MAE 412
Machines & Mechanisms II

Final Project:
Catapult System

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Project Goal

The main objective in this project is to develop a “catapult system” that will be used to throw a squash ball. The device must use at least a 4-bar mechanism in order to complete the task and can be powered only by a standard motor that will be provided for every group. The success of each mechanism will be judged based on how far it is able to throw a squash ball and how well it is able to be adjusted so that the squash ball is thrown at targets with accuracy.
Constraints and Rules

There are several constraints on exactly how each group may go about designing and building their catapult device. First, the entire device must be made of wood. While each link in the mechanism must be made of wood, however, the joints used to connect links may be made of other materials such as metal pins or bearings. Second, the device will have a 2’ by 2’ operating window that it may never leave. The base plate for the device must fit within this window as well. Next, the mechanism should be mounted onto its base plate in such a way that it may be quickly and firmly clamped onto a table using a pair of C-clamps). Finally, no part of the mechanism is allowed to cross the start plane from which the judges will measure the distance traveled by the squash ball.

The rules of the competition dictate that the squash ball is to be hand-loaded onto the device. Afterwards, teams are required to set the mechanism in motion by turning on the switch connecting power to the motor. Students are not allowed to manually store potential energy in the mechanism either by raising weights or compressing springs as part of setting it up prior to a throw. From the moment the motor is turned on, each device will have up to 30 seconds to throw the squash ball. During the competition, each group will have its catapult mounted on a table and the edge of this table will serve as the start plane. Distances for each throw will be measured in centimeters from this start plane to the ball’s point of impact on the ground. Each team will get three tries and the sum of the three distances will serve as their score for the distance portion of the competition. The accuracy portion of the competition will feature waste baskets at a minimum distance of 10 feet from the start plane and increasing in distance in 5 foot increments (15 feet, 20 feet, etc.). Students are expected to be able to adjust their
mechanisms so that the squash ball may be thrown into the waste baskets at different distances.

All parts used to construct the device should be taken from students’ houses, apartments, or garages, as well as machine shops and scrap yards. Purchases of new items should be kept to a minimum. Students should take special care in not burning out the motor that is given to them and in designing their mechanism with safety in mind.
Design Strategies

One of the first steps our group took in determining how to build our mechanism was to examine the projectile it would be throwing. In other words, we tested a squash ball to determine what would be the best way to throw it. If the ball had a noticeable ability to bounce (a high modulus of elasticity), we theorized that one of the best ways to maximize the distance it traveled based on the force applied would be to design a four-bar that could hit or swat the ball into the air. If the squash ball had a minimal ability to bounce, the best way to proceed would be through the use of a conventional catapult. After obtaining a squash ball and running just a few tests on it, it was clear that it had not been designed to bounce and that we therefore would be designing our mechanism as a conventional catapult. Potential energy would be stored as the throwing arm was set down and released once the arm was allowed to spring forward, propelling the ball. The next step was to determine exactly how potential energy could be stored and used to move the throwing arm.

After some discussion, the group determined that the two best ways of storing potential energy with the motor would be through compressing springs or raising a weight. In the case of springs, the motor would be used to bring either the throwing arm or a mounted slider back against a spring, thereby compressing it. Upon release, the spring would expand and the arm or slider would be propelled forward and throw the ball. In each case, multiple springs could be used on different links both in tension or compression for maximum energy storage. The spring mechanism could be comprised of just four bars but also could include several more for increased storage of potential energy. On the other hand, a simple pulley system could be used in conjunction with the
standard motor to raise a weight. This arrangement would make use of a very simple four-bar mechanism. Once the weight is at a given height, it could release and fall onto one of the links. This link, which would be pivoted to the ground near its midpoint and serve as the input link in this case, would cause a chain reaction through the coupler whereby the follower link, the throwing arm, would experience rapid angular acceleration and would throw the ball. The dimensions of the input link and coupler could be designed to ensure that the limiting conditions on the follower link are such that the ball is released at the optimum time (at a 45º angle, theoretically).

Several factors were taken into account when deciding which of the two design concepts was the best to proceed with but after all was said and done, the four-bar mechanism that used stored potential energy in the form of a raised weight appeared to be the best way to go. It would be easy to estimate the force that a falling weight could impart on a four-bar. In contrast, the forces exerted upon release of springs in tension and compression would be much more difficult to estimate and test. Its design was such that it could easily be built, tested, and modified for maximum performance. The simplicity of the design will also be beneficial when constructing it and simulating its motion in Solid Edge/Dynamic Designer. Finally, when it comes to throwing the squash ball at targets that are set distances from the start plane, it seemed clear that a simple design that made use of a raised weight would be the easiest to adjust during the competition to this end.

