CE 400 / CE 500

Process Safety Management

Lecture 22 Probability and Statistics I

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What Do Probability and Statistics Have to Do With PSM?

- We can go a long way with qualitative analysis in our risk assessments

Probability is the likelihood of the hazard occurring and it is often ranked on a five point scale:

- **Frequent - 5** Likely to occur often in the life of an item
- **Probable - 4** Will occur several times in the life of an item
- **Occasional - 3** Likely to occur some time in the life of an item.
- **Remote - 2** Unlikely but possible to occur in the life of an item.
- **Improbable - 1** So unlikely, it can be assumed occurrence may not be experienced.

- Example from Vector Solutions

- This is a simple and easy to understand approach
- It is sufficient for most cases
- When we start talking about C3 and C4 incidents we often need to be a bit more quantitative

What Do Probability and Statistics Have to Do With PSM?

- Suppose one valve in the wrong position will lead to an incident?
  - That is certainly likely to be a 5 on this scale
- Suppose two specific valves must be in the wrong position?
  - Is that a 4 or a 5?
- Suppose two specific valves must be in the wrong position and a dike has to fail?
  - ???

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Probability Basics

- The set of possible outcomes is the sample space
  - Coin Flip {Heads, Tails}
  - Roll of a single di {1, 2, 3, 4, 5, 6}
- Can specify a subset of the possible outcomes
  - H = {Heads}, coin toss lands in heads
  - A = {1, 3, 5}, roll of di is an odd number
  - B = {2, 4, 6}, roll of di is an even number

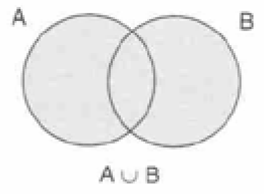
Probability and Statistics

- The probability of an event is expressed as P(X) where X is the subset of events
  - Coin Toss

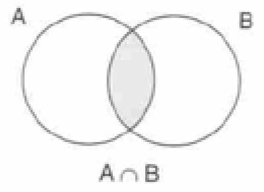
    P(H) = 0.5. There are two equally likely outcomes (heads and tails), 50% of these equally likely outcomes satisfy the subset H and the probability is therefore 0.5
  - Di

    Each number on di is equally likely to come up and there are 6 possible results

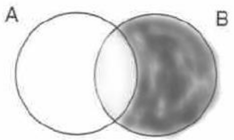
    P(1) = 1/6
- Example with Two possible subsets: A = {1,3,5,4}, B= {2,3,6}
- Union – The result is in one (or both) of the two subsets
  - A ∪ B = {1,2,3,4,5,6}
  - Think of ∪ as being a “U” in union



- Intersection – The result is in BOTH of the two subsets
  - A ∩ B = {3}



- Complement – The result is NOT in a given subset
  - $\bar{A}$ = {2,6} or “the non-occurance” of A



Rules for Probabilities – Rule 1

- **P($\bar{E}$) = 1 – P(E)**

- This just states that the probability of something ***NOT*** happening is:

1 – the probability of it actually happening

Example – Rule 1

- 11% of people are Left Handed

$$P(LH) = 0.11$$

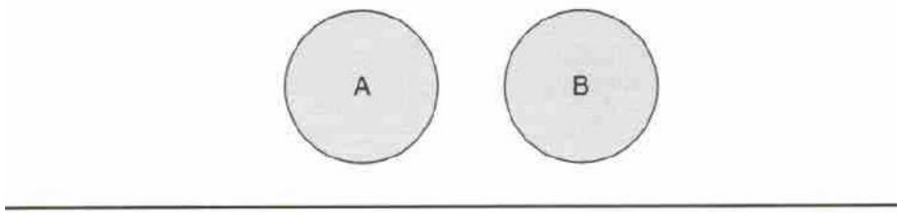
- The probability of being non-Left Handed

$$P(\overline{LH}) = 1 - 0.11 = 0.89$$

Rules for Probabilities – Mutually Exclusive Events

- Mutually Exclusive Events – if one happens, the other cannot happen:
  - Coin cannot simultaneously land on heads and tails
  - Di cannot simultaneously land on two different numbers

- Assume set A and B are mutually exclusive
  - $A \cap B = \{\ \}$ (empty set)
  - $P(A \cap B) = 0$



Mutually exclusive events.

