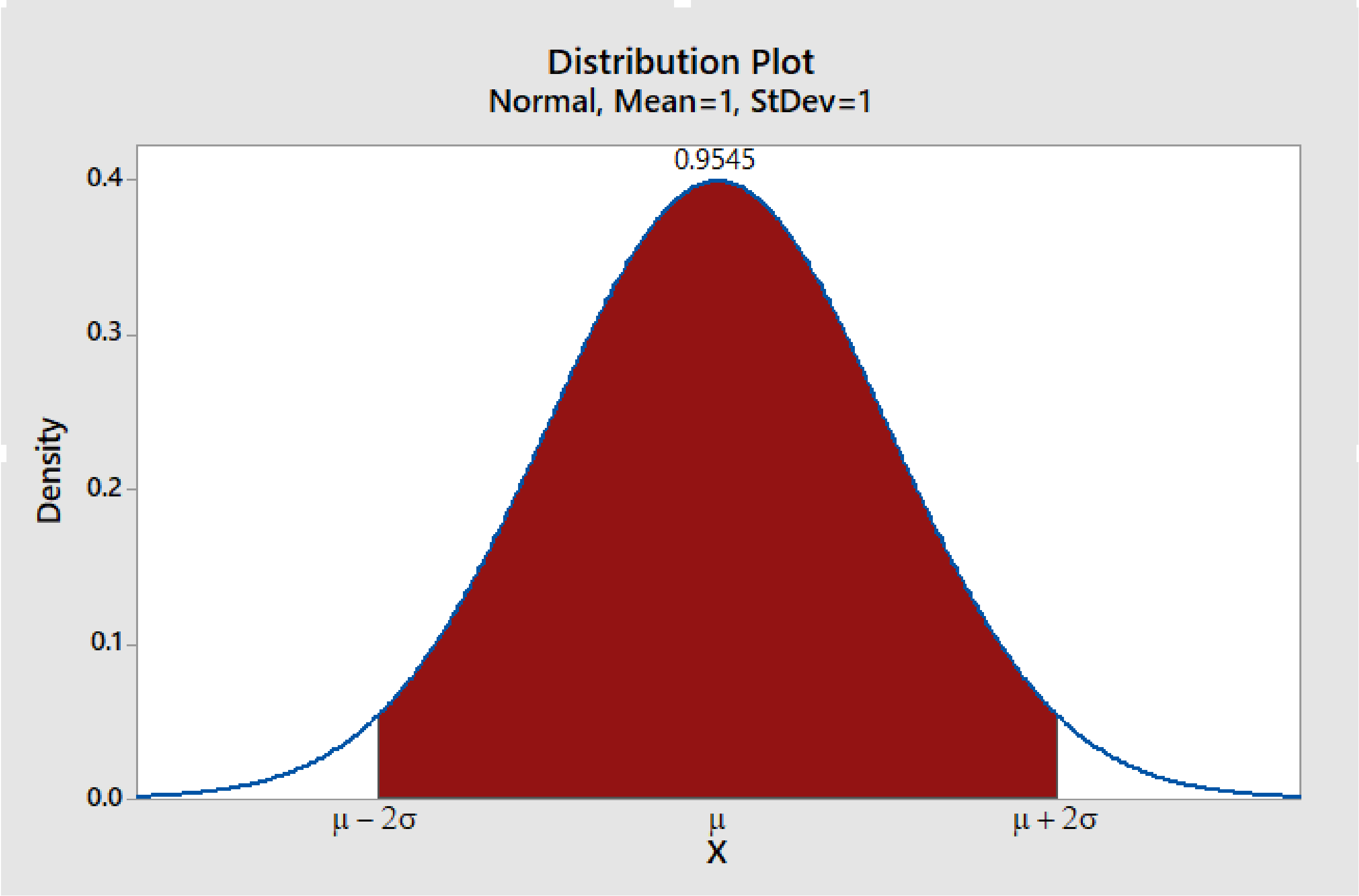
Normal Distribution

- Approximately 68.3% of data falls within one standard deviation of the mean



Normal Distribution

- Approximately 95.5% of data falls within two standard deviations of the mean



Normal Distribution

- Approximately 99.7% of data falls within three standard deviations of the mean