Lastly, with the basic concept behind the mechanism decided upon, the early prototype for the release mechanism was discussed. In raising a weight, there must be a set height at which the weight stops moving upward and is released so that it may fall
onto the input link. The best way to achieve this goal would be to make the string used to raise the weight in the pulley system to be dependent on a hook that itself is attached to the base via string. At a given height, there will be tension in the string connecting the hook to the base and the weight will stop moving upwards. The motor will continue to wind the string around the pulley, however, and this will cause the hook to rotate until it drops the string attached to the weight. The weight will then fall onto the input link and the catapult will spring into action.

Figure 1: A basic sketch of the catapult design to be used for our mechanism.

Figure 2: Sketch of the Release Mechanism
Solid Edge Simulations

After settling on the basic concept behind our four-bar mechanism, the next step was to construct it with Solid Edge and simulate how it would respond when asked to throw a squash ball as a weight falls onto the input link. Figures 3 through 8 show the Dynamic Designer sequence used to test the motion of the four-bar. A mass is dropped from 20 inches above the ground link and lands on the far end of the input link, thereby causing all of the links in the four-bar mechanism to respond and the ball at the end of the four-bar to be propelled through the air. The mass that was dropped was made 2.72 kg (6 pounds) and was dropped from 20 inches based on some initial tests that had been performed to determine the strength of the motor and how high it could lift various masses. The squash ball was given a mass of 28g (the mass of all regulation squash balls) and was modeled as a small cube at the end of the throwing arm.

The input and coupler links were given densities of 640 kg/m³ (that of yellow pine) while the output link had a density of 140 kg/m³ (that of balsa wood). Iterative tests were performed simulating the mass falling onto the input link when the link lengths were altered. It was important to measure the angular velocity and acceleration of the throwing arm (output link) along with its angle. Theoretically, the ball would separate from the arm as soon as the arm’s velocity began to decrease. To achieve a maximum distance, then, the maximum velocity of the throwing arm should be achieved at the point where it makes a 45° angle with the horizontal (135° angle from the front horizontal reference used in this class).

The first length that was altered was link 4, the distance between where the throwing arm is pivoted to the ground and where it is connected via joint to the coupler. After a
few trials, a length of 3 inches (7.62 cm) was found to be optimum. The next dimension to optimize was that between where the input link was grounded and where it was joined with the coupler. A length of 2.787 inches (7.08 cm) was eventually chosen. Lastly, the total length of the throwing arm was tested in order to determine if there was a length at which its weight would result in a lessened angular velocity and, as a result, the ball not being thrown as far. It was important to remember that the ball’s velocity when released would be the arm’s length multiplied by its angular velocity. After performing simulations where the arm was both long and short, a total arm length of 20 inches was found to be ideal.

The final lengths for links 1 through 4 were 7.222, 2.787, 4.5, and 3 inches respectively, while the total lengths of the input and output links were 11 and 20 inches respectively. The angle between the two ground pivots, \( \theta_1 \), was 17.45º. The input link was created in Solid Edge in such a way that it was offset after being pivoted to the ground. This ensured that its mass within the program was similar to the actual link we would eventually construct.

Graphs 1 through 3 show the change in \( \theta_4 \) along with its angular velocity and acceleration respectively. It must be noted that Dynamic Designer measures \( \theta_4 \) with respect to the closest horizontal while the convention used in this class was to measure \( \theta_4 \) with respect to its front horizontal. In addition, Graph 1 is the change in \( \theta_4 \) from its initial angle (15º from the horizontal in our case). At \( t = 0.28 \) seconds in the graphs, the angle \( \theta_4 \) has changed 30º (meaning it is now 45º from the horizontal) and its angular velocity has begun to decrease from its maximum of 962 deg/sec (16.79 rad/sec). This arrangement, then, seemed ideal for throwing the squash ball a maximum distance.
Graph 1: This is a graph of how much $\theta_4$ changes with respect to time as the mass falls onto the input link and the throwing arm reaches its limiting condition (at approximately $t = 3.05$ seconds). This change is with respect to the horizontal so the $\theta_4$ is actually decreasing (as shown in Figures 3 through 8). The magnitude of the change, however, is correct and simply negating the projected angle values shows how the $\theta_4$ used in common four-bar problem solving changes.