Rules for Probabilities – Rule 2

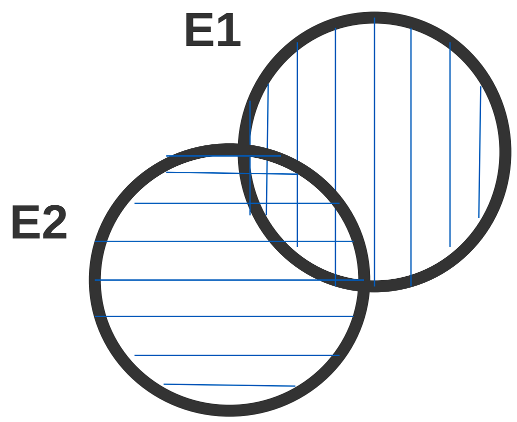
- Probability of Either E_1 or E_2 Occurring

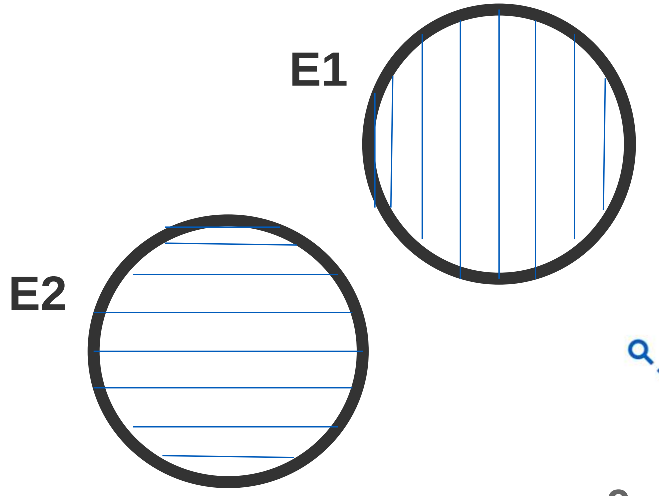
  - General Addition Rule:

    $$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

  - Mutually Exclusive Addition Rule:

    $$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$





Example - Rule 2

- 2 out of 100 people have Green Eyes

$$P(GE) = 0.02$$

- 17 out of 100 people have Blue Eyes

$$P(BE) = 0.17$$

- Eye color is mutually exclusive (assuming both eyes are the same color)
- The probability of being Blue Eyed **and** Green Eyed is Zero

$$P(GE \cap BE) = 0$$

- The probability of being Blue Eyed or Green Eyed

$$P(GE \cup BE) = 0.02 + 0.17 = 0.19$$

Example – Rule 2

- Roll of a Di
- Even Numbers, $E_1 = \{2,4,6\}$: Multiples of 3, $E_2 = \{3,6\}$

$$P(E_1) = 0.5 \quad = 3/6$$

$$P(E_2) = 0.3333 \quad = 2/6$$

- Intersection $(E_1 \cap E_2) = \{6\}$, $P(E_1 \cap E_2) = 0.1667 = 1/6$

- The probability of being Even Numbered or Multiple of 3

$$\text{Union } (E_1 \cup E_2) = \{2,3,4,6\}$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$P(E_1 \cup E_2) = 0.5 + 0.3333 - .1667 = 0.667$$

Independent Variables

- $(B|A)$ is the event B occurring given that A has already occurred
- P$(B|A)$ is the probability of B occurring GIVEN that A has already occurred
- Two events are independent if the occurrence of one does NOT change the probability that the other will occur
- For Independent Variables:

$$P(B|A) = P(B)$$

Rules for Probabilities – Rule 3

- General Multiplication Rule:

$$P(A \cap B) = P(B|A) \times P(A)$$

- Independent Variable Multiplication Rule:

$$P(A \cap B) = P(A) \times P(B)$$

Example – Rule 3

- Eye color is Independent of Handedness

- The probability of being Green Eyed is NOT affected by handedness

$$P(GE|LH) = P(GE) = 0.02$$

$$P(GE|RH) = P(GE) = 0.02$$

- The probability of being Green Eyed AND Left Handed

$$P(GE \cap LH) = 0.02 \times 0.11 = 0.0022$$

Example – Rule 3

- Left Handedness is known to be more prevalent amongst Mathematicians

- The probability of being Left Handed is affected by Mathematician Status

$$P(LH|Math) = 0.21^{*}$$

P(LH) = 0.11 for General Population

P(Math) = 0.02**

- The probability of being Left Handed AND a Mathematician

$$P(LH \cap Math) = P(LH|Math) * P(Math) = 0.21 \times 0.02 = 0.0042$$

- If the two were independent then

$$P(LH \cap Math) = P(LH) * P(Math) = 0.11 \times 0.02 = 0.0022$$

* Not actual data

** % of BS degrees that are given in mathematics