Graph 2: This figure shows the angular velocity of the throwing arm. Again, Dynamic Designer measures the $\theta_4$ with respect to how close it is to the horizontal so the magnitude of these values is correct, but the sign needs to be flipped in order to obtain angular velocities that agree with the sign convention used in this class.
Graph 3: This Figure shows the angular acceleration of the throwing arm and, like Graphs 1 and 2, requires that the angular acceleration values returned by Dynamic Designer have their signs flipped so that they are accurate. This figure makes it clear that when the mass hits the input link (at approximately $t = 0.245$ seconds), the throwing arm experiences rapid acceleration and deceleration.
Orthogonal View of the Four-Bar Mechanism Created in Solid Edge. The mass dropped onto the Input Link is suspended in the air while the squash ball is seated on the far end of the Throwing Arm.
Matlab Simulations

Two Matlab programs were created in order to determine how far the ball would travel through the air once it was released by the mechanism at a 45° angle. These programs were extremely helpful in determining what results we could expect from our four-bar mechanism and how we could optimize and adjust those results.

One can place different release velocities into the ‘projection’ m-file to obtain the angular velocity required for a desired distance. This program can also be used to track the ball’s path through the air. The angular velocity obtained from this program can then be inserted into the ‘posivelace1’ m-file to find the required input angular velocity for the desired distance.

Figures 9 and 10 are plots of the ball’s path through the air for two different release velocities. The initial height of the ball is different in each case because Figure 9 is a plot used to determine maximum distance while Figure 10 is used to illustrate throwing the ball into a waste basket. The initial height of the ball was made an input for the ‘projection’ program in order to ensure that accurate results could be obtained in each case.
Matlab Program for Position, Velocity and Acceleration Analysis

function [x,y]=posi(t4,td4,tdd4)
% calculates position, velocity and acceleration of links two and three
% given t4=theta4, td4=theta4dot, tdd4=theta4doubledot
r1=7.222;
r2=3;
r3=4.5;
r4=3;
th1=17.45*pi/180;
th4=t4*pi/180;

% Position
m=r1*cos(th1)+r4*cos(th4);
n=r1*sin(th1)+r4*sin(th4);
A=-2*r2*m
B=-2*r2*n
C=r2^2+n^2+m^2-r3^2
t1=(-B+(B^2+A^2-C^2)^.5)/(C-A);
th2a=2*atan(t1);
t2=(-B-(B^2+A^2-C^2)^.5)/(C-A);
th2b=2*atan(t2);
th3a=asin((n-r2*sin(th2a))/r3);
th3b=asin((n-r2*sin(th2b))/r3);
theta2a=th2a*180/pi;
theta2b=th2b*180/pi;
theta3a=th3a*180/pi;
theta3b=th3b*180/pi

% velocity
theta1dot=0;
theta4dot=-td4;
B=[(-r1*sin(th1)*theta1dot)-
   (r4*sin(th4)*theta4dot);(r1*cos(th1)*theta1dot)+(r4*cos(th4)*theta4dot)]
a=[-r2*sin(th2b) -r3*sin(th3b); r2*cos(th2b) r3*cos(th3b)]
A=inv(a)
x=A*B
theta2dot=x(1,1)
theta3dot=x(2,1)

% Acceleration
theta4ddot=tdd4
E=[-r4*cos(th4)*(theta4dot)^2-
   r4*sin(th4)*theta4ddot+r2*cos(th2b)*(theta2dot)^2+r3*cos(th3b)*(theta3dot)^2; -
   r4*sin(th4)*(theta4dot)^2+r4*cos(th4)*theta4ddot+r2*sin(th2b)*(theta2dot)^2+r3*sin(th3b)*(theta3dot)^2]
d=[-r2*sin(\(\theta_2b\)) -r3*sin(\(\theta_3b\)); r2*cos(\(\theta_2b\)) r3*cos(\(\theta_3b\))] 
D=inv(d) 
y=D*E 
\(\theta_2\)ddot=y(1,1) 
\(\theta_3\)ddot=y(2,1)
Matlab code for calculating distance and angular velocity required for distance

% Projectile motion simulation
Vi=input('Enter Vo in in/s = ')
y0=input('Enter the height of projectile in ft = ')
x0=0; % ft
v0=Vi/12; % ft/s
theta=30; % deg
g=9.81*3.33; % ft/s^2

b=v0*sin(pi*(theta/180));
a=-g/2;
c=y0;

t_flight=(-b-sqrt(b^2-4*a*c))/(2*a);
range=v0*cos(pi*(theta/180))*t_flight;

t=linspace(0,t_flight,30);
xdot0=v0*cos(pi*(theta/180));
ydot0=v0*sin(pi*(theta/180));
x=xdot0*t+x0;
y=-(g/2)*t.^2+ydot0*t+y0;
angvel4=v0/1.6667
plot(x,y)
axis equal
xlabel(sprintf('Distance (ft)= %5.3f',range));
ylabel('Height (ft)');
title(sprintf('Projectile motion: angular velocity_{r4} = %5.3f rad/s',angvel4));
Figure 9: The following graph shows the ball’s path through the air when required to travel a distance of 26.581 feet. The ball’s initial height here is 5 feet (3 feet high because of the table and another 2 feet thanks to being at the top of the mechanism when released). The angular velocity of the throwing arm required for the ball to travel this distance is 16.500 rad/sec.
Figure 10: The following graph reveals the path of the ball through the air as it moves a distance of 10 feet, a distance ideal for throwing the ball into a waste basket. In this case, the initial height of the ball is 2 feet because the table’s height of 3 feet will be offset by the opening of the waste basket also being 3 feet high (approximately). The angular velocity required for throwing the ball this way is 10.05 rad/sec.
Sample of results from posivelacel.m (Used to Check Hand Calculation Analysis)

```matlab
>> posivelacel(120,13.96,3655)
```

\[
\begin{align*}
\theta_2b &= 21.2235 \\
\theta_3b &= 54.8130 \\
\theta_2\dot &= 22.9037 \\
\theta_3\dot &= -16.6252 \\
\theta_2\ddot &= -4.6052 \times 10^3 \\
\theta_3\ddot &= 3.2687 \times 10^3 
\end{align*}
\]
Sample of results from posivelacel.m (gives angular velocity required for a 10 foot throw using the 10.05 given from graph)

```matlab
>> posivelacel(120,10.05,3655)
theta2b =
    21.2235

theta3b =
    54.8130

theta2dot =
    16.4887

theta3dot =
   -11.9687

theta2ddot =
   -5.2755e+003

theta3ddot =
   3.7909e+003

ans =
    16.4887
   -11.9687
```
**Position, Velocity, Acceleration, and Force Analysis**

The following pages show our hand calculations for the four-bar mechanism we created at the point where \( \theta_1 = 17.45^\circ \) and \( \theta_4 = 120^\circ \) (near one of the throwing arm’s limiting conditions). The link lengths are the same as those used in the Solid Edge simulations: \( R_1 = 7.222” \), \( R_2 = 2.787” \), \( R_3 = 4.5” \), \( R_4 = 3” \). Graphs 2 and 3 of the Solid Edge simulations provide the angular velocity and acceleration of link 4. At this position, the throwing arm’s angular velocity is -13.96 rad/sec and its angular acceleration is 3665 rad/sec².

Position analysis is performed using Vector Loop Closure Equations (Method III) and the Law of Intersecting Circles (Method I). In this case, Method I was done graphically. Instantaneous Centers and Vector Loop Closure Equations were used to perform velocity analysis. Acceleration analysis, then, also made use of the Vector Loop Closure Equations method. The values we obtained for position, velocity, and acceleration analysis could be checked using the ‘posivelace1’ Matlab program we created, as well.

For Force Analysis, we were required to set up the pertinent equations. We did this by drawing free body diagrams of the input link, coupler, and output link. Equations were written for sum of forces in the x- and y-directions and the moment about the center of mass for each link. These equations were then put into matrix form where, once values were substituted for each variable, they could easily be solved.
Position Analysis – Method III

<table>
<thead>
<tr>
<th>Length</th>
<th>Input Link</th>
<th>Coupler Link</th>
<th>Output Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>r₁ = 2.222&quot;</td>
<td>r₂ = 2.223&quot;</td>
<td>r₃ = 3&quot;</td>
<td>r₄ = 5&quot;</td>
</tr>
</tbody>
</table>

Angles:
θ₁ = 90.45°, θ₂ = 7°, θ₃ = 7°, θ₄ = 120°

Using Method III (Least Square Equation)

\[
\begin{align*}
\frac{r_1 \cos \theta_1}{r_2 \sin \theta_2} &= \frac{r_3 \cos \theta_3}{r_4} = M = 3.290 \\
\frac{r_1 \sin \theta_1}{r_2 \sin \theta_2} &= N = 4.324
\end{align*}
\]

\[
\begin{align*}
\bar{r}_1 &= \frac{r_1^2 + r_2^2 + r_3^2 + r_4^2}{4} \\
\bar{r}_1 r_2 \cos \theta_2 + \bar{r}_1 r_3 \cos \theta_3 &= 0 \\
\bar{r}_1 r_2 \sin \theta_2 + \bar{r}_1 r_4 &= 0
\end{align*}
\]

\[
\begin{align*}
\frac{\bar{r}_1}{\bar{r}_2} &= \frac{r_1}{r_2} \\
\frac{\bar{r}_1}{\bar{r}_4} &= \frac{r_1}{r_4}
\end{align*}
\]

\[
A = \begin{bmatrix} A^2 & 2B \theta \bar{r}_1 \cos \theta_2 \\
C - A \bar{r}_1 \cos \theta_3 & C + \bar{r}_1 \cos \theta_3 \end{bmatrix}, \quad \begin{bmatrix} A \theta \bar{r}_1 \cos \theta_2 \\
B \theta \bar{r}_1 \cos \theta_2 + C \bar{r}_1 \cos \theta_3 \end{bmatrix} = 0
\]

Solving:
\[
\begin{align*}
\theta &= \frac{1}{2} \sin^{-1} \left( \frac{2B}{1-C} \right) \\
A &= \frac{1}{1+C} \\
B &= \frac{A \bar{r}_1 \cos \theta_2}{1-C} \\
C &= \frac{A \bar{r}_1 \cos \theta_3}{1+C}
\end{align*}
\]

\[
\begin{align*}
A &= 0.964 \\
B &= 0.258
\end{align*}
\]
Position Analysis - Method III (Continued)

\[
(\theta_3)_1 = 7.4^{\circ}
\]

\[
(\theta_3)_2 = 8.5^{\circ}
\]

\[
\theta_1 = 45^{\circ} \cdot \left( \frac{N - 5 \sin \theta_3}{M - 5 \cos \theta_3} \right)
\]

\[
(\theta_3)_1 = 21.3^{\circ}
\]

For uncross configuration: \(
\theta_2 = 21.2^{\circ}
\)

\[
\theta_3 = 74.1^{\circ}
\]
Position Analysis – Intersecting Circles
Velocity Analysis – Instantaneous Centers

<table>
<thead>
<tr>
<th>Length</th>
<th>Tangent Line</th>
<th>Complex Line</th>
<th>Follower Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( r_1 = 7.25^\circ )</td>
<td>( r_2 = 2.85^\circ )</td>
<td>( r_3 = 3^\circ )</td>
</tr>
<tr>
<td>Angles</td>
<td>( \beta_1 = 124.4^\circ )</td>
<td>( \beta_2 = 202^\circ )</td>
<td>( \beta_3 = 58.2^\circ )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
J_{1a} &= (0,0) \\
J_{1a} &= (r_{1a} \cos \theta_1, r_{1a} \sin \theta_1) \\
J_{1a} &= (0.79, 2.16b) \\
J_{1a} &= (2.54h, 1.01) \\
J_{1a} &= (5.14, 4.82) \\
\end{align*}
\]

\[
\begin{align*}
J_{1a} J_{2a} &= 1.01 - 0 = 2.54 - 0 = 0.307 \\
J_{1a} J_{3a} &= 2.16b - 0 = 0.304 \\
J_{1a} J_{4a} &= 0.79 - 0 = 0.79 \\
J_{1a} J_{2a} &= 1.01 - 0 = 1.01 \\
\end{align*}
\]

\[
\begin{align*}
\eta_2 &= \frac{a \times b}{b \times c} \\
\eta_3 &= a \times b \\
\eta_4 &= \frac{a \times b}{b \times c} \\
\eta_5 &= \frac{a \times b}{b \times c} \\
\end{align*}
\]

\[
\begin{align*}
\eta_2 &= 0.79a \\
\eta_3 &= a \\
\eta_4 &= \frac{a \times b}{b \times c} \\
\eta_5 &= \frac{a \times b}{b \times c} \\
\end{align*}
\]

\[
\begin{align*}
\eta_2 &= 0.79a - 0.79 \\
\eta_3 &= a - 0.79 \\
\eta_4 &= \frac{a \times b}{b \times c} \\
\eta_5 &= \frac{a \times b}{b \times c} \\
\end{align*}
\]
Velocity Analysis – Instantaneous Centers (Continued)

\[ \frac{I_{1u} - I_{3u}}{I_{2u}} = \frac{I_{1u} - I_{2u}}{I_{3u}} = -1.46 \]

\[ \frac{I_{1u} - I_{3u}}{I_{2u}} = \frac{I_{1u} - I_{2u}}{I_{3u}} = -1.46 \]

\[ I_{1u} = \text{Intersection of } I_{2u} \text{ and } I_{3u} \]

\[ \frac{y - 0.344}{2.16} = 1.1124 - 3.674 \]

\[ x = 2.16 \]

\[ y = 0.344 \]

\[ I_{2u} = \text{Intersection of } I_{2u} \text{ and } I_{3u} \]

\[ \frac{y - 0.344}{2.16} = 1.1124 - 3.674 \]

\[ x = 2.16 \]

\[ y = 0.344 \]

\[ \frac{I_{1u} - I_{3u}}{I_{2u}} = \frac{I_{1u} - I_{2u}}{I_{3u}} = -1.46 \]

\[ \frac{I_{1u} - I_{3u}}{I_{2u}} = \frac{I_{1u} - I_{2u}}{I_{3u}} = -1.46 \]

\[ \frac{I_{1u} - I_{3u}}{I_{2u}} = \frac{I_{1u} - I_{2u}}{I_{3u}} = -1.46 \]

\[ \frac{I_{1u} - I_{3u}}{I_{2u}} = \frac{I_{1u} - I_{2u}}{I_{3u}} = -1.46 \]

\[ \omega_2 = 28.26 \text{ rad/s} \]

\[ \omega_3 = 16.31 \text{ rad/s} \]
Velocity and Acceleration Analysis – Method III

<table>
<thead>
<tr>
<th>Length</th>
<th>Input Link</th>
<th>Output Link</th>
<th>Transmission Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ell_1 )</td>
<td>7.21&quot;</td>
<td>2.67&quot;</td>
<td>4.15&quot;</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>17.45°</td>
<td>21.2°</td>
<td>59.9°</td>
</tr>
<tr>
<td>Velocity</td>
<td>( \dot{\theta}_1 )</td>
<td>( \dot{\theta}_2 )</td>
<td>( \dot{\theta}_3 )</td>
</tr>
<tr>
<td>Acceleration</td>
<td>( \ddot{\theta}_1 )</td>
<td>( \ddot{\theta}_2 )</td>
<td>( \ddot{\theta}_3 )</td>
</tr>
</tbody>
</table>

Differentiate with respect to time:

\[
\begin{bmatrix}
-\rho_x \\
\rho_y \\
\rho_z
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
-\rho_x \ddot{\theta}_1 \\
-\rho_y \ddot{\theta}_1 \\
-\rho_z \ddot{\theta}_1
\end{bmatrix}
\]

Differentiate again with respect to time:

\[
\begin{bmatrix}
-\rho_x \\
\rho_y \\
\rho_z
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\ddot{\theta}_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
-\rho_x \dddot{\theta}_1 \\
-\rho_y \dddot{\theta}_1 \\
-\rho_z \dddot{\theta}_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dddot{\theta}_1 \\
\dddot{\theta}_2 \\
\dddot{\theta}_3
\end{bmatrix}
= 
\begin{bmatrix}
-0.2425 \\
2.199 \\
-1.939
\end{bmatrix}
\]

\( \dot{\theta}_2 = -5.13 \text{ rad/s}^2 \)

\( \dot{\theta}_3 = 2450.14 \text{ rad/s}^2 \)
Force Analysis

\[ \text{Link 2} \]

\[ \text{Force Analysis} \]

\[ x : F_{23,y} = F_{32,y} + m_3 \ddot{y}_{32} = 0 \]

\[ y : F_{23,x} = F_{32,x} + m_3 \ddot{x}_{32} = 0 \]

\[ \text{Moment} : -F_{23,x} (a_2 \sin \theta_2) + F_{32,x} (a_3 \sin \theta_3) + \theta_3 = 0 \]

\[ \text{Link 3} \]

\[ x : F_{33,y} = F_{34,y} + m_4 \ddot{y}_{34} = 0 \]

\[ y : F_{33,x} = F_{34,x} + m_4 \ddot{x}_{34} = 0 \]

\[ \text{Moment} : -F_{33,x} (a_4 \sin \theta_4) + F_{34,x} (a_5 \sin \theta_5) + \theta_5 = 0 \]
Force Analysis (Continued)

\[ a: F_{x_1} = F_{x_2} - m_1 \ddot{x}_{1c} = 0 \]

\[ b: F_{y_1} + F_{y_2} + m_1 \ddot{y}_{1c} = T \]

\[ \text{moment: } F_{x_1} \cdot (a, \rho \phi) + F_{y_1} \cdot (y_1, \rho \phi) - \tau_{1c} = 0 \]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{a_1}{b_1} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
F_{x_1} \\
F_{y_1} \\
F_{x_2} \\
F_{y_2} \\
F_{x_3} \\
F_{y_3} \\
F_{x_4} \\
F_{y_4} \\
F_{x_5} \\
F_{y_5} \\
\end{bmatrix}
= \begin{bmatrix}
m_1 \ddot{x}_{2c} \\
m_1 \ddot{y}_{2c} \\
m_2 \ddot{x}_{3c} \\
m_2 \ddot{y}_{3c} \\
m_3 \ddot{x}_{4c} \\
m_3 \ddot{y}_{4c} \\
in_{1c} \ddot{\phi} \\
in_{2c} \ddot{\phi} \\
in_{3c} \ddot{\phi} \\
in_{4c} \ddot{\phi} \\
\end{bmatrix}
\]

\[ A b = \bar{b} \]
\[ \bar{x} = A^{-1} \bar{b} \]
Construction and Testing

A four-bar linkage was constructed to the specifications that had given us the best results in Dynamic Designer. The total length of the throwing arm, or output link, was 20 inches from the point where it was pivoted to the ground to its end (where the ball would be released). The total length of the input link, onto which the mass would fall, was 11 inches. The lengths of links 1, 2, 3, and 4 were 7.25 inches, 2.75 inches, 4.5 inches, and 3 inches respectively. The input link, then, was pivoted to the ground 2.75 inches from the end where it was linked to the link 3, the coupler. The angle, \( ?1 \), was set to 17.5\(^\circ\). On the side of this pivot opposite that where the link was connected to the coupler, link 2 is offset 6 inches to the side. This offset gave the far end of the link its own space away from the rest of the mechanism where a mass could fall onto it. The entire four-bar mechanism was constructed on a 2’ by 2’ piece of 1/4 –inch thick plywood.

Figure 11: The Completed Four-Bar Catapult System
Selecting the mass that would be lifted and dropped by the motor required running some tests to see just how much the motor was capable of lifting. The chief constraint here was that the machine had only 30 seconds to throw the squash ball from the time the motor was turned on. This meant we had to discover the greatest mass the motor was capable of consistently lifting about 20 inches high in just under half a minute. The motor was able to slowly lift masses of nearly 10 pounds but these masses were not raised very high and caused an unacceptable amount of strain on the motor. The motor was able to consistently lift a 6-pound cylindrical metal mass about 20 inches high in 28 seconds, however, so this was chosen for our mass. A hole was drilled in its top and a hook was inserted so that the hook being lifted by the motor would have something to latch onto.

Creating a means of controlling the mass’s path as it is being lifted and finally dropped called for the construction of a tower around the far end of link 2. The motor is
mounted on top of this tower and pulls the mass upwards through a hole in the top. Four
dowels surround the cylindrical mass, and therefore the end of the input link, so that it
may only move up and down. The very top of this tower is 23-1/2 inches tall, ensuring
that it stays within the height constraints of the project.

Figure 13: The Tower used to raise the Mass and drop it onto the Input Link

Figure 14: Close-Up View of Motor and Switch atop Tower
Small nails were placed halfway into the offset part of the coupler arm and on the side of the tower. A rubber band was wrapped around these nails so that as the mass is raised from its rest position, on top of the input link, the far end of the input link will rise. This, in turn, will cause the angle \( \theta_2 \) to decrease. The four-bar mechanism reacts with an increase in \( \theta_4 \) that results in the throwing arm, along with the ball, being lowered. The mechanism is now in the arrangement shown in most of our sketches (Figure 1 of the Design Strategies, for example). Once the mass is dropped, it will fall on the now-raised far end of the input link and cause the throwing arm to rapidly eject the squash ball.

Directly behind the tower is a small rectangular piece of wood with ten nails spaced 1cm apart lining its side vertically. The fishing line tied to the end of the hook being lifted by the motor can be looped around any one of these ten nails. The motor lifts this hook, which is hooked into the one protruding from the top of the mass, until there is tension in this fishing line. At that point, the hook being lifted by the motor no longer
moves vertically but rotates until it finally drops the mass. Therefore, by altering which of the nails the line is looped onto, the height that the mass is dropped at may be changed and the machine’s range may easily be adjusted.

One extra addition that allowed for the range of the machine to be adjusted was a small rectangular piece of wood with a threaded bolt protruding from its top being placed underneath the joint connecting the input link and coupler. The piece provided a means of controlling the values of \( \theta_2 \) by limiting how high the rubber band could lift the far end of the input link. At a certain point the shorter end of the input link (connected to the coupler) would be lowered onto this piece and the value of \( \theta_2 \) prior to the mass falling on the mechanism was set. This obviously limited the values of other angles in the mechanism, most notably \( \theta_4 \), and therefore controlled how high the ball would be prior to its launch. Adjusting how far out of the piece of wood the bolt was changed the limiting values for \( \theta_2 \) and therefore the rest position of the entire mechanism. Our testing showed this to be the most reliable way to adjust the range of the throwing arm.

![Figure 16: The pieces of wood behind the tower and below the input link responsible for adjustability.](image)
<table>
<thead>
<tr>
<th>Initial Height of Squash Ball (in.)</th>
<th>Distance Traveled by Squash Ball (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>13.5</td>
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<tr>
<td>16.25</td>
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</tr>
<tr>
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</tr>
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<tr>
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<td>18.75</td>
</tr>
<tr>
<td>21.5</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Table 1: Testing showed that there was a substantial difference in how far the mechanism was able to throw just based on how high the squash ball was prior to launch. In addition, as the mechanism threw shorter distances, the ball traveled through the air with a higher arc, which was more suitable for hitting horizontal targets such as a waste basket. These tests were performed on the ground because the machine’s height on the table should be comparable to that of a waste basket. The results shown here are for when the mass was raised to its maximum height of about 20 inches before being dropped.

<table>
<thead>
<tr>
<th>Initial Height of Squash Ball (in.)</th>
<th>Distance Traveled by Squash Ball (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>7.5</td>
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<tr>
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<td>21</td>
<td>13</td>
</tr>
<tr>
<td>21.5</td>
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</table>

Table 2: The results shown here are the distances the squash ball traveled when the release mechanism only allowed the mass to obtain a height of approximately 16 inches before dropping it. As with Table 1, the mechanism was placed on the ground for these tests and there was a clear difference in both the ball’s arc and the distance it traveled through the air based on its initial height.
The input and output links were originally made from balsa wood due to its light weight. Making the masses of the links as small as possible would allow for greater angular acceleration once the mass fell onto the input link. The concerns about the balsa wood’s strength, however, turned out to be well-founded. The force of the six-pound mass falling on the input link managed to break it in half after only a few trials and the throwing arm snapped at the point where it was pivoted to the coupler shortly thereafter. These two links were replaced by stronger versions made from pine. This type of wood was still fairly light and was strong enough that there was no longer any concern about the mechanism links breaking.
## Project Competition Results

### Farthest Distance Shooting

<table>
<thead>
<tr>
<th>Group</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Farthest</th>
<th>Ranking</th>
<th>Number of Successful Shots</th>
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<td>37 ft</td>
<td>X</td>
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<td>2</td>
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<td>30 ft</td>
<td>X</td>
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<td>2</td>
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<td>30 ft</td>
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### Precision Shooting

Grades by Judges

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<td>90</td>
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The charts above show the final results from the competition held on December 6th.

For distance, our four-bar mechanism was not as successful as some other groups but was able to throw the squash ball a fairly impressive 27 feet (good enough for 8th place among
the 12 teams). Our design was extremely successful, however, when it came to accuracy.

We were able to throw the squash ball into a waste basket in two of our three attempts, placing our group in a first place tie for precision with four other groups. The only missed precision throw overshot the waste basket by only six inches, so this portion of the competition was very successful for our group.

The second chart shows the grades awarded by the five judges for the compactness, construction, aesthetics, and originality of the final prototype. Here our group rose up above the competition to finish in first place with an average grade of 96.4, more than three points higher than our closest competition. In addition, we were the only group to receive a perfect score of 100 from any single judge (Judge #3 in our case). The originality of our design, the care in its construction, and the many nuances we used to make it adjustable paid off. This part of the competition was clearly a resounding success.
Future Improvements

The first and possibly most obvious way to have made our mechanism throw farther is to decrease the masses of the links. Unfortunately, the balsa wood we used initially for the throwing arm and input link did not last more than a few trials. However, if there had been a way to reinforce this wood and perhaps decrease the stress on it, there is no doubt that the four-bar would have been able to throw the ball farther because less force would have been required to move the arms. Even if balsa wood could not be made to work, making the pine links that we did use thinner would be an easy means of decreasing their weight and, hopefully, not sacrificing much of their strength.

Another possible improvement would be to use pulleys that would allow the motor to lift more than 6 pounds. Doubling or tripling the mass that falls onto the input link would similarly increase the input force and result in the squash ball’s initial velocity increasing greatly. This innovation would most likely require stronger links however.

Finally, bearings could be used at each joint in an effort to decrease the friction each link experiences at each joint. Our current model had bolts placed through holes drilled in each wooden link. Bearings could allow for smoother movement between links and a better translation of force from the input link to the throwing arm.